The population of a city doubled from 1950 to 1985, going from 2.5 million to 5 million people. Using the exponential model, $P = P_0 e^{rt}$, where P is population, P_0 is initial population, and t is time in years, find the annual growth rate r for that period. Although the annual growth rate has declined slightly to 1.68% annually, the population of the city is still growing at a tremendous rate. Using the initial population of 5 million in 1985 and an annual rate of 1.68%, estimate the population of the city in the year 2010.

The annual growth, r, between 1950 and 1985 was 1.98 %.
(Round to two decimal places as needed.)

The population of the city in the year 2010 should be 7.6 million people.
(Round to one decimal place as needed.)

Let t = number of years after 1950

Doubles in 35 yrs

$P(t) = P_0 e^{kt}$

Find k: (Don't need the 2.5 million and 5 million, here, because doubling time is a gift.)

w/ Doubling Time

$P(35) = 2P_0$

$P_0 e^{k \cdot 35} = 2P_0$

$e^{35k} = 2$

$\ln(e^{35k}) = \ln(2)$

$35k = \ln(2)$

$\boxed{k = \frac{\ln(2)}{35}}$

$P(t) = 2.5e^{kt}$, where $k =$

w/ actual data

$P(35) = 5$

$2.5e^{kt} = 5$

$e^{kt} = 2$

$\approx .019842052$

Growth rate is approximately 1.98%

Relative Growth Rate is 1.68% after 1985,

Using 1985 as $t_0 = 0$, then

$2010 - 1985 = 25 = t$, where t = time, in years, measured after 1985.

$P(t) = P_0 e^{kt} = 5e^{.0168t}$

$P(25) = 5e^{(.0168)(25)} = ?$

Express as a difference of logarithms.

$$\log_2\left(\frac{7}{3x}\right) = \log_2(7) - \log_2(3x) =$$

$$= \log_2(7) - \left[\log_2(3) + \log_2(x)\right]$$

$$= \log_2(7) - \log_2(3) - \log_2(x)$$

Oct 24-7:41 AM

Find all real and imaginary solutions to the equation.

$w^4 + 216w = 0$

$w(w^3+216) = 0 \Rightarrow w = 0$ oR $w^3+216=0$

$w^3 = -216$

$w = \sqrt[3]{-216} = -\sqrt[3]{2\cdot2\cdot2\cdot3\cdot3\cdot3}$

$= -2\cdot3 = -6$

$2 \,|\, 216$, $2 \,|\, 108$, $2 \,|\, 54$, $3 \,|\, 27$, $3 \,|\, 9$, 3

$$\begin{array}{r|rrrr} -6 & 1 & 0 & 0 & 216 \\ & & -6 & 36 & -216 \\ \hline & 1 & -6 & 36 & 0 \end{array}$$

$(w^3+216) \div (w+6)$

This gives us: $w(w+6)(w^2-6w+36)$

$w^2-6w+36=0$

$a=1, b=-6, c=36$

$b^2-4ac = (-6)^2-4(1)(36)$

$= 36-144$

$= -108$

$2 \,|\, 108$, $2 \,|\, 54$, $3 \,|\, 27$, $3 \,|\, 9$, 3

$x = \frac{6 \pm \sqrt{-108}}{2} = \frac{6 \pm 2\cdot3i\sqrt{3}}{2} = \frac{\cancel{2}(3\pm3i\sqrt{3})}{\cancel{2}} = 3\pm3\sqrt{3}i$

$w(w+6)(w-3+3i\sqrt{3})(w-3-3i\sqrt{3})$

Zeros: $-6, 0, 3\pm3i\sqrt{3}$

Solve the equation below.

$2 \cdot 2^{3x} = 8^x + 64$

x = 2 (Simplify your answer.)

$2 \cdot 2^{3x} = 2 \cdot (2^3)^x = 2 \cdot 8^x$

$2 \cdot 8^x = 8^x + 64$

$-8^x = -8^x$

$8^x = 64 = 8^2$

$x = 2$

$2 \cdot 8^x - 8^x =$

$8^x(2-1) = 8^x \cdot 1$

$2 \cdot 2^{3x} = (2^3)^x + 64$

$2 \cdot 2^{3x} = 2^{3x} + 64$

$2^{3x} = 64 = 2^6$

$3x = 6$

$x = 2$

$2 \,|\, 64$
$2 \,|\, 32$
$2 \,|\, 16$
$2 \,|\, 8$
$2 \,|\, 4$
2

$\frac{1}{2}$ life of Millsium is 50 years.

