Rewrite the series using the new index j as indicated.

$$\sum_{i=7}^{14}(-i+9) = \sum_{j=1}(-j+3)$$

$$-7+9 + -8+9 + \cdots + -14+9$$

$j=1$ $\quad$ $j=2$

$$\underbrace{-(j+6)+9}_{-j-6+9=\boxed{-j+3}} + -(2+6)+9$$

$$\sum_{i=7}^{14}(-i+9) = \sum_{j=1}^{14}(\boxed{-7j+9})$$

$14 \to 14-6=8$

$j=i-6$

The sum from $k=1$ to $k=5$ of the a_k's

$$\sum_{k=1}^{5} a_k = a_1 + a_2 + a_3 + a_4 + a_5$$

Σ = Sigma = Greek letter "S" for "sum"

$$\sum_{k=1}^{4}(2k-1) = 2(1)-1 + 2(2)-1 + 2(3)-1 + 2(4)-1$$

$= 16$

Think of $a_k = 2k-1 = k^{th}$ term as a function from the integers into the reals.

$a_k = f(k)$.

Ⓒ 8 : My Main Goal is for you to understand Compound Interest:

1. Savings Account - Lump Sum
2. Annuity - Stream of Payments.

Compound Interest

$$A = P\left(1+\frac{r}{m}\right)^{mt}$$

A = Accumulated Amt.
P = Lump Sum principal
r = Annual Interest rate
m = Periods per year (compoundings)
t = time, in years.

\$5,000 compounded monthly for 3 years @ 8% APR

$$A = 5000\left(1+\frac{.08}{12}\right)^{12\cdot 3} \approx \$6351.19$$

```
5000*(1+.08/12)^
(12*3)
     6351.185258
```

-

An Annuity with monthly pmts of \$5,000 for 3 yrs. Pmts made @ end of each month. Interest compounded at the end of each month.

$n = mt = 12 \cdot 3 = 36$ periods

1st: $5000\left(1+\frac{.08}{12}\right)^{35}$ $35 = n-1$

2nd: $5000\left(1+\frac{.08}{12}\right)^{34}$ $34 = n-2$

3rd: $5000\left(1+\frac{.08}{12}\right)^{33}$ $33 = n-3$

$\vdots$

36th: $5000\left(1+\frac{.08}{12}\right)^{0}$ $0 = n-36$

$$FV = 5000 + 5000\left(1+\frac{.08}{12}\right)^{1} + 5000\left(1+\frac{.08}{12}\right)^{2} + \cdots + 5000\left(1+\frac{.08}{12}\right)^{33} + 5000\left(1+\frac{.08}{12}\right)^{34} + 5000\left(1+\frac{.08}{12}\right)^{35}$$

Our Goal: Find a Formula for this.

Geometric Growth

" Sums

" Series.

Sequences

Series

Geometric Sums & Sequences.

1, 3, 9, 27, 81, ...

$a = a_1 = 1 = 1 \cdot 3^0$

$a_2 = 3 = 3 \cdot a_1 = 3^1 \cdot 1$

$a_3 = 9 = 3 \cdot a_2 = 3^2 \cdot a_1$

$a_n = 3^{n-1} \cdot 1 = 1 \cdot 3^{n-1}$

$r = 3$ is the common ratio

$\frac{a_3}{a_2} = \frac{9}{3} = 3$

$\frac{a_5}{a_4} = \frac{81}{27} = 3$

2, 6, 18, 54, 162, ...

$2 \cdot 3^0,\ 2 \cdot 3^1,\ 2 \cdot 3^2,\ 2 \cdot 3^3,\ 2 \cdot 3^4$

$\boxed{a_n = 2 \cdot 3^{n-1}}$

$a = 2,\ r = 3$

Common ratio: $r = 3$

$a = 2$ is the 1st term

Geometric Sums

S_8 = 8th Partial Sum.

Add 'em up!

