Teacher Scratch Build a Question

$(x-1)^2(x-(2+i))(x-(2-i))$

Zeros: $x = 1,\ 2+i,\ 2-i$

Multi: $2,\ 1\ \ 1$

$(x-1)^2 = x^2-2x+1$

$(x-(2+i))(x-(2-i))$

$= (x-2-i)(x-2+i)$

$= x^2-2x+ix-2x+4-2i-ix+2i-i^2$

$= x^2-4x+5$

$(x^2-2x+1)(x^2-4x+5)$

$x^4-4x^3+5x^2$

$-2x^3+8x^2-10x$

x^2-4x+5

$f(x) = x^4-6x^3+14x^2-14x+5$

is what we're going to analyze.

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Find all zeros, their multiplicities, and graph it.

$f(x) = x^4 - 6x^3 + 14x^2 - 14x + 5$

$\frac{p}{q} = \pm 1, \pm 5$

We split off a linear factor

$$\begin{array}{r|rrrrr} 1 & 1 & -6 & 14 & -14 & 5 \\ & & 1 & -5 & 9 & -5 \\ \hline 1 & 1 & -5 & 9 & -5 & 0 \\ & & 1 & -4 & 5 & \\ \hline & 1 & -4 & 5 & 0 & \end{array}$$

$f(x) = (x-1)(\underbrace{x^3 - 5x^2 + 9x - 5}_{\text{depressed polynomial.}})$

$x^3 - 5x^2 + 9x - 5 = 0$ is the depressed equation

This gives $f(x) = (x-1)^2(x^2 - 4x + 5)$

and the depressed equation

$x^2 - 4x + 5 = 0$

$x^2 - 4x + 2^2 = -5 + 4$

$(x-2)^2 = -1$

$x - 2 = \pm\sqrt{-1} = \pm i$

$x = 2 \pm i$

Zeros: $x = 1, 2+i, 2-i$

Multiplicities: $m = 2, 1, 1$, respectively

Split into linear factors:

$$(x-1)^2(x-(2+i))(x-(2-i))$$

That's how it factors over the field of complex numbers, $\mathbb{C}$.

Here's how it factors over the field of Real numbers, $\mathbb{R}$:

$$(x-1)^2(x^2-4x+5)$$

is an irreducible quadratic factor over the reals.

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Graph it

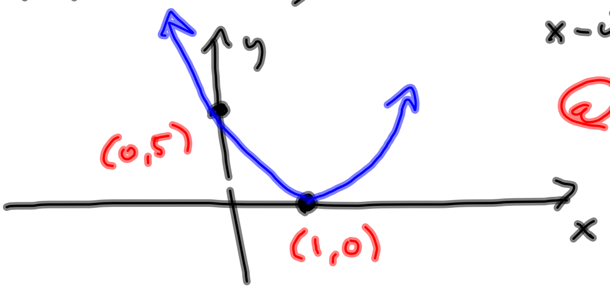
$f(x) = x^4 - 6x^3 + 14x^2 - 14x + 5$

$(x-1)^2(x^2 - 4x + 5)$

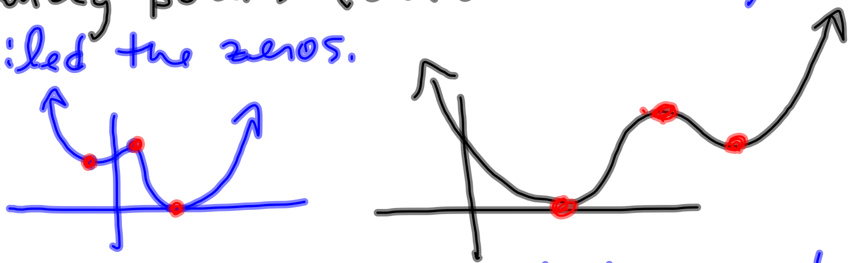
y-int: $(0,5)$

x-int: $(1,0)$

@ $x = 1$, $m = 2$ is even ⟶ graph kisses x-axis.



It might have as many as two more turning points (local extremos), but we've nailed the zeros.



we don't know about those, without technology or calculus. So we don't care.

