Form a polynomial with real coefficients that has the given zeros and degree.

a. Zeros: 3, multiplicity 2; -2, multiplicity 3; 6, multiplicity 1. Degree 6

$(x-3)^2(x+2)^3(x-6)^1$ is degree 6 ✓

$2+3+1=6$ ✓

... with leading coefficient 5:

$5(x-3)^2(x+2)^3(x-6)$

Expand $(x-(2+5i))(x-(2-5i))$

$= (x-2-5i)(x-2+5i)$

$= x^2 - 2x + \cancel{5ix}$
$\quad - 2x + 4 \cancel{-10i}$
$\quad \cancel{-5ix} \cancel{+10i} - 25i^2$

$-25i^2 = -25(-1) = 25$

$= \boxed{x^2 - 4x + 29}$

Let $f(x) = 3(x-2)^3(x+4)(x-5)^2$

$f(0) = 3(-2)^3(4)(-5)^2 =$

a. List each real zero and its multiplicity. Determine whether the graph of $f(x)$ touches or crosses the x-axis at each x-intercept.

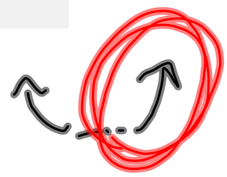
$x=2, m=3$ cross (change sign)

$x=-4, m=1$ " "

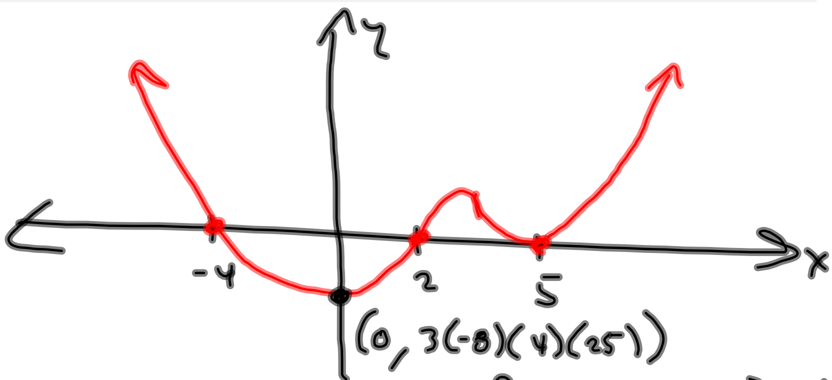
$x=5, m=2$ Don't change sign

b. Determine the power function that $f(x)$ resembles as $x \to \pm\infty$. This is the End Behavior part of the question. (i.e. determine $\lim_{x\to\infty} f(x)$ and $\lim_{x\to-\infty} f(x)$)

$3(x)^3(x)(x)^2 = 3x^{3+1+2} = 3x^6$



d. Use the information you reported to obtain a rough graph of $f(x)$.



$(0, 3(-8)(4)(25))$

$f(0) = 3(-2)^3(4)(-5)^2 =$

5. Find the asymptotes (i.e. vertical, and/or horizontal and/or oblique). Reminder: you find the vertical asymptotes by finding where the denominator equals zero. For Part ii, you will need to use long division to find the slant asymptote.

i) $R(x) = \frac{120x^4+5594x^2-0.009x+2}{-12x^4+x^3}$

H.A. $\frac{120x^4}{-12x^4} = \boxed{-10 = y}$

V.A. $-12x^4+x^3 = -x^3(12x-1)$

$x=0, \quad x=\frac{1}{12}$

Nothing to cancel with, so expect no holes.

ii) $G(x) = \frac{x^3-8}{x^2-5x+6} = \frac{x^3-2^3}{(x-3)(x-2)} = \frac{\cancel{(x-2)}(x^2+2x+4)}{(x-3)\cancel{(x-2)}}$

$= \boxed{\frac{x^2+2x+4}{x-3}}$ in lowest terms.

