

Solve the system of equations.

$$5x + 3y + z = 13$$

$$x - 3y + 2z = 17$$

$$14x - 2y + 3z = 58$$

ST

Augmented Coefficient Matrix

$$\left[\begin{array}{ccc|c} 1 & -3 & 2 & 17 \\ 5 & 3 & 1 & 13 \\ 14 & -2 & 3 & 58 \end{array} \right] \begin{array}{l} R1 \\ -5R1 + R2 \\ -14R1 + R3 \end{array}$$

$$\left[\begin{array}{ccc|c} 1 & -3 & 2 & 17 \\ 0 & 18 & -9 & -72 \\ 0 & 40 & -25 & -180 \end{array} \right]$$

$$-5(17) = \frac{-85}{13} \text{ (1)}$$

$$\begin{array}{l} (-3)(14) = -42 \\ \frac{-2}{-44} \\ (-3)(-14) = 42 \\ \frac{-2}{40} \text{ (2)} \end{array}$$

$5/2x$ \rightarrow $5x/2$?
 $5/2x$

$$\begin{array}{r} -14 \\ 17 \\ \hline 98 \\ 146 \\ \hline -238 \end{array} \quad \begin{array}{r} -238 \\ +58 \\ \hline -180 \end{array}$$

TRIANGULAR !!

$$\begin{array}{l} R1 \\ \frac{1}{18}R2 \\ R3 \end{array} \left[\begin{array}{ccc|c} 1 & -3 & 2 & 17 \\ 0 & 1 & -\frac{1}{2} & -4 \\ 0 & 40 & -25 & -180 \end{array} \right] \begin{array}{l} R1 \\ R2 \\ -40R2 + R3 \end{array}$$

$$\left[\begin{array}{ccc|c} 1 & -3 & 2 & 17 \\ 0 & 1 & -\frac{1}{2} & -4 \\ 0 & 0 & -5 & -20 \end{array} \right]$$

$$\left(\frac{1}{18}\right)(-9) = -\frac{1}{2}$$

$$\left(\frac{1}{18}\right)(-72) = -4$$

$$x - 3y + 2z = 17$$

$$y - \frac{1}{2}z = -4$$

$$z = 4$$

$$-5z = -20$$

$$z = 4$$

Now, Back-substitute

$$z = 4 \Rightarrow$$

$$y - \frac{1}{2}z = y - \frac{1}{2}(4) = y - 2 = -4 \Rightarrow y = -2$$

$$z = 4, y = -2 \Rightarrow$$

$$x - 3y + 2z = x - 3(-2) + 2(4) = x + 6 + 8 = x + 14 = 17$$

$$x = 3$$

What we just saw was Gaussian Elimination which involves Back-substitution.

Gauss-Jordan involves Back-Elimination

$$\begin{aligned}
 & \left[\begin{array}{ccc|c} 1 & -3 & 2 & 17 \\ 0 & 1 & -\frac{1}{2} & -4 \\ 0 & 0 & 1 & 4 \end{array} \right] \xrightarrow{-2R_3 + R_1} \left[\begin{array}{ccc|c} 1 & -3 & 0 & 9 \\ 0 & 1 & -\frac{1}{2} & -4 \\ 0 & 0 & 1 & 4 \end{array} \right] \xrightarrow{+\frac{1}{2}R_3 + R_2} \left[\begin{array}{ccc|c} 1 & -3 & 0 & 9 \\ 0 & 1 & 0 & -2 \\ 0 & 0 & 1 & 4 \end{array} \right] \checkmark \\
 & \text{Note: } \left[\begin{array}{ccc|c} 1 & -3 & 2 & 17 \\ 0 & 1 & -\frac{1}{2} & -4 \\ 0 & 0 & 1 & 4 \end{array} \right] \xrightarrow{R_3} \text{should be } +\frac{1}{2} \quad (4)(\frac{1}{2}) = +2 \\
 & \text{Operations: } \begin{array}{l} 3R_2 + R_1 \\ R_2 \\ R_3 \end{array} \left[\begin{array}{ccc|c} 1 & 0 & 0 & 3 \\ 0 & 1 & 0 & -2 \\ 0 & 0 & 1 & 4 \end{array} \right] \begin{array}{l} x=3 \\ y=-2 \\ z=4 \end{array} \\
 & \text{Calculation: } \begin{array}{l} -4 \\ -4 \\ \hline -8 \end{array} \rightarrow -2
 \end{aligned}$$