$S_8 = 2 + 6 + 18 + 54 + \cdots + 2\cdot 3^7$

$= 2\cdot 3^0 + 2\cdot 3^1 + 2\cdot 3^2 + 2\cdot 3^3 + \cdots + 2\cdot 3^7$

$3\cdot S_8 = 2\cdot 3^1 + 2\cdot 3^2 + 2\cdot 3^3 + 2\cdot 3^4 + \cdots + 2\cdot 3^8$

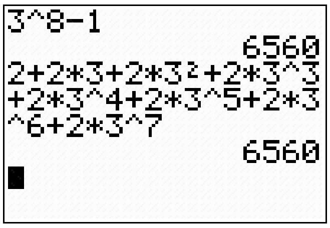
$= -2S_8$

$S_8 - 3\cdot S_8 = S_8(1-3) = 2\cdot 3^0 - 2\cdot 3^8 = 2(1-3^8)$

$S_8 = \dfrac{2\cdot(1-3^8)}{1-3} = \dfrac{2(1-3^8)}{-2}$

$= -(1-3^8) = 3^8 - 1 = 6560$

$\displaystyle\sum_{k=1}^{8} 2\cdot 3^{k-1} = \frac{2(1-3^8)}{1-3}$


```
3^8-1
                6560
2+2*3+2*3²+2*3^3
+2*3^4+2*3^5+2*3
^6+2*3^7
                6560
```

$S_n = a + a\cdot r + a\cdot r^2 + \cdots + a\cdot r^{n-1} = \displaystyle\sum_{k=1}^{n} a\cdot r^{k-1}$

$= \dfrac{a\cdot(1-r^n)}{1-r} = \displaystyle\sum_{k=1}^{n} a\cdot r^{k-1}$

Capper!

Next time Sigma Notation

$\displaystyle\sum_{k=1}^{\infty} ar^{n-1} = \frac{a}{1-r}$ if $-1 < r < 1$

$R + R(1+\frac{r}{m}) + R(1+\frac{r}{m})^2 + \cdots + R(1+\frac{r}{m})^{n-1} = ?$

$$R + R(1+\tfrac{r}{m}) + R(1+\tfrac{r}{m})^2 + \cdots + R(1+\tfrac{r}{m})^{n-1} = ?$$

$n = mt$ = total # of periods

m = # of periods per year

t = " " years

R = payment

$i = \frac{r}{m}$ = interest rate per period.

$$R + R(1+\tfrac{r}{m}) + R(1+\tfrac{r}{m})^2 + \cdots + R(1+\tfrac{r}{m})^{n-1}$$

$$R + R(1+i) + R(1+i)^2 + \cdots + R(1+i)^{n-1} = ?$$

$$a + ar + ar^2 + \cdots + ar^{n-1} = \frac{a(1-r^n)}{1-r}$$

$$FV = ? = \frac{R(1-(1+i)^n)}{1-(1+i)} = \frac{R(1-(1+i)^n)}{1-1-i} =$$

$$= \frac{R(1-(1+i)^n)}{-i} = \frac{R((1+i)^n-1)}{i} = FV$$

= Future Value of an annuity.

An Annuity with monthly pmts of \$5,000 for 3 yrs. Pmts made @ end of each month. Interest compounded at the end of each month.

Assume interest rate is 7% annually. "7% APR"

$R = 5000$

$t = 3\text{yrs}$

$m = 12$

$$FV = \frac{R((1+i)^n - 1)}{i}$$

Floor

$r = .07$

$n = mt = 12 \cdot 3 = 36$

$i = \frac{r}{m} = \frac{.07}{12}$

```
5000*36
                180000
5000*((1+.07/12)
^36-1)/(.07/12)
           199650.5036
```

$$FV = \frac{5000\left(\left(1+\frac{.07}{12}\right)^{36} - 1\right)}{\frac{.07}{12}} \approx \$199{,}650.50$$

5000 * ((1+.07/12)^36 - 1)/(.07/12)

