Evaluate the following exponential expression without using a calculator.

$-1000^{-5/3} = -\frac{1}{1000^{\frac{5}{3}}}$

$-1000^{-5/3} = \square$

(Type an integer or a simplified fraction.)

$$2 \,|\, 1000 \quad 2 \,|\, 500 \quad 2 \,|\, 250 \quad 5 \,|\, 125 \quad 5 \,|\, 25 \quad 5$$

$$= -\frac{1}{(2\cdot2\cdot2\cdot5\cdot5\cdot5)^{5/3}}$$

$$= -\frac{1}{\left((2\cdot2\cdot2\cdot5\cdot5\cdot5)^{\frac{1}{3}}\right)^5}$$

$$\frac{5}{3} = \frac{1}{3}\cdot\frac{5}{1} = \frac{1}{3}\cdot5$$

$$1000^{\frac{5}{3}} = 1000^{\frac{1}{3}\cdot5} = \left((1000)^{\frac{1}{3}}\right)^5$$

$$\left(\sqrt[3]{2\cdot2\cdot2\cdot5\cdot5\cdot5}\right)^5$$

$$2\cdot5\sqrt[3]{1\cdot1}$$

$$= -\frac{1}{(2\cdot5)^5}$$

$$= -\frac{1}{2^5\cdot5^5}$$

$$125 \times 5 \qquad 5^4 = 625 \qquad 625 \times 5 \qquad 5^5 = 3125$$

$$= -\frac{1}{32\cdot3125}$$

= etc.

Use transformations to graph $= -0.2 \cdot 2^{x+1}$. $= g(x)$

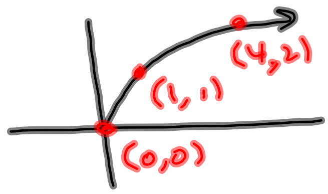
$-.2\sqrt{x+1}$

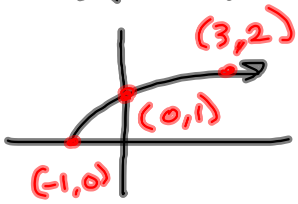
$f(x) = \sqrt{x} \longrightarrow \sqrt{x+1} \longrightarrow -.2\sqrt{x+1} = g(x)$

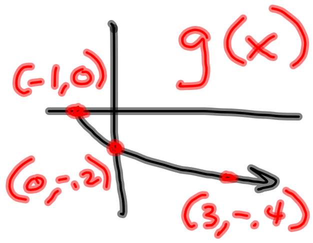
left 1

$-.2$ times y-vals.

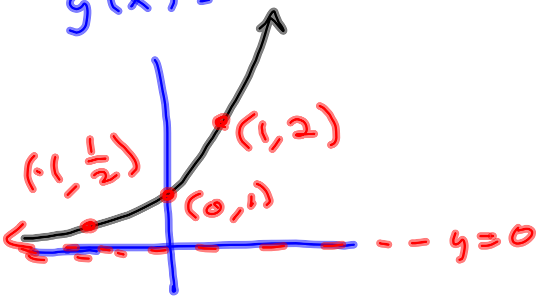
$f(x+1) \longrightarrow -.2 f(x+1)$







$g(x) = -.2 \cdot 2^{x+1}$



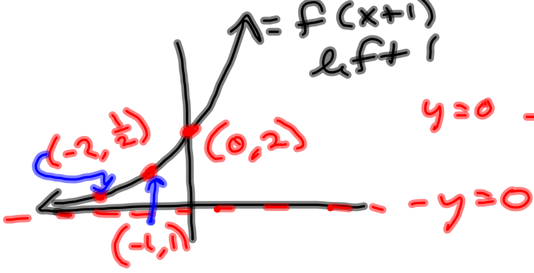
$f(x) = 2^x \longrightarrow 2^{x+1} \longrightarrow -.2 \cdot 2^{x+1}$

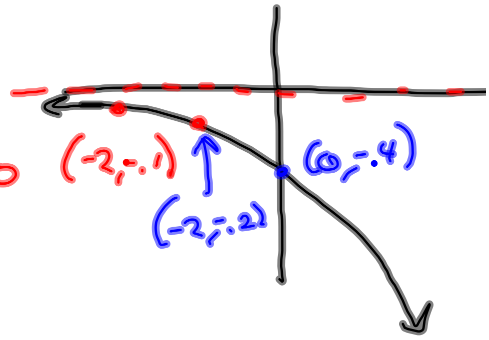
$= f(x+1)$

left 1

$(-.2)\left(\frac{1}{2}\right) = -.1$

$-\frac{2}{10} \cdot \frac{1}{2} = -.1$





Recall Inverse Functions

§4.2x is Xtra credit.

§4.2 Logarithmic Functions

log to the base b of x is $\log_b x$, and it's the inverse of b^x

$$f(x)=3^x \Longrightarrow f^{-1}(x)=\log_3(x)=g(x)$$

$f(g(x))=x$ is what that means.

$$3^{\log_3(x)} = x$$

$$\log_3(3^x) = x$$

$$\log_3(27) = \log_3(3^3) = 3$$

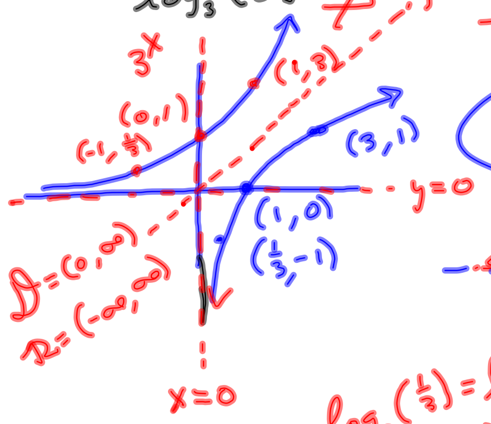
Handy if you can write the argument as a power of 3.

