From Test 5

$$\left[\begin{array}{ccc|c} 1 & 1 & 3 & 8 \\ 2 & 3 & 8 & 17 \\ 3 & 3 & 10 & 26 \end{array}\right] \begin{array}{l} -2R1+R2 \\ -3R1+R3 \end{array} \left[\begin{array}{ccc|c} 1 & 1 & 3 & 8 \\ 0 & 1 & 2 & 1 \\ 0 & 0 & 1 & 2 \end{array}\right]$$

$\boxed{z=2}$

$y+2z=1$

$y+2(2)=1$

$y+4=1$

$\boxed{y=-3}$

$x+y+3z=8$

$x-3+3(2)=8$

$x-3+6=8$

$x+3=8$

$\boxed{x=5}$

$(x,y,z)=(5,-3,2)$

Sol'n Set $(x,y,z) \in \{(5,-3,2)\}$

$-2\cdot 5^{-3x+9}$ Coming to a Trig Class near you.

① Hor. shrink/stretch
② " shift

} opposite of what a beginner thinks

③ vert shrink/stretch
④ vert. shift

} Just what a beginner thinks.

$\pi x - \pi = \pi(x-1)$

$-2\sin(\pi x - \pi) + 1$

$\sin(x) \longrightarrow \sin(\pi x) \longrightarrow \sin(\pi(x-1))$

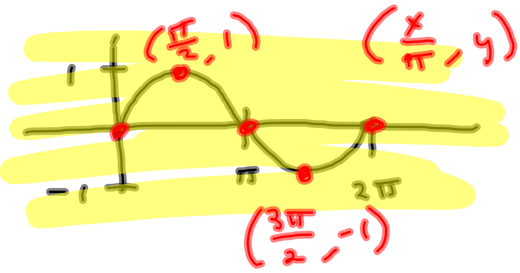
$(x,y) \longrightarrow (x+1, y)$

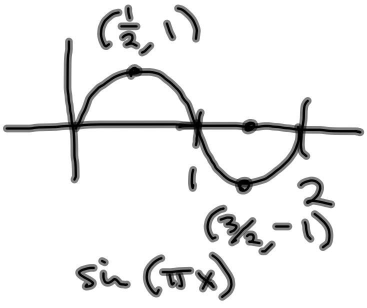
$\longrightarrow -2\sin(\pi(x-1))$

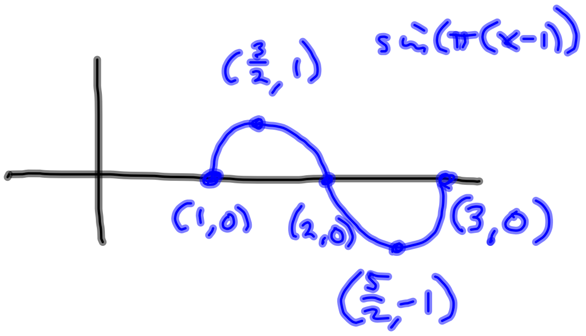
$(x,y) \longrightarrow (x, -2y)$

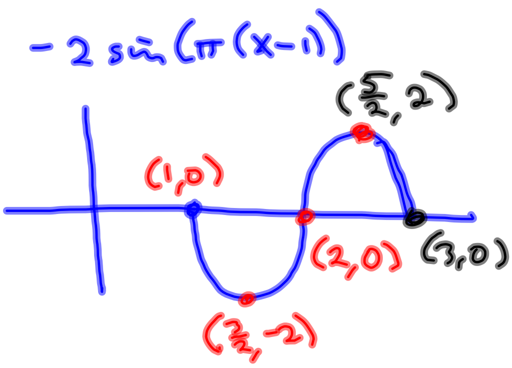
$\longrightarrow -2\sin(\pi(x-1)) + 1$

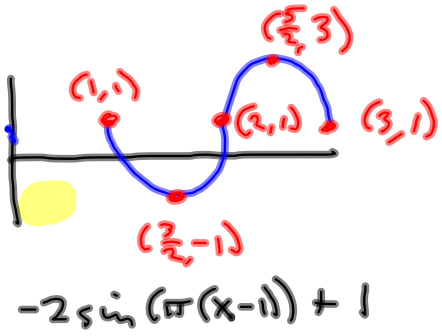
$(x,y) \longrightarrow (x, y+1)$











C 8 : My Main Goal is for you to understand Compound Interest:

① Savings Account – Lump Sum
② Annuity – Stream of Payments.

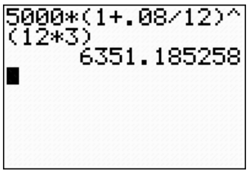
Compound Interest

$$A = P\left(1+\frac{r}{m}\right)^{mt}$$

A = Accumulated Amt.
P = Lump Sum principal
r = Annual Interest rate
m = Periods per year (compoundings)
t = time, in years.

\$5,000 compounded monthly for 3 years @ 8% APR

$$A = 5000\left(1+\frac{.08}{12}\right)^{12\cdot 3} \approx \underline{\$6351.19}$$



```
5000*(1+.08/12)^
(12*3)
        6351.185258
█
```

An Annuity with monthly pmts of \$5,000 for 3 yrs. Pmts made @ end of each month. Interest compounded at the end of each month.

