# Bounds on Real Roots.

Show that $x=4$ is an upper bound on real roots for $f(x)=2x^3-5x^2-6x+4=0$

$$\begin{array}{r|rrrr} 4 & 2 & -5 & -6 & 4 \\ & & 8 & 12 & 24 \\ \hline & 2 & 3 & 6 & 28 \end{array}$$

$(x-4)(2x^2+3x+6)+28$

Bottom row all nonnegative

So $x=4$ is an upper bound on real zeros.

Show that $x=-2$ is a lower bound on real zeros:

$$\begin{array}{r|rrrr} -2 & 2 & -5 & -6 & 4 \\ & & -4 & 18 & -24 \\ \hline & 2 & -9 & 12 & -20 \end{array}$$

0's in bottom row are ok.

Bottom row alternates.

So $x=-2$ is a lower bound on real zeros.

$f(x) = 2x^3 - 5x^2 - 6x + 4$

Because $x = 4$ is U.B. and
$x = -2$ is L.B. ...

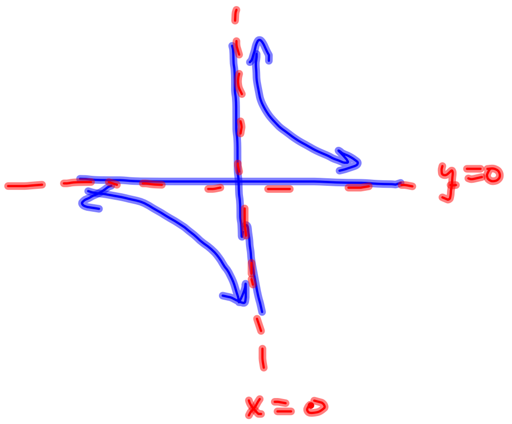
$p: 4$
$q: 2$

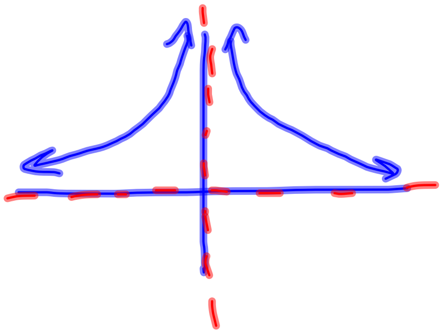
$\pm 1, \pm \frac{1}{2}, \pm 2, \pm 4$

By our knowledge, we KNOW that $x = -4$ doesn't need to be tested.

All x-intercepts in here

$f(x) = \frac{1}{x}$ , $f(x) = \frac{1}{x^2}$



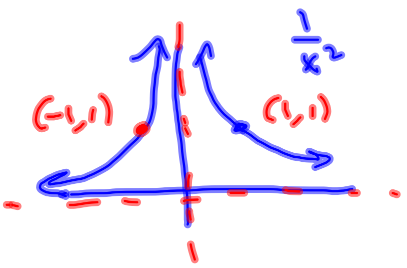



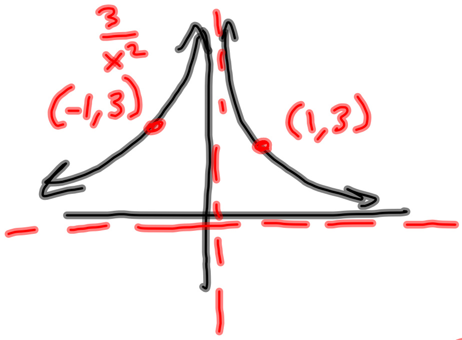
$g(x) = \frac{3}{(x-2)^2} + 7$

$\frac{1}{x^2} \longrightarrow \frac{3}{x^2} \longrightarrow \frac{3}{(x-2)^2} \longrightarrow \frac{3}{(x-2)^2} + 7$