$(x-3)^1$

Hole: $x=2$

V.A.: $x=3$

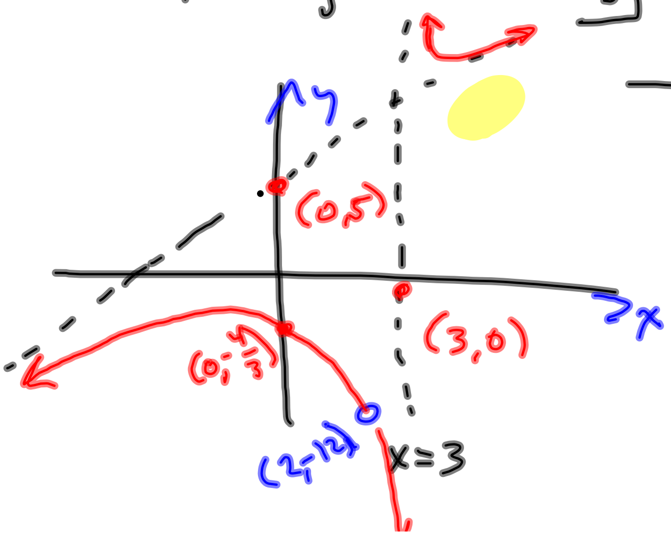
H.A.: None

Denominator is linear, so synthetic division may be used for the oblique asymptote: slant.

S.A.: $y=x+5$

$$\begin{array}{r|rrr} 3 & 1 & 2 & 4 \\ & & 3 & \cancel{15} \\ \hline & 1 & 5 & \cancel{19} \end{array}$$

$x+5$



y-int: (0,

Hole:

$\frac{2^2+2(2)+4}{2-3} = \frac{12}{-1}$

$\rightsquigarrow (2,-12)$

Carbon-14 has a half-life of 7500 yrs (suppose). How old is charcoal from an ancient fire pit if 30% of the normal amount of C-14 is present?

$P(t) = P_0 e^{kt}$

$\frac{1}{2}$-life is 7500 yrs

$$P_0 e^{k \cdot 7500} = \frac{1}{2} P_0$$

$$e^{7500k} = \frac{1}{2}$$

$$\ln(e^{7500k}) = \ln\left(\frac{1}{2}\right)$$

$$7500k = \ln\left(\frac{1}{2}\right)$$

$$k = \frac{\ln\left(\frac{1}{2}\right)}{7500} = \frac{-\ln(2)}{7500}$$

$$\ln\left(\frac{1}{2}\right) = \ln(2^{-1}) = -\ln(2)$$

How old is the sample? Keep doing it as symbolically as possible:

30% of radioactive C-14 remains)

$$P_0 e^{kt} = .3 P_0$$ Solve for t (We know k.)

$$e^{kt} = .3$$

$$kt = \ln(.3)$$

$$t = \frac{\ln(.3)}{k} = \frac{\ln(.3)}{-\frac{\ln(2)}{7500}} = -\frac{\ln(.3)}{\ln(2)} \cdot 7500$$

```
ln(.3)*7500/ln(2
)
           -13027.24196
```

Forgot the "−"

About 13,027 yrs.

$\frac{1}{2}$-life is 7500:

$\frac{1}{2}$ in 7500

$\frac{1}{2} \cdot \frac{1}{2} = \frac{1}{4}$ in 15,000 yrs.

→ 25 %

So common sense agrees.

We wanted something between 7500 & 15000, since 30% is between 50% & 25%

one $\frac{1}{2}$-life two half-lives.

Oct 31-9:04 AM

What interest rate will cause an investment to triple in 10 yrs, if interest is compounded weekly?

P

It triples in 10 yrs.

$$\cancel{P}\left(1+\frac{r}{52}\right)^{52\cdot 10} = 3\cancel{P}$$

$$\left(1+\frac{r}{52}\right)^{520} = 3$$

There's a power, but the variable is Not in the exponent.

$$\sqrt[520]{\left(1+\frac{r}{52}\right)^{520}} = \sqrt[520]{3}$$

$$\left(\left(1+\frac{r}{52}\right)^{520}\right)^{\frac{1}{520}} = 3^{\frac{1}{520}}$$

$$1+\frac{r}{52} = \sqrt[520]{3} = 3^{\frac{1}{520}}$$

$$\frac{r}{52} = 3^{\frac{1}{520}} - 1$$

$$r = 52\left(3^{\frac{1}{520}} - 1\right) = 52*(3\verb|^|(1/520)-1)$$

$\approx .1099773634$

Rounded to $\frac{1}{100}$ of a percent:

10.99773634%

$\approx$ 11.00%

```
52*(3^(1/520)-1)
         .1099773634
```

```
52*(3^(1/520)-1)
         .1099773634
5000(1+Ans/52)^(
52*10)
               15000
```

Good Check.

