$$\begin{matrix}\text{Row1}\\ \text{Row2}\end{matrix}\begin{bmatrix}9 & 2\\ -9 & -9\end{bmatrix}\begin{bmatrix}1 & 1\\ 0 & 1\end{bmatrix}=\begin{bmatrix}(9)(1)+(2)(0) & (9)(1)+(2)(1)\\ (-9)(1)+(-9)(0) & (-9)(1)+(-9)(1)\end{bmatrix}$$

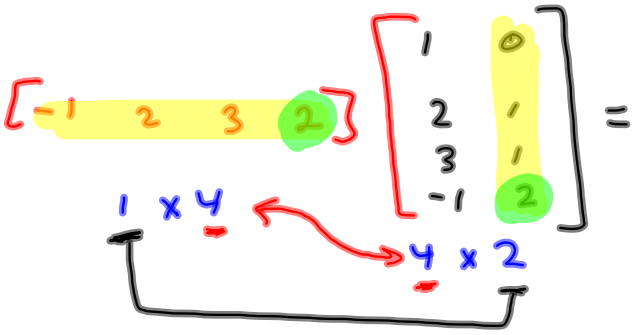
C C
0 0
1 1
1 2

2 Rows x 2 Cols Times 2 rows x 2 cols

2x2 ⟷ 2 x 2 Legal

Result is 2x2

$$=\begin{bmatrix}9 & 11\\ -9 & -18\end{bmatrix}\text{ is } 2\text{x}2$$



$$= \begin{bmatrix} (-1)(1)+(2)(2)+(3)(3)+(2)(-1) & (-1)(0)+(2)(1)+(3)(1)+(2)(2) \end{bmatrix}$$

$$= \begin{bmatrix} 10 & 9 \end{bmatrix}$$

The connection between systems and matrix products.

Consider the system:

$$x + 2y = 3$$
$$-x + y = 0$$

Consider the matrix equation

$$\begin{bmatrix} 1 & 2 \\ -1 & 1 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 3 \\ 0 \end{bmatrix}$$

$2\times 2 \quad 2\times 1$

$$\begin{bmatrix} 1x + 2y \\ -1x + 1y \end{bmatrix} = \begin{bmatrix} 3 \\ 0 \end{bmatrix}$$, which holds only if

$$x + 2y = 3$$
$$-x + y = 0$$

Solve the system with matrices:

$$\begin{bmatrix} 1 & 2 & 3 \\ -1 & 1 & 0 \end{bmatrix} \overset{R1+R2}{\underset{R1}{\rightarrow}} \begin{bmatrix} 1 & 2 & 3 \\ 0 & 3 & 3 \end{bmatrix}$$

$3y = 3$

$y = 1$

$x + 2y = 3$

$x + 2(1) = 3$

$x = 1$

Check:

$x + 2y = 3$?

$1 + 2(1) = 3$? Yes ✓

$-x + y = 0$?

$-1 + 1 = 0$? Yes ✓

Check:

$$\begin{bmatrix} 1 & 2 \\ -1 & 1 \end{bmatrix}\begin{bmatrix} 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 1+2 \\ -1+1 \end{bmatrix} = \begin{bmatrix} 3 \\ 0 \end{bmatrix}$$ ✓

Word Problems:

Let $x =$ .....

$y =$ ..... is 30% of any word problem.

Nonlinear Systems!

Emphasis on those in the notes.

Also nonlinear systems such as

The logarithmic & exponential equations from Chapter 4.

$2^{3x-1} = 5^{x+7}$

$y = 2^{3x-1}$

$y = 5^{x+7}$

How much 20% alcohol to be mixed with 10 gallons of 37% alcohol to obtain a mixture of 25% alcohol?

Let x = the amt of 20% alcohol (in gallons)
or x = the # of gallons of 20% alcohol

	Vol	conc.	Pure Alcohol
20%	x	.2	$.2x$
37%	10	.37	$+ .37(10)$
25%	$x+10$	.25	$.25(x+10)$

$.2x + .37(10) = .25(x+10)$, etc.

Let x = the amt of 20% alcohol (in gallons)
y = ... 25% .. .. ..

	Vol	conc.	Pure Alcohol
20%	x	.2	$.2x$
37%	10	.37	$+ .37(10)$
25%	$y = x+10$	.25	$.25y$

Pure Alcohol $.2x + .37(10) = .25y$

$y = x + 10$

$.2x - .25y = -.37(10)$
$-x + y = 10$

Looking at it as truly a system of 2 equations in 2 variables

How much 20% & 37% alcohol must be used to obtain 100 gallons of 25% alcohol?