$\log_3(9)=2$, b/c $9=3^2$

$$\log_3(3) = 1$$

$\log_3(0)\ \nexists$ says $0=3^?$

Nevah!



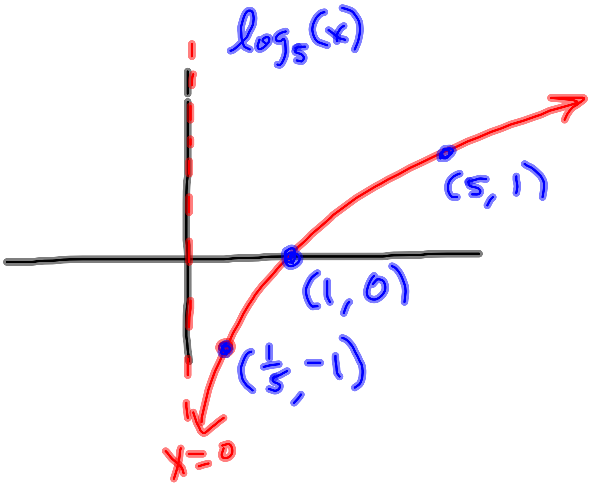
$$\log_3(\tfrac{1}{3}) = \log_3(3^{-1}) = -1$$

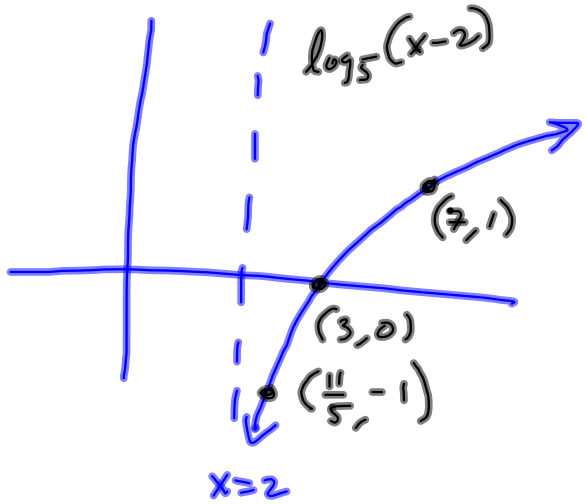
$$\log_3(1) = \log_3(3^0) = 0$$

$$\log_3(3) = \log_3(3^1) = 1$$

Graph $\log_5(x-2) = g(x)$

$\boxed{f(x) = \log_5(x)} \xrightarrow[\text{RIGHT 2}]{} f(x-2) = g(x)$





$$25^{x-1} = 5^{3x}$$
$$(5^2)^{x-1} = 5^{3x}$$
$$(5)^{2(x-1)} = 5^{3x}$$

Manipulating in order to write both sides in terms of the same base.

$2(x-1) = 3x$, etc.

$\log_3(81) = \log_3(3^4) = 4$

$\quad \log_3(3^1)$

$= 4 \cdot 1 = 4$

Sledgehammer

$$25^{x-1} = 5^{3x}$$
$$\log_5(25^{x-1}) = \log_5(5^{3x})$$
$$(x-1)\log_5(25) = 3x\log_5(5)$$
$$(x-1)\cdot 2 = 3x\cdot 1$$, etc.

Bird finds a new perch.

Oct 19-8:29 AM

$$\log_b(xy) = \log_b(x) + \log_b(y)$$

$$b^x \cdot b^y = b^{x+y}$$

$$3^2 \cdot 3^5 = 3^{2+5} = 3^7$$

Computers use this trick to turn multiplication into addition of exponents.

$327 \cdot 598$

$$\log(327 \cdot 598) = \log(327) + \log(598) =$$

```
log(327)+log(598
)
        5.291248937
10^Ans
             195546
327*598
             195546
```

Oct 19-8:37 AM

$\log_b(x^y) = y \cdot \log_b(x)$

Powers inside become products outside

Products inside become sums outside

$\log_b(xy) = \log_b(x) + \log_b(y)$.

Change of Base (So you can do $\log_7(x)$ on a calculator)

$\log(x) = \log_{10}(x)$

is common log

$\left\{ \begin{array}{l} \ln(x) = \log_e(x) \\ \text{is natural log.} \end{array} \right\}$ Euler's "e"

e^x is exactly as steep as it is tall.

Derivation of change of base

Step 2 $M = b^x$

Step 1 want: $\log_b(M) = x$

$= \dfrac{\log_c(M)}{\log_c(b)}$

$\log_c(M) = \log_c(b^x)$

$\log_c(M) = x \log_c(b)$

$\boxed{\dfrac{\log_c(M)}{\log_c(b)}} = x$

$\boxed{\log_7(25) = \dfrac{\log(25)}{\log(7)}}$

Calculator method

$\log_5(125)$

Solve $e^{5x} = 7$

Using logs to solve exponential equations

$$\log_e(e^{5x}) = \ln(e^{5x}) = \ln(7)$$

$$5x = \ln(7)$$

$$\boxed{x = \frac{\ln(7)}{5}}$$

Oct 19-8:58 AM