121-G11 Trad. Lec.

MAT 121 – Fall, 2010 Test 1

Chapter 1 Name KEY

Solve the following equations. For each equation, state whether it is conditional, identity or inconsistent.

1. $\frac{3}{2}x+\frac{1}{3}=\frac{1}{4}x-\frac{1}{6}$ LCD = 24

5 pts

$24\left(\frac{3}{2}x\right)+24\left(\frac{1}{3}\right)=24\left(\frac{1}{4}x\right)-24\left(\frac{1}{6}\right)$

$12(3x)+8(1)=6x-4(1)$

$36x+8=6x-4$

Conditional

$30x=-12$

$x=-\frac{12}{30}=\boxed{-\frac{2}{5}=x}$ $x\in\{-\frac{2}{5}\}$

5 pts

2. $\frac{4}{x-1}-\frac{9}{x+1}=\frac{3}{x^2-1}$ LCD = $(x-1)(x+1)$

$(x-1)(x+1)\left(\frac{4}{x-1}\right)-(x-1)(x+1)\left(\frac{9}{x+1}\right)=(x-1)(x+1)\left(\frac{3}{(x-1)(x+1)}\right)$

$(x+1)(4)-(x-1)(9)=3$

$4x+4-9x+9=3$

$-5x+13=3$

$-5x=-10$

$\boxed{x=2}$ $x\in\{2\}$

Conditional

3. (5 pts bonus) Give an example of an equation in the variable x that is...

a. ... an identity. $3x+5=3x+5$

b. ... inconsistent. $3x+5=3x-2$

10

+5

5pts

4. The old combine can harvest the crop in 100 hours, but a new one can do it in 92 hours. How many hours will it take the two of them operating at the same time to harvest the crop?

Let x = the # of hours it takes them to finish if they work together.

Then $\frac{1}{100}x + \frac{1}{92}x = 1$

LCD: $2|92$, $2|46$, 23

$2300\left(\frac{1}{100}x\right) + 2300\left(\frac{1}{92}x\right) = 2300$

$2|48$, $2|24$, $2|12$, $2|6$, 3 — $2|100$, $2|50$, $5|25$, 5

$23x + 25(x) = 2300$

$23x + 25x = 2300$

$48x = 2275 \longrightarrow x = \frac{2300}{48} = \frac{575}{12}$ hrs $= 47.916\overline{6}$

LCD $= 2 \cdot 2 \cdot 5 \cdot 5 \cdot 23 = 2300$

5pts

5. How much 60% antifreeze solutions should be added to 4 quarts of 15% antifreeze solution to yield a 50% antifreeze solution?

Let x = the amt of 60% antifreeze soln used (quarts)

Then $.6x + .15(4) = .50(x+4)$

$60x + 15(4) = 50(x+4)$

$60x + 60 = 50x + 200$

$10x + 60 = 200$

$10x = 140$

$x = \frac{140}{10} = 14$ qts

$(.6)(14) + (.15)(4) = ?$

$.5(14+4)$

$9 = 9$ ✓

6. Find the distance between (3, 2) and (-3, 5).

5pts $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

$= \sqrt{(-3-3)^2 + (5-2)^2}$

$= \sqrt{(-6)^2 + 3^2}$

$= \sqrt{36+9} = \sqrt{45} = 3\sqrt{5}$

$2|90$, $5|45$, $3|9$, 3

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7. Determine the center and radius of the circle given by