Let x = the amt of 20% alcohol (in gallons)
& y = " " " 37% " " "

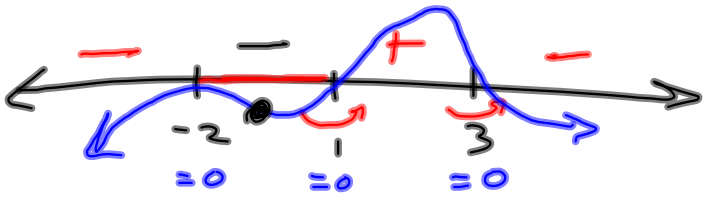
	Vol.	Conc.	Pure Alcohol
20%	x	.2	$.2x$
37%	y	.37	$.37y$
25%	100	.25	$.25(100) = 25$

$x + y = 100 \Rightarrow y = 100 - x$

$.2x + .37y = 25 \Rightarrow .2x + .37(100 - x) = 25$

Solve for x.
Substitute to find y.

$-50(x-1)(x+2)^2(x-3) \geq 0$



$x=1, m=1$

$x=-2, m=2$

$x=3, m=1$

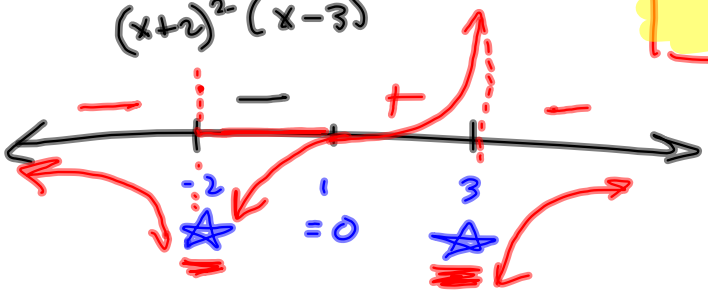
$x=0$:

$-50(-1)(2)^2(-3) < 0$

$(-)(-)(+)(-) = -$

$[-2] \cup [1,3]$

$\frac{(x-1)}{(x+2)^2(x-3)} \geq 0$



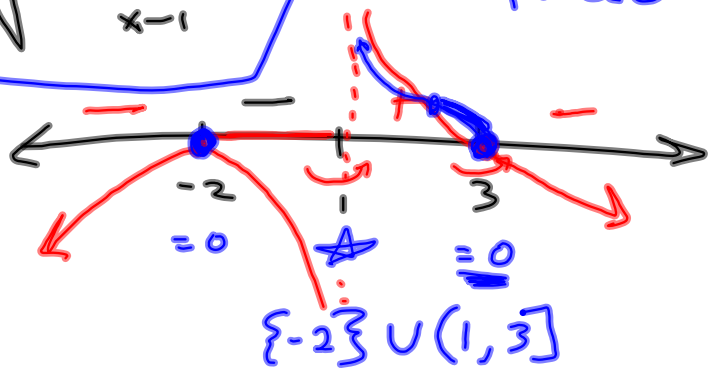
Can't include -2 or +3, because they're asymptotes.

$[1,3)$

Find the domain:

$$\sqrt{\frac{(x+2)^2(x-3)}{x-1}}$$

$\Rightarrow$ Need $\frac{(x+2)^2(x-3)}{x-1} \geq 0$

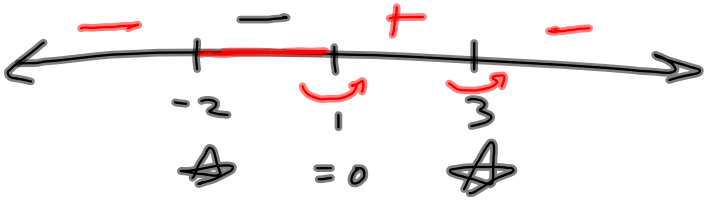


$\{-2\} \cup (1, 3]$

Superimpose graph of $\frac{(x+2)^2(x-3)}{x-1}$

Find the domain: $\ln\left(\frac{x-1}{(x+2)^2(x-3)}\right)$

$\Rightarrow$ Need $\frac{x-1}{(x+2)^2(x-3)} > 0$



$(1, 3)$