How old is a sample with 10% of the radioactive Millsium remaining?

$\frac{1}{2}$-life $= 50$

$$\boxed{P_0 e^{50k} = \frac{1}{2} P_0}$$

$$\boxed{e^{50k} = \frac{1}{2}}$$

$$50k = \ln\left(\frac{1}{2}\right) = \ln(1) - \ln(2) = -\ln(2)$$

$$k = -\frac{\ln(2)}{50} \approx -.0138629436$$

$$\boxed{P(t) \approx P_0 e^{-.0138629436t}}$$

```
      5733.148035
4000e^(.06*6)
      5733.317658
ln(2/50
     -3.218875825
ln(2)/50
      .0138629436
```

Oct 24-8:58 AM

Test 3 Take-home Due Wednesday!

Polynomial with zeros @ $x = 1, m = 2$; $x = 3, m = 1$; $x = -2, m = 3$. Leading coefficient of 3.

$$3(x-1)^2(x-3)(x+2)^3 = f(x)$$

Sketch it

E.B.

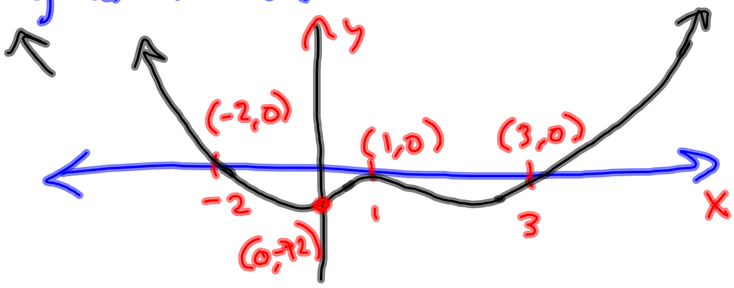
End behavior: $3(x)^2(x)^1(x)^3 = 3x^6$

y-int: $3(-1)^2(-3)(2)^3 = -72$

$x=3$ cross

$x=1$ touch

$x=-2$

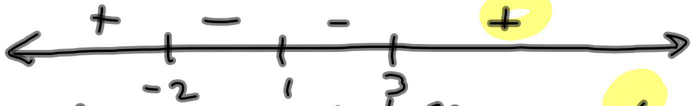


New prob: $3(x-1)^2(x-3)(x+2)^3 \geq 0$

$x=-2$ $m=3$

$x=1$ $m=2$

$x=3$ $m=1$



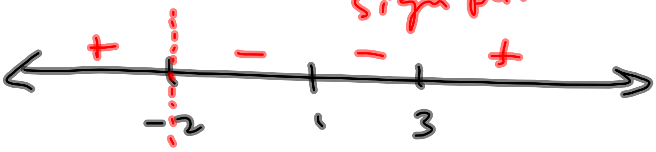
E.B. or plugging $x=4$ tells you $+$ on $(3, \infty)$

$$(-\infty, -2] \cup \{1\} \cup [3, \infty)$$

It's in lowest terms: No Holes.

$$\frac{(x-1)^2(x-3)}{(x+2)^3} \geq 0$$

Same exact sign pattern



$x=-2 \quad m=3$

$x=1 \quad m=2$

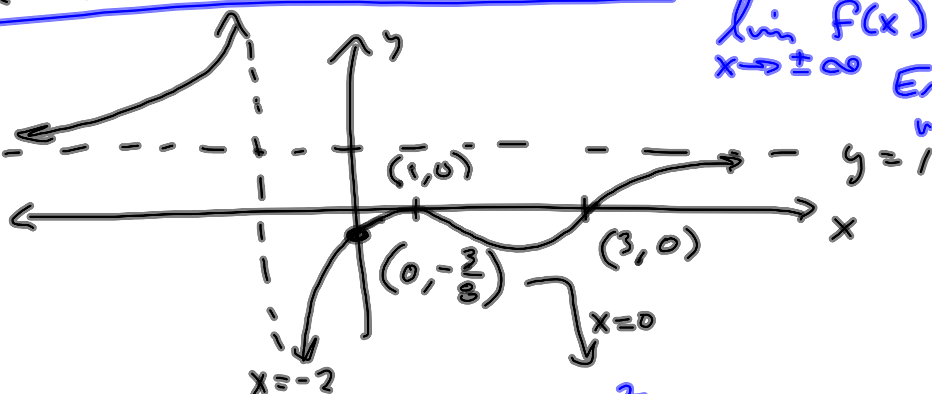
$x=3 \quad m=1$

$$\boxed{(-\infty,-2) \cup \{1\} \cup [3,\infty)}$$

Horizontal Asymptote.