5 pts

~~$x^2+6x+25+y^2-8y=121$~~ $x^2+6x+y^2-8y=96$

$x^2+6x+\quad+y^2-8y \quad=96$

$x^2+6x+3^2+y^2-8y+4^2=96+9+16$

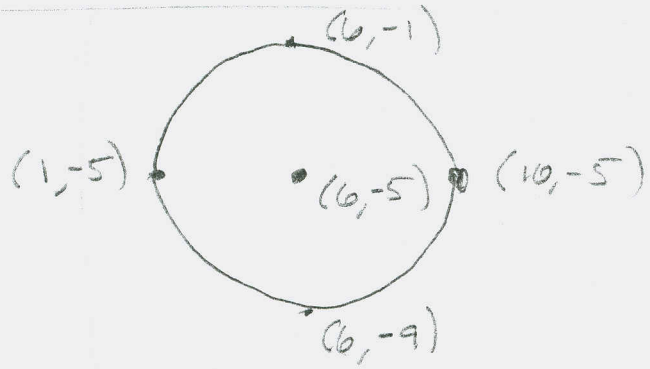
$(x+3)^2+(y-4)^2=121$

$(h,k)=(-3,4)$

$r=11$

8. Sketch the graph of $(x-6)^2+(y+5)^2=16$.

5 pts

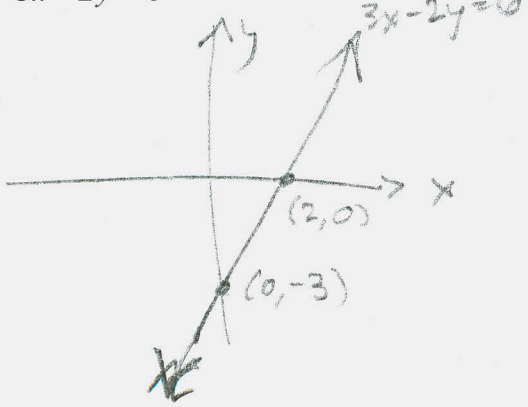


9. Sketch the graph of each of the following equations. Main points *I* want to see are the intercepts.

5 pts

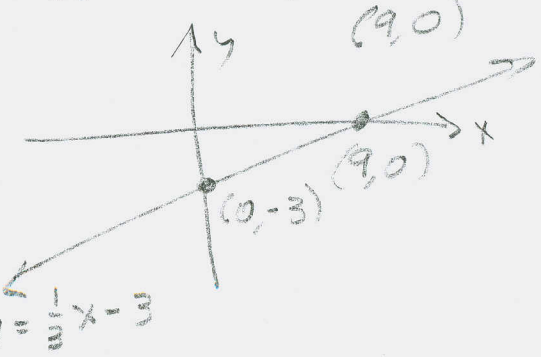
a. $3x-2y=6$

x	y
0	-3
2	0



b. $y=\frac{1}{3}x-3$ (0,-3)

SET=0 $\rightarrow \frac{1}{3}x=3 \Rightarrow x=9$



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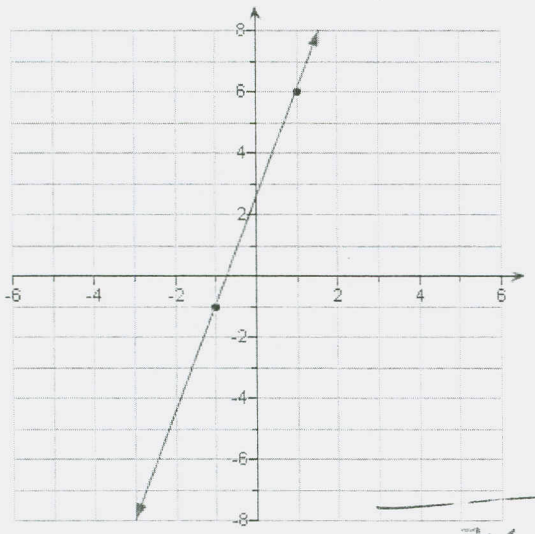
10. Find three forms of the equation of the line in the graph (Hint: Use the two points given.)



$(x_1, y_1) = (-1, -1)$

$(x_2, y_2) = (1, 6)$

$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{6-(-1)}{1-(-1)} = \frac{7}{2}$ 3pts

$y = m(x - x_1) + y_1$

$y = \frac{7}{2}(x-(-1)) - 1$

$y = \frac{7}{2}x + \frac{7}{2} - 1$

3pts a. Point-Slope form: $y = \frac{7}{2}(x+1) - 1$

3pts b. Slope-Intercept form: $y = \frac{7}{2}x + \frac{5}{2}$

$y = \frac{7}{2}x + \frac{5}{2}$

$2y = 7x + 5$

$-7x + 2y = 5$

3pts c. Standard form (with integer coefficients): $-7x + 2y = 5$

11. Based on your answer to the previous question, find an equation of the line through (-1, -1) that is perpendicular to the one in the graph. Give your answer in slope-intercept form.

3pts $m_\perp = -\frac{2}{7}$, $(x_1, y_1) = (-1, -1)$

$y = -\frac{2}{7}(x+1) - 1$ ↳ Point-slope

$y = -\frac{2}{7}x - \frac{9}{7}$ ↳ final Ans.

12. Compute the discriminant for each of the following quadratic equations, and determine the *nature* of the solutions (How many, and whether it/they is/are real or non-real):

a. $3x^2 - 22x + 35$ $b^2 - 4ac = (-22)^2 - 4(3)(35)$

3pts $= 484 - 420 = 64$ 2 real solns

b. $4x^2 + 8x + 13$ $b^2 - 4ac = 8^2 - 4(4)(13) = 64 - 208 = -144$

3pts 2 nonreal solns

c. $4x^2 + 12x + 9$

3pts $b^2 - 4ac = 12^2 - 4(4)(9) = 144 - 144 = 0$ 1 real soln

2pts

13. Find all real or non-real solutions of the following quadratic equations by completing the square AND by quadratic formula. Write in the lowest terms possible, e.g. $\sqrt{12} = 2\sqrt{3}$. Do *not* use decimal approximations via calculator.

