Find the vertex of the graph of the quadratic function.

MN
3.1 #11
Text #25

$y = -\frac{1}{2}x^2 - \frac{1}{4}x$

Cheat!

$(h,k) = \left(-\frac{b}{2a}, f\left(-\frac{b}{2a}\right)\right)$

$a = -\frac{1}{2}, b = -\frac{1}{4}, c = 0$

$2\left(-\frac{1}{2}\right) = -\frac{2}{1}\cdot\frac{1}{2} = \frac{-2\cdot 1}{1\cdot 2} = -1$

$-\frac{b}{2a} = -\frac{-\frac{1}{4}}{2\left(-\frac{1}{2}\right)} = -\frac{-\frac{1}{4}}{-1} = -\frac{1}{4} \Longrightarrow$ Lost the negative, dipstick.

$(h,k) = \left(\frac{1}{4}, -\frac{3}{32}\right)$

$f\left(-\frac{b}{2a}\right) = f\left(\frac{1}{4}\right) = -\frac{1}{2}\left(\frac{1}{4}\right)^2 - \frac{1}{4}\left(\frac{1}{4}\right)$ → Nope → Nope → OK

$= -\frac{1}{2}\cdot\frac{1^2}{4^2} - \frac{1\cdot 1}{4\cdot 4}$ → Nope

Should be $-\frac{1}{32} + \frac{1}{16} = \frac{-1+2}{32} = \frac{1}{32}$ ✓

$= -\frac{1}{2}\cdot\frac{1}{16} - \frac{1}{16}$

$= -\frac{1}{32} - \frac{1}{16}\cdot\frac{2}{2} = \frac{-1-2}{32} = -\frac{3}{32}$

$-\frac{1}{2}x^2 - \frac{1}{4}x =$

$-\frac{1}{2}\left(x^2 + \frac{1}{2}x \qquad\qquad \right)$ Factor out coefficient of x^2.

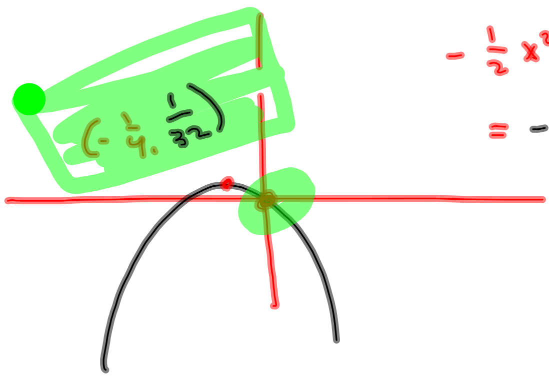
$= -\frac{1}{2}\left(x^2 + \frac{1}{2}x + \left(\frac{1}{4}\right)^2 - \left(\frac{1}{4}\right)^2\right)$ complete square inside

$= -\frac{1}{2}\left(x^2 + \frac{1}{2}x + \left(\frac{1}{4}\right)^2\right) - \left(\frac{1}{2}\right)\left(-\frac{1}{16}\right)$ Bring the $-\left(\frac{1}{4}\right)^2$ out

$= -\frac{1}{2}\left(x + \frac{1}{4}\right)^2 + \frac{1}{32}$ write the result

This says $(h,k) = \boxed{\left(-\frac{1}{4}, \frac{1}{32}\right)}$ Interpret.

Doesn't agree with $-\frac{b}{2a}$ cheat.



$-\frac{1}{2}x^2 - \frac{1}{4}x$

$= -x\left(\frac{1}{2}x + \frac{1}{4}\right)$

SET $= 0$

$-x = 0$ OR $\frac{1}{2}x + \frac{1}{4} = 0$

$x = 0$

Write the quadratic function in the form $y = a(x-h)^2 + k$ and ~~sketch~~ its graph.

$y = -2x^2 + 7x - 1$

ST
3.1 #8
Text #19

$(a+b)^2 = a^2 + 2ab + b^2$

$x \qquad x$

$= -2\left(x^2 - \frac{7}{2}x \qquad\qquad \right) - 1$

$\frac{7x}{-2} = -\frac{7}{2}x$

$= -2\left(x^2 - \frac{7}{2}x + \left(\frac{7}{4}\right)^2 - \left(\frac{7}{4}\right)^2\right) - 1$

$\frac{\frac{7}{2}}{\frac{2}{1}} = \frac{7}{2} \cdot \frac{1}{2}$

$= -2\left(x^2 - \frac{7}{2}x + \left(\frac{7}{4}\right)^2\right) - 2\left(-\frac{7}{4}\right)^2 - 1$

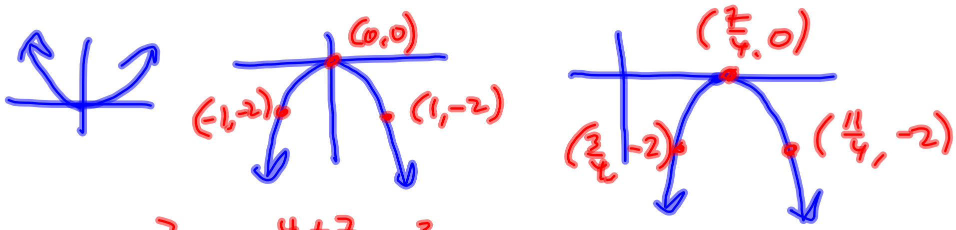
$= -2\left(x - \frac{7}{4}\right)^2 - \frac{57}{8}$