$\underline{n = \underline{mt} = 12\cdot 3 = 36 \text{ periods}}$

$\underline{1}^{st}$: $5000\left(1+\frac{.08}{12}\right)^{35}$ $35 = n-\underline{1}$

$\underline{2}^{nd}$: $5000\left(1+\frac{.08}{12}\right)^{34}$ $34 = n-\underline{2}$

$\underline{3}^{rd}$: $5000\left(1+\frac{.08}{12}\right)^{33}$ $33 = n-\underline{3}$

⋮

$\underline{36}^{th}$: $5000\left(1+\frac{.08}{12}\right)^{0}$ $0 = n-\underline{36}$

$$\left\{ \begin{array}{l} FV = 5000 + 5000\left(1+\frac{.08}{12}\right)^{1} + 5000\left(1+\frac{.08}{12}\right)^{2} \\ \quad + \cdots + 5000\left(1+\frac{.08}{12}\right)^{33} + 5000\left(1+\frac{.08}{12}\right)^{34} + 5000\left(1+\frac{.08}{12}\right)^{35} \\ \text{Our Goal: Find a Formula for this.} \end{array} \right.$$

Geometric Growth
.. Sums
.. Series.

Sequences
Series

Arithmetic Sequences : Common Difference

1, 3, 5, 7, 9, 11, 13, ...

Next term is 15

start with $n=1$

$a_1 = 1 = f(1) = 2(1) - 1 = 1$

$a_2 = 3 = f(2) = 2(2) - 1 = 3$

$a_3 = 5 = f(3) = 2(3) - 1$

$\vdots$

$a_7 = 13 = f(7)$

$\vdots$

$a_n = \quad = f(n) = \boxed{2n - 1 = a_n}$

Common Difference is 2

1, 5, $\underset{n=3}{9}$, 13, 17, ...

$4n$

$n=1$, $f(n) = 1 = 4n-3$ $= 4(1)-3 = 1$

$n=3$, $f(n) = 9 \overset{?}{=} 4(3)-3$? Yes.

$a_n = 4n-3$

13, 21, 29, 37, 45

common Difference is 8

$f(1) = 13 = 8n \underline{+5}$ $= 8(1) + 5$

$a_n = 8n+5$

1, 3, 9, 27, 81, ...

$a_1 = 1 = 1 \cdot 3^0$

$a_2 = 3 = 3 \cdot a_1 = 3^1 \cdot 1$

$a_3 = 9 = 3 \cdot a_2 = 3^2 \cdot a_1$

$a_n = 3^{n-1} \cdot 1$

r = 3 is the common ratio

$\frac{a_3}{a_2} = \frac{9}{3} = 3$

$\frac{a_5}{a_4} = \frac{81}{27} = 3$

2, 6, 18, 54, 162, ...

$2 \cdot 3^0, 2 \cdot 3^1, 2 \cdot 3^2, 2 \cdot 3^3, 2 \cdot 3^4$

$\boxed{a_n = 2 \cdot 3^{n-1}}$

Common ratio: r = 3

a = 2 is the 1st term

Geometric Sums

Add 'em up!

$$S_8 = 2+6+18+54+\dots+2\cdot 3^7$$

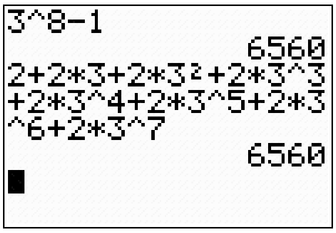
$$= 2\cdot 3^0 + 2\cdot 3^1 + 2\cdot 3^2 + 2\cdot 3^3 + \dots + 2\cdot 3^7$$

$$3\cdot S_8 = 2\cdot 3^1 + 2\cdot 3^2 + 2\cdot 3^3 + 2\cdot 3^4 + \dots + 2\cdot 3^8$$

$$S_8 - 3\cdot S_8 = S_8(1-3) = 2\cdot 3^0 - 2\cdot 3^8 = 2(1-3^8)$$

$$S_8 = \frac{2\cdot(1-3^8)}{1-3} = \frac{2(1-3^8)}{-2}$$

$$= -(1-3^8) = 3^8-1 = 6560$$



```
3^8-1
            6560
2+2*3+2*3²+2*3^3
+2*3^4+2*3^5+2*3
^6+2*3^7
            6560
```

$$S_n = a + a\cdot r + a\cdot r^2 + \dots + a\cdot r^{n-1} = \sum_{k=1}^{n} a\cdot r^{k-1}$$

$$= \frac{a\cdot(1-r^n)}{1-r}$$

Capper!

Next time Sigma Notation

$$\sum_{k=1}^{\infty} ar^{n-1} = \frac{a}{1-r} \quad \text{if } -1<r<1$$

$$R + R\left(1+\frac{r}{m}\right) + R\left(1+\frac{r}{m}\right)^2 + \dots + R\left(1+\frac{r}{m}\right)^{n-1} = ?$$