$\lim_{x\to\pm\infty} f(x)$ End Behavior.

Sketch:



$$\frac{(x-1)^2(x-3)}{(x+2)^3} \xrightarrow{x\to \text{Big}} \frac{x^3}{x^3} = \boxed{1 = y}$$

is H.A.

$$f(x) = 3x^5 \overset{1}{-} 17x^4 \overset{2}{+} 25x^3 + 65x^2 \overset{3}{-} 128x \overset{4}{+} 52$$

Descartes: 4, 2, 0 possible positive zeros.

$$f(-x) = -3x^5 - 17x^4 - 25x^3 \overset{1}{+} 25x^2 + 128x + 52$$

Exactly one negative zero.

$2 \underline{|52}$
$2 \underline{|26}$
13

Rational Zeros:

$\frac{p}{q}: \frac{52}{3}$ $\pm 1, \pm \frac{1}{3}, \pm 2, \pm \frac{2}{3}, \pm 4, \pm \frac{4}{3},$
$\pm 13, \pm \frac{13}{3}, \pm 26, \pm \frac{26}{3}, \pm 52, \pm \frac{52}{3}$

On scratch, we make some bad guesses

Here's the synthetic divisions, with no false moves: Rest of the work is on the next page.

1	$3x^5$	-17	25	65	-128	52
		3	-14	11	76	-52
-2	3	-14	11	76	-52	
		-6	40	-102	52	
$\frac{2}{3}$	3	-20	51	-26	sweet	
		2	-12	26		
	3	-18	39			

Oct 24-8:08 AM

Scratch

$x=1$

$f(x) = (x-1)(3x^4-14x^3+11x^2+76x-52)$

$$\begin{array}{r|rrrrrr}
1 & 3x^5 & -17 & 25 & 65 & -128 & 52 \\
 & & 3 & -14 & 11 & 76 & -52 \\ \hline
 & 3 & -14 & 11 & 76 & -52 & 0
\end{array}$$

$$\begin{array}{r|rrrrr}
1 & 3 & -14 & 11 & 76 & -52 \\
 & & 3 & 11 & 0 & 76 \\ \hline
 & 3 & -11 & 0 & 76 & \text{Nope}
\end{array}$$

$$\begin{array}{r|rrrrr}
-1 & 3 & -14 & 11 & 76 & -52 \\
 & & -3 & 17 & -28 & -48 \\ \hline
 & 3 & -17 & 28 & 48 & \text{Nope}
\end{array}$$

Keep guessing...

$f(x) = (x-1)(x+2)(3x^3-20x^2+51x-26)$

$$\begin{array}{r|rrrrr}
-2 & 3 & -14 & 11 & 76 & -52 \\
 & & -6 & 40 & -102 & 52 \\ \hline
 & 3 & -20 & 51 & -26 & \text{Sweet}
\end{array}$$

$$\begin{array}{r|rrrr}
\frac{2}{3} & 3 & -20 & 51 & -26 \\
 & & 2 & -12 & 26 \\ \hline
 & 3 & -18 & 39 &
\end{array}$$

$(x-1)(x+2)(x-\frac{2}{3})(3x^2-18x+39)$

So this is done being factored over the reals.

$3x^2-18x+39$ is irreducible over the reals.

$3x^2-18x+39=0 \Rightarrow$

$3(x^2-6x+13)=0 \Rightarrow$

$x^2-6x+13=0 \Rightarrow$

$a=1, b=-6, c=13$

$b^2-4ac = (-6)^2-4(1)(13)$

$= 36-52$

$= -16 \Rightarrow$

No real zeros

Handle the non-real stuff:

$$x = \frac{6\pm\sqrt{-16}}{2} = \frac{6\pm i\sqrt{16}}{2} = \frac{6\pm 4i}{2} = \frac{\cancel{2}(3\pm 2i)}{\cancel{2}}$$

$$= 3\pm 2i$$

Factored form: $f(x) = 3(x-1)(x+2)(x-\frac{2}{3})(x-3+2i)(x-3-2i)$

Zeros: $x = 1, -2, \frac{2}{3}, 2\pm 3i$.