Today: § 3.2, 3.3

Polynomial Breakdown

Rational Zeros Theorem

Conjugate Pairs Theorem

Divide.

$(4s^4+5s^3-24s^2-5s+20)\div(s^2-2)$

Divisor is degree 2
Long division.

$$\begin{array}{r} -24 \\ +8 \\ \hline -16 \end{array} \qquad \frac{4s^4}{s^2}=4s^2 \qquad \frac{5s^3}{s^2}=5s \qquad -\frac{16s^2}{s^2}=-16$$

$$\begin{array}{rl} & 4s^2+5s-16 \text{ r } 5s-12 \\ s^2-2 & \overline{)\,4s^4+5s^3-24s^2-5s+20} \\ & -(4s^4 \qquad\quad -8s^2) \\ \hline & 5s^3-16s^2-5s+20 \\ & -(5s^3 \qquad\quad -10s) \\ \hline & -16s^2+5s+20 \\ & -(-16s^2 \qquad +32) \\ \hline & 5s-12 \end{array}$$

$$\begin{array}{rl} & 94 \text{ r } 1 \\ 3 & \overline{)\,283} \\ & -270 \\ \hline & 13 \\ & -12 \\ \hline & 1 \end{array}$$

Interpret:
$283=3\cdot 94+1$
OR
$\frac{283}{3}=94+\frac{1}{3}$

Interpret:

$4s^4+5s^3-24s^2-5s+20=(s^2-2)(4s^2+5s-16)+5s-12$

Later:

$$\frac{4s^4+5s^3-24s^2-5s+20}{s^2-2}=4s^2+5s-16+\frac{5s-2}{s^2-2}$$

IMPROPER RATIONAL FUNCTION = Polynomial + PROPER RATIONAL FUNCTION.

Use synthetic division to find the quotient and remainder when the first polynomial is divided by the second.

$9x^3 - 7x + 2, \quad x - \frac{1}{3}$

So, $x - \frac{1}{3}$ is a FACTOR of $9x^3 - 7x + 2$

The quotient is $9x^2 + 3x - 6$ and the remainder is 0.

$$\begin{array}{c|cccc} \frac{1}{3} & 9 & 0 & -7 & 2 \\ & & 3 & 1 & -2 \\ \hline & 9 & 3 & -6 & 0 \\ & x^2 & x & c & r \end{array}$$

$\left(\cancel{9}^{3}\right)\left(\frac{1}{\cancel{3}_{1}}\right)$

$(3)\left(\frac{1}{3}\right)$

$(-6)\left(\frac{1}{3}\right) = -2$

Determine whether the binomial $x-2$ is a factor of the polynomial $x^3+8x^2+4x-48$.

If it is a factor, $\boxed{\text{then factor the polynomial completely}}$.

The breakdown :

2	1	8	4	-48
		2	20	48
	1	10	24	0

Yes. It's a factor

This says $x^3+8x^2+4x-48=(x-2)(x^2+10x+24)$

24	10
6·4	6+4 = 10 sweet.
8·3	11
12·2	14

$1x^2+10x+24$

$= x^2+6x+4x+24$

$= x(x+6)+4(x+6)$

$= (x+6)(x+4)$

Final Answer: $\boxed{(x-2)(x+6)(x+4)}$

Find all possible rational zeros for the polynomial function.

$P(x) = 22x^3 - 36x^2 + 46x - 21$

We need the rational zeros theorem

Scratch WRITE MUCH, THINK LITTLE

$(2x+3)(3x-10) = 6x^2 - 20x + 9x - 30$

$= 6x^2 - 11x - 30$

$-180 \mid -11$

$(-20)(9) \mid -20+9$

$= 6x^2 - 20x + 9x - 30$

$= 2x(3x-10) + 3(3x-10)$

$= (3x-10)(2x+3)$

2 ⌊180
2 ⌊90
3 ⌊45
3 ⌊15
5

$a = 6,\ b = -11,\ c = -30$

$b^2 - 4ac = 121 - 4(6)(-30)$

$= 121 + 720 = 841$

$x = \dfrac{11 \pm \sqrt{841}}{2(6)} = \dfrac{11 \pm 29}{12}$

$\dfrac{40}{12} = \dfrac{20}{6} = \dfrac{10}{3}$

$\dfrac{-18}{12} = -\dfrac{9}{6} = -\dfrac{3}{2}$

This says

$6x^2 - 11x - 30 = 6\left(x - \frac{10}{3}\right)\left(x + \frac{3}{2}\right)$

$x - \frac{10}{3}$

$= 3 \cdot 2 \left(x - \frac{10}{3}\right)\left(x + \frac{3}{2}\right)$

$= 3\left(x - \frac{10}{3}\right)(2)\left(x + \frac{3}{2}\right)$

$= (3x - 10)(2x + 3)$

This is the quadratic formula "CHEAT" for factoring.

Oct 3-7:45 AM

$6x^2 - 11x - 30 = 6\left(x - \frac{10}{3}\right)\left(x + \frac{3}{2}\right)$

$\frac{10}{3}$ is a zero. Notice 10 is a factor of 30
and 3 is a factor of 6

$-\frac{3}{2}$ is a zero Notice 3 is a factor of 30
and 2 is a factor of 6.

This is the Rational Zeros Theorem at work.

Find all possible rational zeros for the polynomial function.

