Studying for the Final:

Tests 1 and 2:
- Absolute value equations and inequalities
- Solving Quadratic Equations by 3 methods:
  - Quadratic Formula
  - Completing the Square
  - Factoring
- Solve an equation that's quadratic in form.

Test 3:
- One simple question on evaluating $P(2)$ using synthetic division

Test 4:
- Graph an exponential or logarithmic function by transformations. Solving exponential/logarithmic equations will be in the context of finding the x-intercept.

Test 5:
- Set up a word problem in 3 variables.

Word problems: 2 or 3. Areas of interest:
- Shared work, Mixtures, Finance (Annuity-related problem).

Pascal's Triangle

$0! = 1! = 1$

$n! =$ "n factorial" $= n \cdot (n-1)(n-2) \cdots (3)(2)(1)$

$5! = 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1$

$3! = 3 \cdot 2 \cdot 1$

Binomial Coefficient: "n choose k"

The number of subsets with k elements that can be formed from a set of n elements.

$\{a,b,c\}$ $n=3$ elements.

How many subsets with $\underbrace{\text{2 elements}}_{k=2}$?

$\{a,b\}, \{a,c\}, \{b,c\}$

"n choose k" $= \binom{n}{k} = {}_nC_k = C^n_k = C(n,k)$

= "combinations of n things taken k at a time."

$$= \frac{n!}{(n-k)!\,k!}$$

$$\binom{3}{2} = \frac{3!}{1!\,2!} = \frac{3 \cdot \cancel{2 \cdot 1}}{1 \cdot \cancel{2 \cdot 1}} = 3$$

$$\binom{5}{2} = \frac{5!}{3!\,2!} = \frac{5 \cdot \overset{2}{\cancel{4}} \cdot \cancel{3 \cdot 2 \cdot 1}}{\cancel{(3 \cdot 2 \cdot 1)}(\cancel{2} \cdot 1)} = 10$$

Handy way to build these binomial coefficients.

$$\begin{array}{ccccccccccc}
 & & & & & 1 & & & & & \\
 & & & & 1 & & 1 & & & & \\
 & & & 1 & & 2 & & 1 & & & \\
 & & 1 & & 3 & & 3 & & 1 & & \\
 & 1 & & 4 & & 6 & & 4 & & 1 & \\
1 & & 5 & & 10 & & 10 & & 5 & & 1 \\
\binom{5}{0} & & \binom{5}{1} & & \binom{5}{2} & & \binom{5}{3} & & \binom{5}{4} & & \binom{5}{5}
\end{array}$$

Write the complete binomial expansion for $(r+s)^3$.

CH

$(r+s)^3 = \square$

(Simplify your answer.)

Binomial Theorem

$$(r+s)^n = \binom{n}{0} r^n s^0 + \binom{n}{1} r^{n-1} s^1 + \binom{n}{2} r^{n-2} s^2 + \cdots + \binom{n}{n-1} r^1 s^{n-1} + \binom{n}{n} r^0 s^n$$

$$(r+s)^3 = \binom{3}{0} r^3 s^0 + \binom{3}{1} r^{3-1} s^1 + \binom{3}{2} r^{3-2} s^2 + \binom{3}{3} r^0 s^3$$

$$= r^3 + 3r^2 s + 3rs^2 + s^3$$

1

1 1

$\boxed{1 \quad 2 \quad 1}$

1 3 3 1

$\binom{3}{0} \binom{3}{1} \binom{3}{2} \binom{3}{3}$

$$(r+s)^2 = \underline{1}r^2 + \underline{2}rs + \underline{1}s^2$$

Write the complete binomial expansion for $(4s+5t^2)^4$.

$(4s+5t^2)^4 = \square$

(Simplify your answer.)

1
1 1
1 2 1
1 3 3 1
1 4 6 4 1

$= 1(4s)^4(5t^2)^0 + 4(4s)^3(5t^2)^1 + 6(4s)^2(5t^2)^2$
$+ 4(4s)^1(5t^2)^3 + 1(4s)^0(5t^2)^4$

$= 4^4 s^4 + 4\cdot 4^3 s^3 \cdot 5 \cdot t^2 + 6 \cdot 4^2 s^2 \cdot 5^2 \cdot t^4$
$+ 4 \cdot 4s \cdot 5^3 t^6 + 5^4 t^8$

Solve in 3 ways

$x^2 + 4x - 5 = 0$

$(x+5)(x-1) = 0$

$x \in \{-5, 1\}$

If you get two different solutions by two different methods, make note of the fact.