Gauss-Jordan

$$\begin{aligned}
 & \left[\begin{array}{ccc|c} A & B & C & D \\ G & F & G & H \\ I & J & K & L \end{array} \right] \sim \left[\begin{array}{ccc|c} 1 & A & B & C \\ D & E & F & G \\ H & I & J & K \end{array} \right] \sim \left[\begin{array}{ccc|c} 1 & A & B & C \\ 0 & D & E & F \\ 0 & G & H & I \end{array} \right] \\
 & \sim \left[\begin{array}{ccc|c} 1 & A & B & C \\ 0 & 1 & D & E \\ 0 & F & G & H \end{array} \right] \sim \left[\begin{array}{ccc|c} 1 & A & B & C \\ 0 & 1 & D & E \\ 0 & 0 & 1 & F \end{array} \right] \\
 & \sim \left[\begin{array}{ccc|c} 1 & A & 0 & B \\ 0 & 1 & 0 & C \\ 0 & 0 & 1 & D \end{array} \right] \sim \left[\begin{array}{ccc|c} 1 & 0 & 0 & A \\ 0 & 1 & 0 & B \\ 0 & 0 & 1 & C \end{array} \right]
 \end{aligned}$$

$$\begin{aligned} 4x + 3y + z &= 6 \\ x - 3y + 2z &= 7 \\ 11x - 2y + 3z &= 27 \end{aligned}$$

Sorry, your answer is not correct.
 Correct answer: A: 2, -1, 1
 Your answer: A: -1, 2, 1

Select the correct choice below and, if necessary, fill in the answer

MN

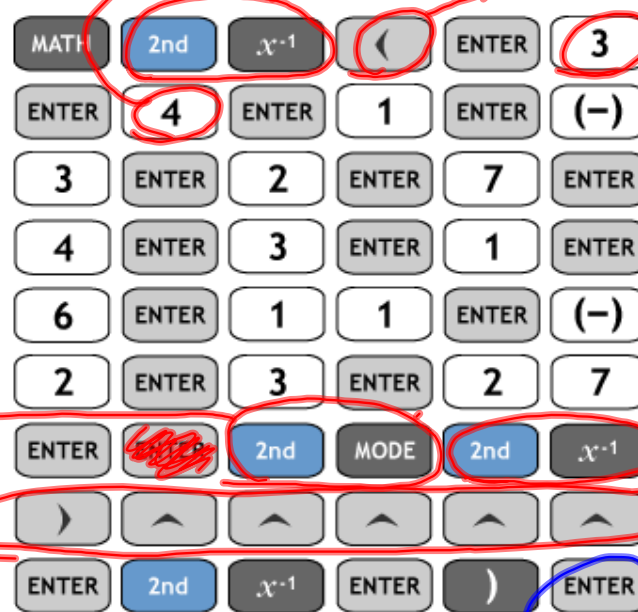
- A. There is one solution. The solution set is $\{(-1, 2, 1)\}$.

$$\left[\begin{array}{ccc|c} 1 & -3 & 2 & 7 \\ 4 & 3 & 1 & 6 \\ 11 & -2 & 3 & 27 \end{array} \right] \sim$$

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5000(1+Ans/52)^(
52*10)
15000
rref([A])
[[1 0 0 2 ]
 [0 1 0 -1]
 [0 0 1 1 ]]
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$$\begin{aligned} x &= 2 \\ y &= -1 \\ z &= 1 \end{aligned}$$

Quit



$$\begin{aligned} x &= 3y - 1 \\ y &= 2z + 2 \\ z &= 4x - 3 \end{aligned}$$

MN

$$\begin{aligned} x - 3y &= -1 && \text{Standard} \\ y - 2z &= 2 && \text{form} \\ -4x + z &= -3 \end{aligned}$$

ct the correct choice below and, if necessary, fill in the answer box to c

The system is independent. The solution set is $\left\{ \left(\frac{217}{23}, \frac{80}{23}, \frac{17}{23} \right) \right\}$.