3pts a. $x^2 - 2x - 1 = 0$

$a=1, b=-2, c=-1$

$b^2-4ac = (-2)^2-4(1)(-1)$

$= 4+4 = 8$

$x = \frac{2 \pm \sqrt{8}}{2} = \frac{2 \pm 2\sqrt{2}}{2}$

$= \frac{2(1 \pm 1\sqrt{2})}{2} = \boxed{1 \pm \sqrt{2} = x}$

3pts $x^2 - 2x = 1$

$x^2 - 2x + 1^2 = 1 + 1^2$

$(x-1)^2 = 2$

$x - 1 = \pm\sqrt{2}$

$\boxed{x = 1 \pm \sqrt{2}}$

$x^2 - x + \frac{3}{4} = 0$

b. ~~$x^2 - \frac{2}{3}x + \frac{1}{3} = 0$~~

$b^2-4ac = (-1)^2-4(1)(\frac{3}{4})$

$= 1-3 = -2$

$x = \frac{1 \pm \sqrt{-2}}{2} = \boxed{\frac{1 \pm i\sqrt{2}}{2}}$

3pts

$x^2 - x = -\frac{3}{4}$

$x^2 - x + (\frac{1}{2})^2 = -\frac{3}{4} + \frac{1}{4}$

$(x - \frac{1}{2})^2 = -\frac{2}{4} = -\frac{1}{2}$

$x - \frac{1}{2} = \pm\sqrt{-\frac{1}{2}} = \pm i\frac{\sqrt{1}}{\sqrt{2}}$

$x = \frac{1}{2} \pm i\frac{1}{\sqrt{2}} \cdot \frac{\sqrt{2}}{2} = \frac{1}{2} \pm \frac{i\sqrt{2}}{2}$

$x = \frac{1 \pm i\sqrt{2}}{2}$

3pts

12

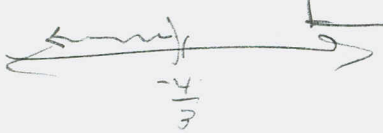
14. Solve the following inequalities: write solution set in interval notation.

a. $2-3x>6$

3pts $-3x > 4$

$x < -\frac{4}{3}$

$(-\infty, -\frac{4}{3})$



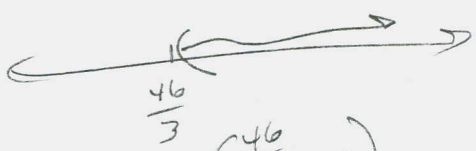
b. $\frac{x}{2}-7>\frac{2}{3}$ LCD = 6

3pts $6(\frac{x}{2}) - 6(7) > 6(\frac{2}{3})$

$3x - 42 > 4$

$3x > 46$

$x > \frac{46}{3}$



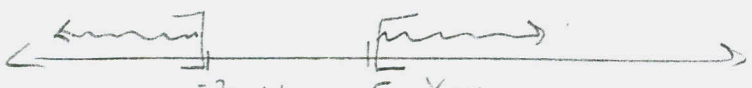
$(\frac{46}{3}, \infty)$

c. $|2x-3| \geq 7$

3pts $2x-3 \geq 7$ OR $2x-3 \leq -7$

$2x \geq 10$ $2x \leq -4$

$x \geq 5$ $x \leq -2$



Yes NO Yes

$(-\infty, -2] \cup [5, \infty)$

$x \in (-\infty, -2] \cup [5, \infty)$

d. $|2x-3| > -7$

3pts $(-\infty, \infty)$

e. $|2x-3| \leq -7$ $\emptyset$

3pts

f. $|2x-3| - 8 \leq -7$

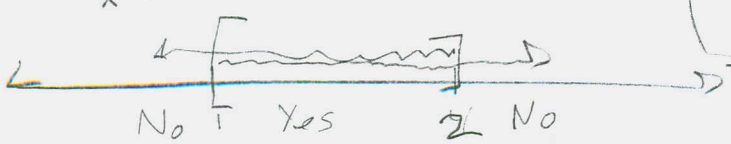
4pts $|2x-3| \leq 1$

$2x-3 \leq 1$ AND $2x-3 \geq -1$

$2x \leq 4$ $2x \geq 2$

$x \leq 2$ AND $x \geq 1$

$x \in [1, 2]$



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