$x^2 + 4x = 5$

$x^2 + 4x + 2^2 = 5 + 2^2$

$(x+2)^2 = 9$

$|x+2| = \sqrt{9}$

$x + 2 = \pm\sqrt{9} = \pm 3$

$x = -2 \pm 3$

$-2+3 = 1$

$-2-3 = -5$

$x \in \{-5, 1\}$

$a = 1,\ b = 4,\ c = -5$

discriminant $= b^2 - 4ac = 4^2 - 4(1)(-5) = 16 + 20 = 36$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-4 \pm \sqrt{36}}{2(1)} = \frac{-4 \pm 6}{2}$$

$\frac{2}{2} = 1$

$\frac{-10}{2} = -5$

$x \in \{-5, 1\}$

Dec 5-8:14 AM

Quadratic in Form

Equation	quadratic in	New Version
① $x + 4\sqrt{x} - 5 = 0$	$u = \sqrt{x}$	$u^2 + 4u - 5 = 0$
② $x^4 + 4x^2 - 5 = 0$	$u = x^2$	$u^2 + 4u - 5 = 0$
③ $(3x-1)^2 + 4(3x-1) - 5 = 0$	$u = 3x - 1$	$u^2 + 4u - 5 = 0$

$u^2 + 4u - 5 = 0 \longrightarrow u = -5$ OR $u = 1$

① $u = \sqrt{x} = -5$ No way (only looking for REAL solutions)

$u = \sqrt{x} = 1$

$x = 1^2$ $\boxed{x \in \{1\}}$

② $u = x^2 = -5$ No way

$u = x^2 = 1$

$x = \pm 1$ $x \in \{-1, 1\}$

Coming in Trig.

$\sin(x)^2 + 2\sin(x) - 3 = 0$

③ $u = 3x - 1 = -5$

$3x = -4$

$x = -\frac{4}{3}$

$u = 3x - 3 = 1$

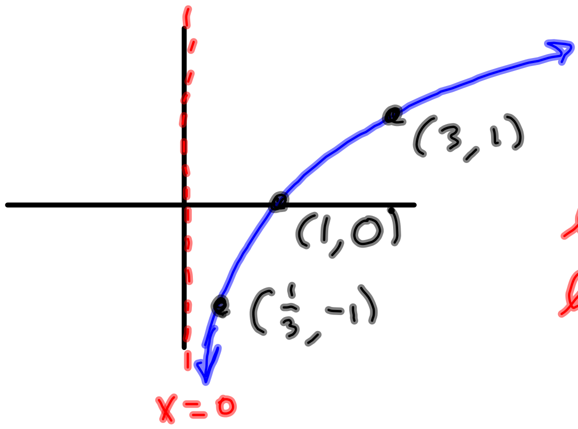
$3x = 4$

$x = \frac{4}{3}$

$x \in \{-\frac{4}{3}, \frac{4}{3}\}$

$\log_3(x)$ says what power of 3 x is.

$\log_3(9) = 2$, b/c $9 = 3^2$



$\log_3(x) = f(x)$

$\log_3(3) = 1 \qquad 3 = 3^1$

$\log_3(1) = 0 \qquad 1 = 3^0$

$\log_3\left(\frac{1}{3}\right) = \log_3\left(3^{-1}\right)$

$\frac{1}{3} = 3^{-1}$

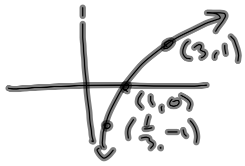
$-2\log_3(2x+4)+5$

$2x+4 \to 2(x+2)$

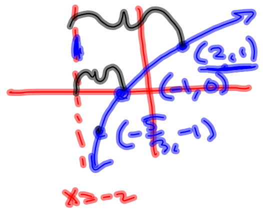
$\log_3(x) \longrightarrow \log_3(x+2) \longrightarrow \log_3(2(x+2))$

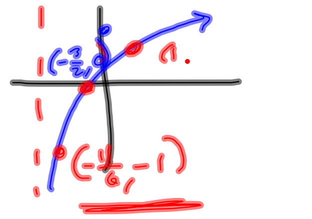
Left 2

cut the distance to the line $x=-2$ by a factor of 2

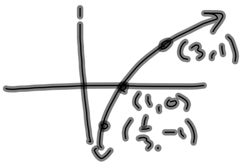


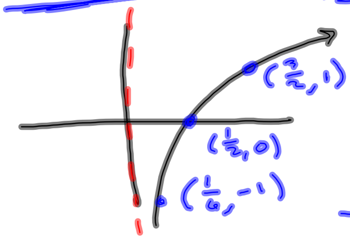
$-\frac{5}{3}-\frac{1}{6}=-\frac{11}{6}$



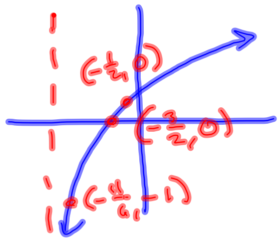


$\log_3(x) \longrightarrow \log_3(2x) \longrightarrow \log_3(2(x+2))$





$\frac{1}{6}-2=\frac{1}{6}-\frac{12}{6}=\frac{-11}{6}$



1 Horizontal Shrink/stretch $x=-2$

$x \longrightarrow \frac{1}{a}x$ in $f(ax)$

2 Horizontal shift $x \longrightarrow x-a$ in $f(x+a)$

3 vertical shrink/stretch

$y \longrightarrow ay$ $\quad af(x)$

4 vertical shift

$y \longrightarrow y+a$ $\quad f(x)+a$

$af(bx+c)+d$

$f(x) \longrightarrow f(bx) \longrightarrow f(b(x+\frac{c}{b}))$

$\longrightarrow af(b(x+\frac{c}{b})) \longrightarrow af(b(x+\frac{c}{b})+d$