$f(x)$ $3f(x)$ $3f(x-2)$ $3f(x-2)+7$

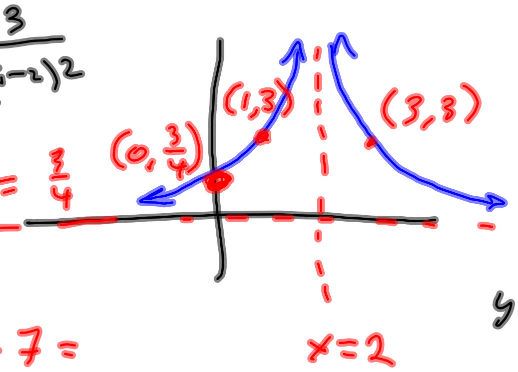
$= g(x)$




$\frac{3}{(x-2)^2}$

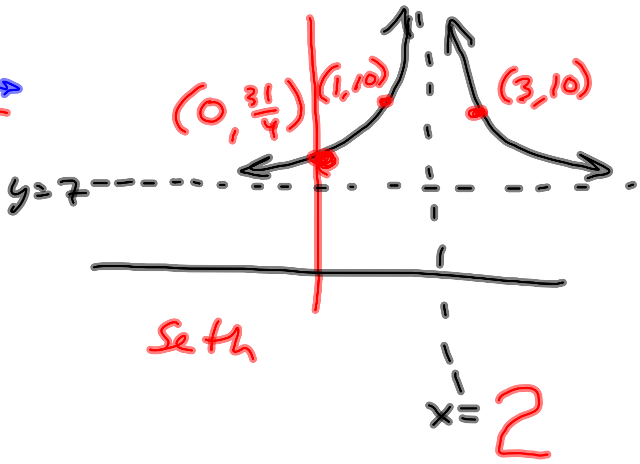
$\frac{3}{(0-2)^2} = \frac{3}{4}$




$\frac{3}{4} + 7 =$

$\frac{3+28}{4} = \frac{31}{4}$

$\frac{3}{(x-2)^2} + 7$




Seth

$$|x^2 - 2x - 16| = 8$$

$$|☺| = \Delta$$

$x^2-2x-16 = 8$ or $x^2-2x-16 = -8$

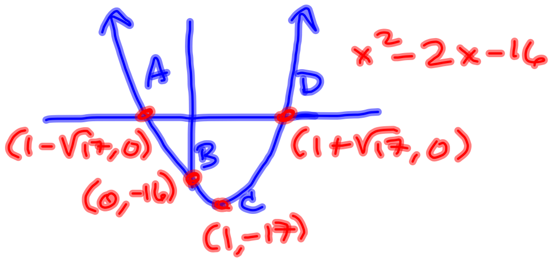
Solve Separately

$☺ = \Delta$ or $☺ = -\Delta$

$|A| = B$

$A = B$ or $A = -B$

Quick Sketch of the situation:




Scratch:

$x^2 - 2x + 1^2 - 1^2 - 16$

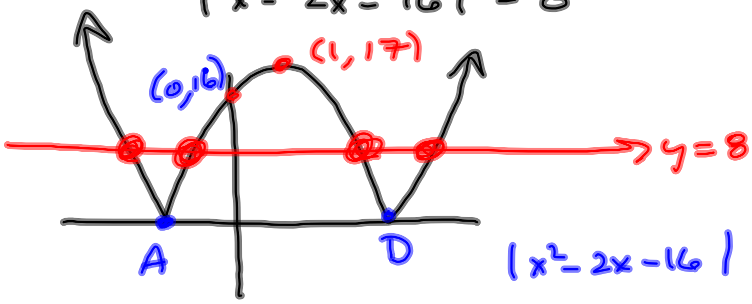
$= (x-1)^2 - 17 \overset{\text{SET}}{=} 0$

$(x-1)^2 = 17$

$x - 1 = \pm\sqrt{17}$

$x = 1 \pm \sqrt{17}$

$$|x^2 - 2x - 16| = 8$$



Equations that are quadratic in form:

$\sqrt{x^2} = |x|$

Note

$x - 4\sqrt{x} - 21 = 0$

$\sqrt{x}^2 = x$

Let $u = \sqrt{x}$, then $u^2 = \sqrt{x}^2 = x$

understood that $x \geq 0$

Then $u^2 - 4u - 21 = 0$

$(u-7)(u+3) = 0$

$u = 7$ or $u = -3$

$\sqrt{x} = 7$  $\sqrt{x} = -3$

Ain't happening

$\sqrt{x}^2 = 7^2$

$\boxed{x = 49}$

$x^4 - 4x^2 - 21 = 0$

Let $u = x^2$, then $u^2 = (x^2)^2 = x^{2\cdot 2} = x^4$

So $u^2 - 4u - 21 = 0$

$(u-7)(u+3) = 0$

$u = 7$ or $u = -3$

$x^2 = 7$ $\quad$ $x^2 = -3$

$x = \pm\sqrt{7}$ $\quad$ $x = \pm\sqrt{-3} = \pm i\sqrt{3}$

2 real & 2 nonreal solutions.