(Two integers or simplified fractions)

To solve a system of three variables in three equations first reduce the problem to a system of two equations in two variables. Look for the easiest variable to eliminate. Use addition or substitution to eliminate the chosen variable from two pairs of the original equations. Solve the system of two equations in two variables. Then find the value of the third variable using one of the original equations. If the system results in a false statement, it is inconsistent and has no solution. If the system results in dependent equations, it has infinitely many solutions.

$$\left[\begin{array}{ccc|c} 1 & -3 & 0 & -1 \\ 0 & 1 & -2 & 2 \\ -4 & 0 & 1 & -3 \end{array} \right] \begin{array}{l} R1 \\ R2 \\ 4R1 + R3 \end{array} \quad \left[\begin{array}{ccc|c} 1 & -3 & 0 & -1 \\ 0 & 1 & -2 & 2 \\ 0 & -12 & 1 & -7 \end{array} \right]$$

$$\begin{array}{l} R1 \\ R2 \\ 12R2 + R3 \end{array} \left[\begin{array}{ccc|c} 1 & -3 & 0 & -1 \\ 0 & 1 & -2 & 2 \\ 0 & 0 & -23 & 17 \end{array} \right] \begin{array}{l} x - 3y = -1 \\ y - 2z = 2 \end{array}$$

$$-23z = 17 \qquad y - 2z = y - 2\left(-\frac{17}{23}\right) = y + \frac{34}{23} = 2$$

$$z = -\frac{17}{23}$$

$$y = 2 - \frac{34}{23} = \frac{46 - 34}{23} = \frac{12}{23} = y$$

$$x - 3y = x - 3\left(\frac{12}{23}\right) = x - \frac{36}{23} = -1$$

$$x = -1 + \frac{36}{23} = \frac{-23 + 36}{23} = \frac{13}{23} = x$$

WORD PROBLEMS GET THE VARIABLES RIGHT!
30% of every word problem!

Let x = length of shortest side (in m)
 y = medium
 z = longest

Perimeter of a triangle is 40 m.

$$x + y + z = 40$$

Sum of two shorter sides is 2 m more than long side.

$$x + y = z + 2$$

Longest side is 11 m more than shortest side.

$$z = x + 11$$

Find the lengths of the sides of the triangle.

SYSTEM, IN STANDARD FORM

$$x + y + z = 40$$

$$x + y - z = 2$$

$$-x + z = 11$$

AUGMENTED COEFFICIENT MATRIX

$$\left[\begin{array}{ccc|c} 1 & 1 & 1 & 40 \\ 1 & 1 & -1 & 2 \\ -1 & 0 & 1 & -11 \end{array} \right]$$

Let
 $x =$ amt invested in stocks (in \$)
 $y =$ " " " " bonds " "
 $z =$ " " " " Mutual Fund " "

Total of \$25,000 invested.

$$x + y + z = 25000$$

Stocks earn 8%

Bonds earn 10%

Mutual fund earns 6%

$$.08x + .1y + .06z = 1860 \text{ \$}$$

Total return was \$1860.

$$\frac{.08 \text{ \$}}{1 \text{ \$ invested}} \times \text{ \$}$$

Invested twice as much in mutual funds as she invested in bonds.

$$x + y + z = 25000$$

$$.08x + .1y + .06z = 1860$$

$2z = y$ is COMMON ERROR

Mutual Fund investment is twice as big as bonds investment

$$-2y + z = 0$$

$z = 2y$ is correct

How much was invested in each instrument?