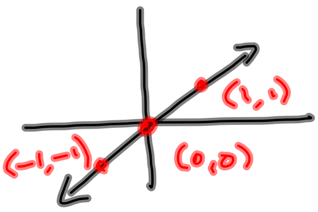
$$y = 6 - 2x = -2(-3 + x) = \boxed{-2(x-3)}$$

$-2x+6$

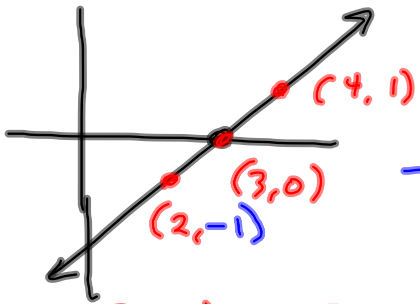
Graph by transforming $f(x) = \underline{x}$

$$f(x) \longrightarrow f(x-3) \longrightarrow -2f(x-3)$$

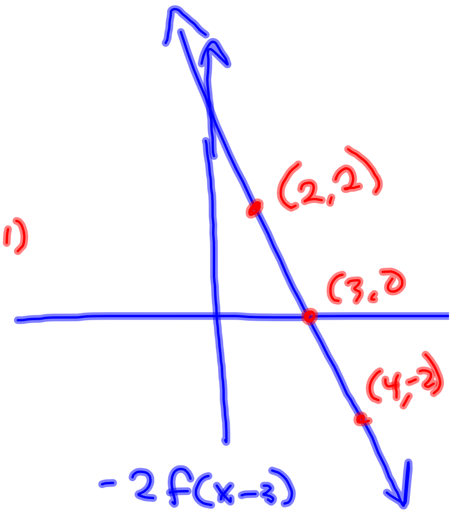
$$x \longrightarrow x-3 \longrightarrow -2(x-3)$$



$f(x) = x$



$f(x-3) = x-3$

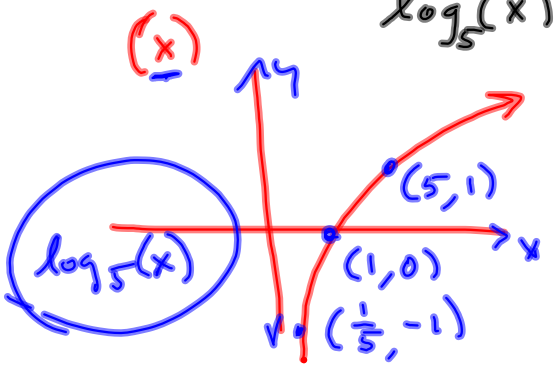


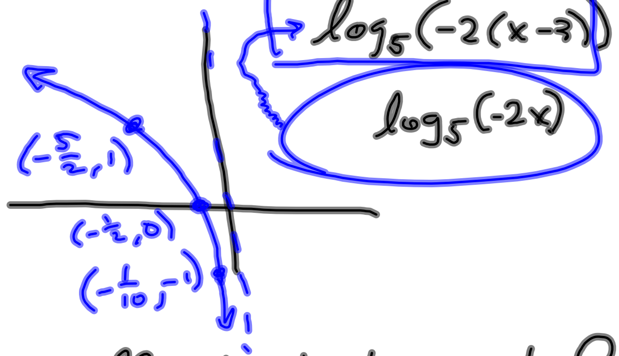
$-2f(x-3)$

$= -2(x-3)$

$$\log_5(6-2x) = \log_5(-2(x-3))$$

$$\log_5(x) \longrightarrow \log_5(-2x) \longrightarrow \boxed{\log_5(-2(x-3))}$$





$-2(x-3)$ is inside, so it's all about horizontal changes

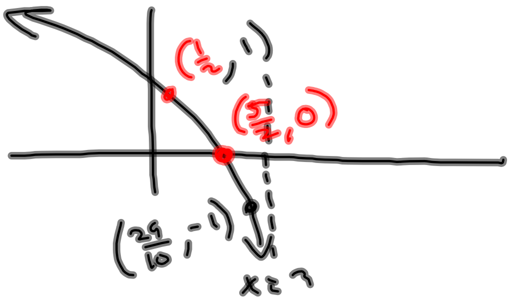
1 Hor stretch $\quad -\frac{1}{10}+\frac{30}{10}$