$P(x) = 22x^3 - 36x^2 + 46x - 21$

$$P(x) = a_n x^n + a_{n-1} x^{n-1} + a_{n-2} x^{n-2} + \dots + a_1 x + a_0$$

$a_n = 22$

$a_0 = -21$

$$2 \,\underline{|22}\quad 11 \qquad 3\,\underline{|21}\quad 7$$

$$\frac{p}{q} = \pm\frac{21}{1},\ \pm\frac{21}{2},\ \pm\frac{21}{11},\ \pm\frac{21}{22},$$

$$\pm\frac{7}{1},\ \pm\frac{7}{2},\ \pm\frac{7}{11},\ \pm\frac{7}{22},$$

$$\pm\frac{3}{1},\ \pm\frac{3}{2},\ \pm\frac{3}{11},\ \pm\frac{3}{22},$$

$$\pm\frac{1}{1},\ \pm\frac{1}{2},\ \pm\frac{1}{11},\ \pm\frac{1}{22}$$

These are all the POSSIBLE rational zeros.

Graphing Calculator is GREAT for making good guesses.

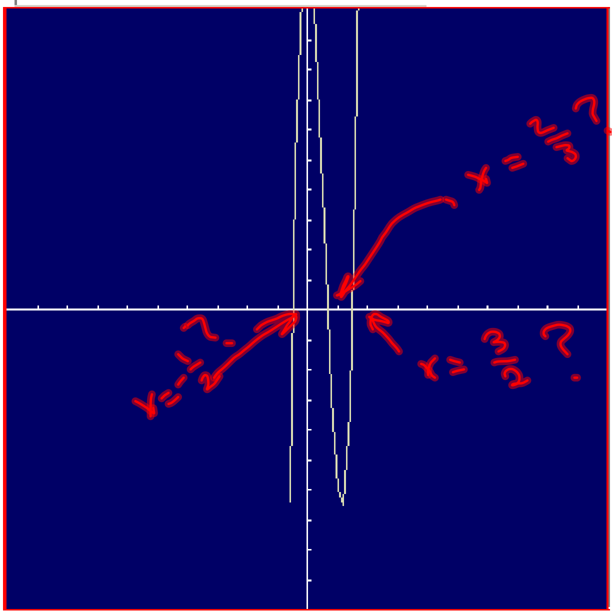
Find all of the real and imaginary zeros for the polynomial function.

$f(x) = 24x^3 - 40x^2 - 2x + 12 = 2(12x^3 - 20x^2 - x + 6)$

Free Online Graphers are handy.

$2\overline{)12}$, $2\overline{)6}$, 3 $\qquad$ $2\overline{)6}$, 3

http://www.coolmath.com/graphit/



Rational zeros

$\pm 1, \pm \frac{1}{2}, \pm \frac{1}{3}, \pm \frac{1}{4}, \pm \frac{1}{6}, \pm \frac{1}{12}$

$\pm 2, \pm \frac{2}{3}, \pm 3, \pm \frac{3}{2}, \pm \frac{3}{4}, \pm 6$

$-\frac{1}{2}$	12	−20	−1	6
		−6	13	−6
	12	−26	12	0

Yes!

http://www.coolmath.com/graphit/

$f(x) = 2(x + \frac{1}{2})(12x^2 - 26x + 12)$

$= 4(x + \frac{1}{2})(6x^2 - 13x + 6)$ Slay the quadratic

Guess: $= 4(x + \frac{1}{2})(6)(x - \frac{2}{3})(x - \frac{3}{2})$ based on the picture.

Check:

$a = 6, b = -13, c = 6$

$b^2 - 4ac = (-13)^2 - 4(6)(6)$

$= 169 - 144 = 25$

$x = \frac{13 \pm 5}{2(6)} = \frac{13 \pm 5}{12}$ → $\frac{18}{12} = \frac{3}{2}$, $\frac{8}{12} = \frac{2}{3}$

Oct 3-7:44 AM

$f(x) = 1x^4 - 5x^3 + 15x^2 - 5x - 26$

$\frac{p}{q} = \pm 1, \pm 2, \pm 13, \pm 26$

$$\begin{array}{r|rrrrr} 1 & 1 & -5 & 15 & -5 & -26 \\ & & 1 & -4 & 11 & \\ \hline & 1 & -4 & 11 & 6 & \end{array}$$

Nope

$$\begin{array}{r|rrrrr} -1 & 1 & -5 & 15 & -5 & -26 \\ & & -1 & 6 & -21 & 26 \\ \hline -1 & 1 & -6 & 21 & -26 & 0 \\ & & -1 & 7 & \text{No} & \\ \hline & 1 & -7 & 28 & & \end{array}$$

$f(x) = (x+1)(x^3 - 6x^2 + 2x - 26)$

$$\begin{array}{r|rrrr} 2 & 1 & -6 & 21 & -26 \\ & & 2 & -8 & 26 \\ \hline & 1 & -4 & 13 & 0 \end{array}$$

Sweet!

Here's where we are

$f(x) = (x+1)(x-2)(x^2 - 4x + 13)$

$x^2 - 4x + 13 = 0$

$x^2 - 4x + 2^2 = -13 + 4$

$(x-2)^2 = -9$

$x - 2 = \pm\sqrt{-9} = \pm 3i$

$x = 2 \pm 3i$

Factored Form:

$f(x) = (x+1)(x-2)(x-(2+3i))(x-(2-3i))$

Next Time Conjugate Pairs Theorem.