Split $x^4 - 4x^2 - 21$ into the product of linear factors

$(x - \sqrt{7})(x - (-\sqrt{7}))(x - i\sqrt{3})(x - (-i\sqrt{3}))$

$x^4 - 4x^2 - 21 = 0$

Use C3 skills to
Find all real zeros
and factor over the
real number field.

p: -21

q: 1

$\frac{p}{q}$: ±1, ±3, ±7, ±21

8 guesses. All sucky.
Recognizing Quadratic Form BIG.

We Know that this guy has NO rational zeros.

It has two irrational (but real) roots and two nonreal (Pure imaginary).

Rational Roots Don't Help!

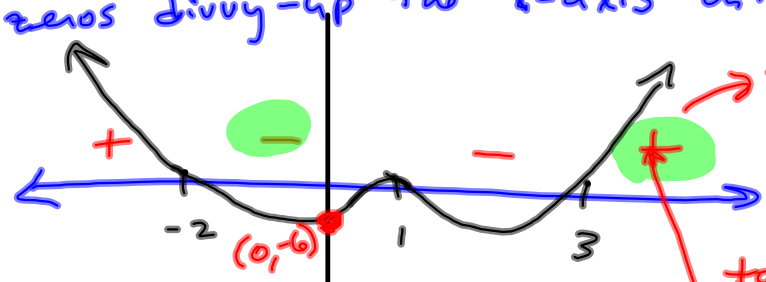
Oct 7-8:28 AM

§3.5 Graphing Polynomial Functions and Solving Polynomial Inequalities.
§3.6

$(x+2)^1(x-1)^2(x-3)^1$

| | | | |
|---|---|---|---|
| zeros: | $x=-2$ | $m=1$ | Cross |
| | $1$ | $m=2$ | Touch |
| | $3$ | $m=1$ | Cross |

The zeros divvy-up the x-axis into intervals.



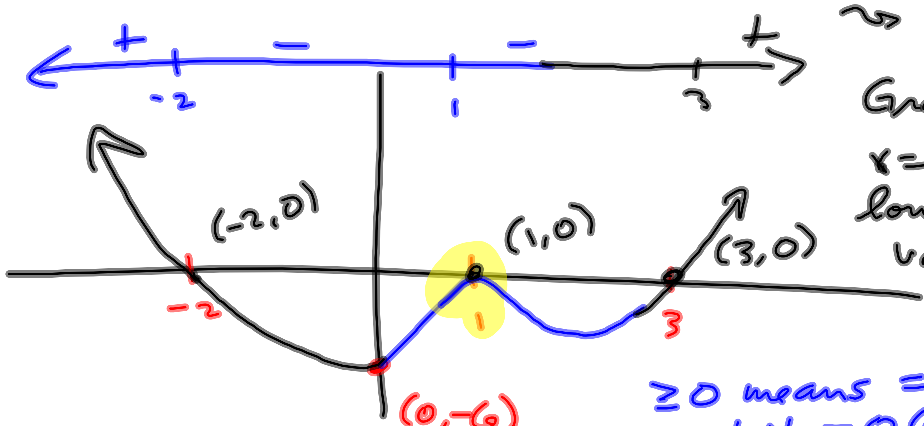

The only spot I'd be likely to use a test value. The rest of the sign pattern is LOGIC. Touch? Cross?

| Test values | Test | Value of f |
|---|---|---|
| $(-\infty,-2)$ | $x=-3$ | |
| $(-2,1)$ | $x=0$ | $(0+2)(0-1)^2(0-3)=-6$ − |
| $(1,3)$ | $x=2$ | |
| $(3,\infty)$ | $x=4$ | $(4+2)(4-1)^2(4-3)=72$ + |

$(x+2)(x-1)^2(x-3) = f(x)$

$f(0) = -6$

$\rightsquigarrow (0,-6)$




Graph using $x=0$ as lone test value

$\geq 0$ means $=$ is OK and it $= 0$ @ $x=1$

$(x+2)(x-1)^2(x-3) \geq 0 \quad (-\infty,-2] \cup \{1\} \cup [3,\infty)$

$(x+2)(x-1)^2(x-3) < 0 \quad (-2,1) \cup (1,3)$

$(x+2)(x-1)^2(x-3) > 0 \quad (-\infty,-2) \cup (3,\infty)$

$(x+2)(x-1)^2(x-3) \leq 0 \quad [-2,1] \cup [1,3]$

$= [-2,3]$

Oct 7-8:46 AM