2 Hor shift

3 ver stretch $\quad -\frac{1}{2}+\frac{6}{2}$

4 ver shift $\quad -\frac{5}{2}+\frac{6}{2}=\frac{1}{2}$

$\log_5(-2(x-3))$



$$\boxed{5 \cdot 2^{-2x-4} - 7}$$

$$= 5 \cdot 2^{-2(x+2)} - 7$$

$-2(x+2)$

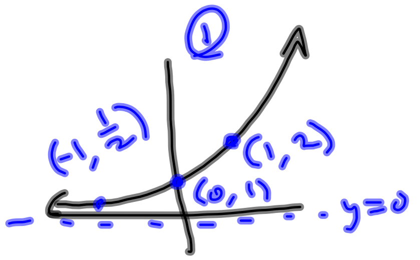
10^{-2x-4} → WRONG!

$$2^{x} \longrightarrow 2^{-2x} \longrightarrow 2^{-2(x+2)}$$

① ② ③

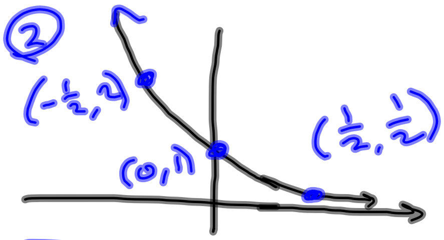
$$5 \cdot 2^{-2(x+2)} \longrightarrow 5 \cdot 2^{-2(x+2)} - 7$$

①



$2^{x} = f(x)$

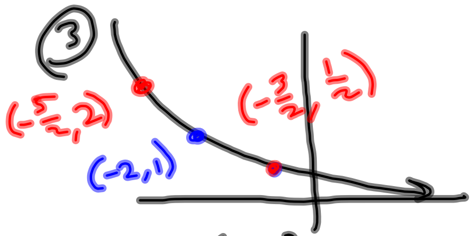
②



$\boxed{2^{-2x} = f(-2x)}$

$-\frac{1}{2}$ times x-values to handle $f(-2x)$

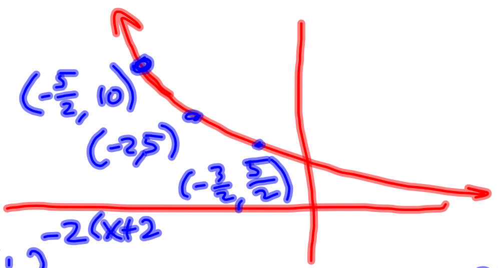
③



$2^{-2(x+2)}$

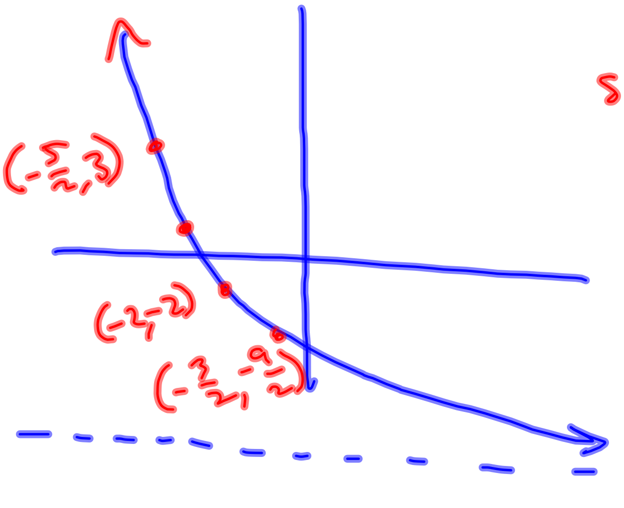
left 2

$$-\tfrac{1}{2} - 2 = -\tfrac{1}{2} - \tfrac{4}{2} = -\tfrac{5}{2}$$



$5 \cdot 2^{-2(x+2)}$

stretch vertically
5 times y-values.



subtract 7 from y-values

$5 \cdot 2^{-2(x+2)} - 7$

$y = -7$

Oct 31-8:51 AM