Notice that f(x) had real coefficients and its nonreal zeros were a conjugate pair: 2+i and 2−i

$$(x-(2+i))(x-(2-i)) = x^2-4x+5$$

Baby version of conjugate pairs theorem:

$$(x-(2+\sqrt{3}))(x-(2-\sqrt{3})) =$$

$$= (x-2-\sqrt{3})(x-2+\sqrt{3})$$

$$= x^2-2x+\sqrt{3}x-2x+4-2\sqrt{3}-\sqrt{3}x+2\sqrt{3}-3$$

$$= x^2-4x+1$$

"Rational" If the coefficients of the polynomial are rational, then irrational zeros occur in "conjugate pairs"

$2\pm\sqrt{3}$

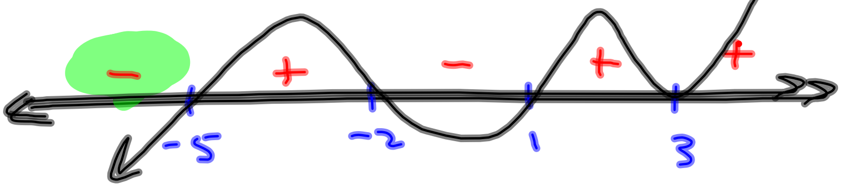
MAIN CONJUGATE PAIRS Theorem:
If the coefficients are REAL, then the NONREAL zeros occur in conjugate pairs

$2\pm i$

$(x-1)(x+2)(x-3)^2(x+5)^1 > 0$

$-5, -2, 1, 3$

Crude Sketch from SIGN PATTERN



Test: $x=4 \implies (4-1)(4+2)(4-3)^2(4+5) > 0$

Analyze the question: want ">0"

" "+"

$(-5,-2) \cup (1,3) \cup (3,\infty)$

New Question!

$(x-1)(x+2)(x-3)^2(x+5) \geq 0$

$[-5,-2] \cup \underbrace{[1,3] \cup [3,\infty)}$

$[-5,-2] \cup [1,\infty)$

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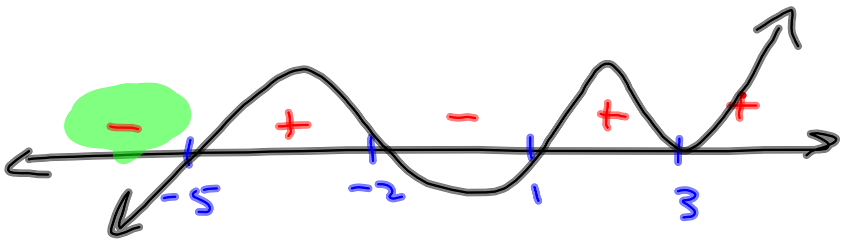


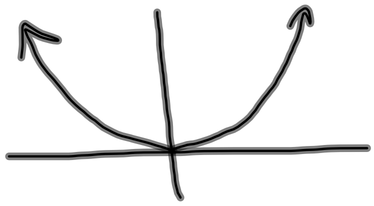
$$(x-1)(x+2)(x-3)^2(x+5) \le 0$$

$$(-\infty,-5] \cup [-2,1] \cup \{3\}$$

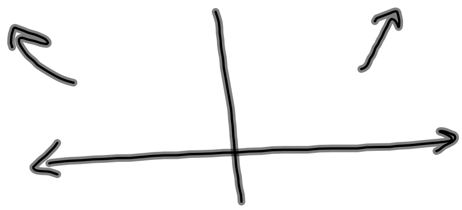
The subtle one.
Where it $= 0$

End Behavior

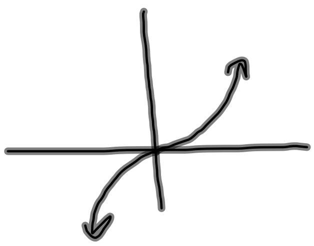
$x^2, x^4, x^6, x^8, \ldots$



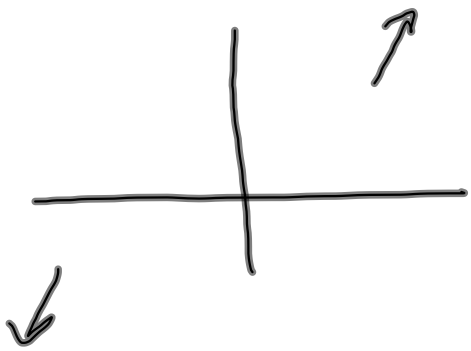
Focus on "end behavior," that is, what $f(x)$ does as $x \to \pm\infty$