Notice $1+i>1$

$$\sum_{i=1}^{\infty} 5000(1+i)^{n-1} = \infty$$

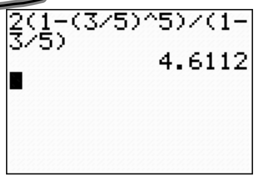
$|r|<1 \Rightarrow \underline{r<1 \text{ and } r>-1}$

<u>But</u> if $\underline{-1<r<1}$, then

$$\sum_{n=1}^{\infty} ar^{n-1} = \frac{a}{1-r}$$

$$\sum_{n=1}^{5} 2\left(\frac{3}{5}\right)^{n-1} = \frac{a(1-r^n)}{1-r} = \boxed{\frac{2\left(1-\left(\frac{3}{5}\right)^5\right)}{\left(1-\frac{3}{5}\right)}} \approx 4.6112$$

$a=2,\ r=\frac{3}{5}$



$$= \frac{2\left(1-\left(\frac{3}{5}\right)^5\right)}{\frac{2}{5}} = \frac{5}{\cancel{2}}\left(\cancel{2}\left(1-\left(\frac{3}{5}\right)^5\right)\right)$$

$$= 5\left(1-\left(\frac{3}{5}\right)^5\right)$$

$$\sum_{n=1}^{6} 2\left(\frac{3}{5}\right)^{n-1} = \frac{2\left(1-\left(\frac{3}{5}\right)^6\right)}{1-\frac{3}{5}}$$

$$\sum_{n=1}^{1000} 2\cdot\left(\frac{3}{5}\right)^{n-1} = \frac{2\left(1-\left(\frac{3}{5}\right)^{1000}\right)}{1-\frac{3}{5}}$$

vewy, vewy small

zero

$$\sum_{n=1}^{\infty} 2\cdot\left(\frac{3}{5}\right)^{n-1} = \frac{2\left(1-\left(\frac{3}{5}\right)^{\infty}\right)}{1-\frac{3}{5}} = \frac{2}{1-\frac{3}{5}} = \frac{2}{1-r}$$

$$= \frac{2}{\frac{2}{5}} = 2\cdot\frac{5}{2} = 5$$

$f(x) = \left(\frac{3}{5}\right)^x$

Give me any positive #.
I can make $\left(\frac{3}{5}\right)^x$
smaller than that number,
just by letting x get big enough.
That's the meaning of $\lim_{x\to\infty}\left(\frac{3}{5}\right)^x = 0$

$$\sum_{k=1}^{n} ar^{k-1} = \frac{a(1-r^n)}{1-r}$$

$$\sum_{k=1}^{\infty} ar^{k-1} = \frac{a}{1-r} \quad (\text{Provided } -1<r<1)$$

$$FV = \frac{R((1+i)^n-1)}{i}$$

J.G. Wentworth Figures the Present Value of an Annuity.

Say, J.G. wants to earn 10% interest from his purchase of an annuity.

Say \$100,000 is the investment. and it's for 10 years.

$$A = P\left(1+\frac{r}{m}\right)^{mt} = P(1+i)^n$$

$m = 12$ compounded monthly.
$t = 10$
$r = .10$

$$100000\left(1+\frac{.1}{12}\right)^{120}$$

Nov 28-8:37 AM

What would the monthly payments be?

$$\text{Annuity: } \frac{R\left(\left(1+\frac{.1}{12}\right)^{120}-1\right)}{\frac{.1}{12}} = 100000\left(1+\frac{.1}{12}\right)^{120}$$

Solve for R to find the payments.

$$\frac{R((1+i)^n-1)}{i} = PV(1+i)^n$$

Present Value / Loan Amount

Loan Payment.

$$\text{Loan Amount} = PV = \frac{R((1+i)^n-1)(1+i)^{-n}}{i} = \boxed{\frac{R(1-(1+i)^{-n})}{i} = PV}$$

$$R = \frac{PV\, i}{1-(1+i)^{-n}} = \text{Loan Payment}$$

Amortization Formula.

$$R = PV\left(\frac{i}{1-(1+i)^{-n}}\right)$$