$x, x^3, x^5, x^7, \ldots$



as $x \to \infty$,



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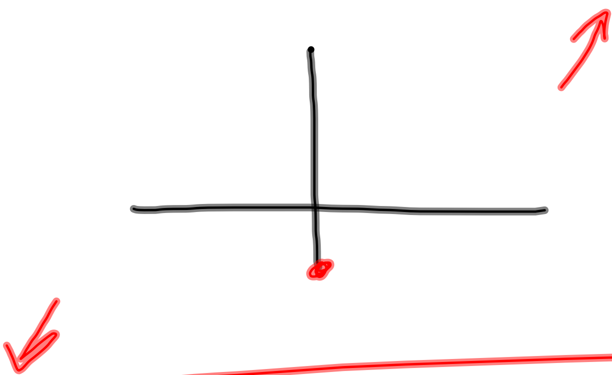
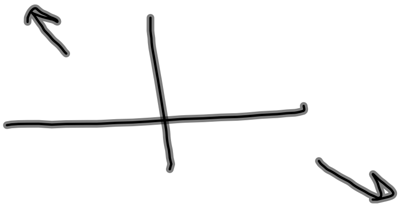
The leading term controls end behavior

$3x^5 - 7x^4 + 379{,}121x^3 - 11$

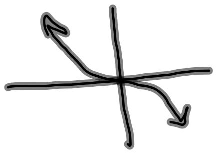


E.B. is controlled by the $3x^5$ term.

$-3x^{17}$ + stuff of lower degree



$-3x^{17}$



Build a polynomial, in factored form that has ...

$x = 1, 2, 3-i$ as zeros.

$(x-1)(x-2)(x-(3-i))$

$x = 1, m=3$ $\quad (x-1)^3(x-2)$

$2, m=1$

Build a polynomial with REAL coefficients that has zeros ...

$x = 1, 2, 3-i$

$(x-1)(x-2)(x-(3-i))(x-(3+i))$

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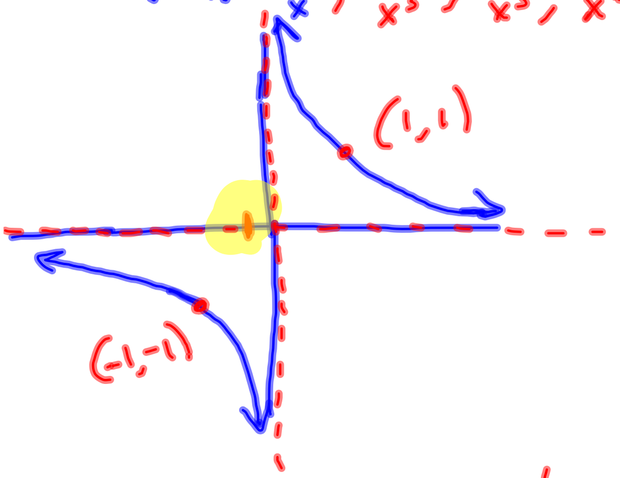
$\frac{1}{x}$, $\frac{1}{x^2}$ Basic Functions

$$\frac{1}{-\frac{1}{1000}} = -1000$$

$f(x) = \frac{1}{x}, \frac{1}{x^3}, \frac{1}{x^5}, \frac{1}{x^7}$



$y=0$ is Horizontal Asymptote.

$x=0$ is Vertical Asymptote

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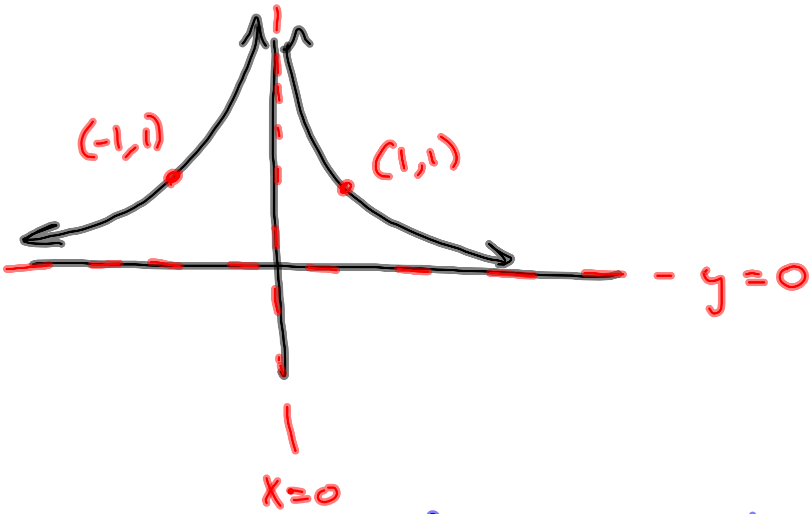
$\frac{1}{x^2}, \frac{1}{x^4}, \frac{1}{x^6}, \frac{1}{x^8}, \ldots$



$\frac{2x-4}{x} = \frac{2(x-2)}{x}$