$-2\left(\frac{49}{16}\right) - 1$

$= -\frac{49}{8} - \frac{8}{8} = -\frac{57}{8}$

$(h,k) = \left(\frac{7}{4}, -\frac{57}{8}\right)$

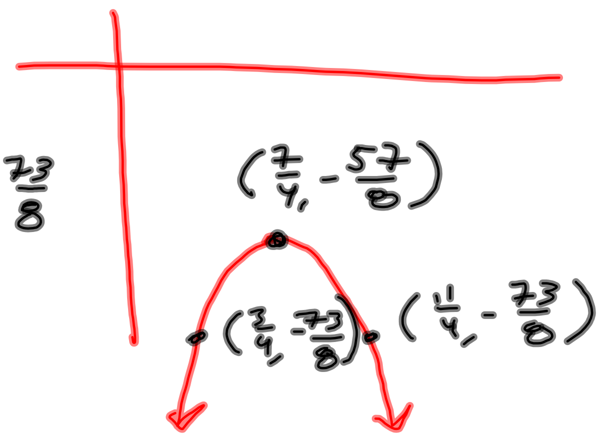
$x^2 \longrightarrow -2x^2 \longrightarrow -2\left(x - \frac{7}{4}\right)^2 \longrightarrow -2\left(x - \frac{7}{4}\right)^2 - \frac{57}{8}$



$-1 + \frac{7}{4} = \frac{-4+7}{4} = \frac{3}{4}$

$1 + \frac{7}{4} = \frac{11}{4}$

$-2 - \frac{57}{8} = \frac{-16-57}{8} = -\frac{73}{8}$



Find the range of the quadratic function and the maximum and minimum value of the function. Identify the intervals over which the function is increasing or decreasing.

I always have to check the book on the precise definition of increasing/decreasing. Some books include the endpoints. Some don't. I prefer not, but I always have to check, because the MyLab expects specific things, based on what the durn book says.

$y = 2x^2 + 3x + 1$

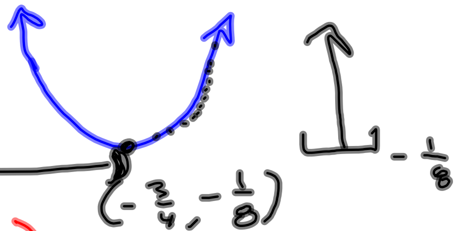
ST
3.1 #15
Text #37

$= 2\left(x^2 + \frac{3}{2}x + \left(\frac{3}{4}\right)^2\right) + 1 - 2\left(\frac{9}{16}\right)$

$= 2\left(x + \frac{3}{4}\right)^2 - \frac{1}{8}$

$1 - \frac{9}{8} = \frac{-1}{8}$

$\left(-\frac{3}{4}, -\frac{1}{8}\right)$



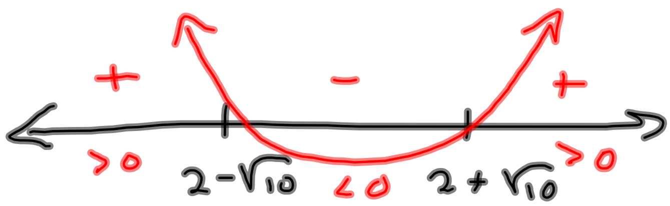
Decreasing: $\left(-\infty, -\frac{3}{4}\right]$ $)$

Increasing $\left[-\frac{3}{4}, \infty\right)$

Most Calculus books won't include the end-points

Range $= \{ y \mid y = f(x) \text{ for some } x \in \mathcal{D} \}$

$= \left[-\frac{1}{8}, \infty\right)$



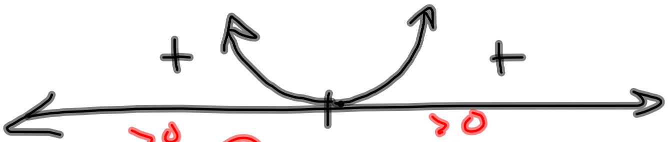
Test Value Method (We just did it by knowing what b^2-4b-6 looks like.) If in doubt about the picture, break it down, using the zeros we found.

	Test		
$(-\infty, 2-\sqrt{10})$	-2	$(-2)^2-4(-2)-6=6>0$	No
$(2-\sqrt{10}, 2+\sqrt{10})$	0	$0^2-4(0)-6=-6<0$	Yes
$(2+\sqrt{10}, \infty)$	$x=6$	$6^2-4(6)-6=6>0$	No

$$4x^2 + 28x + 49 > 0$$

$$\Rightarrow (2x)^2 + 2\cdot 2\cdot 7x + 7^2 > 0$$

$$\Rightarrow (2x+7)^2 > 0$$



@ $-\frac{7}{2}$, it $= 0$

$$(-\infty, -\tfrac{7}{2}) \cup (-\tfrac{7}{2}, \infty) = \{x \mid x \neq -\tfrac{7}{2}\}$$

$4x^2+28x+49 \geq 0$

$(-\infty, \infty)$

$4x^2+28x+49 < 0$

$\varnothing$

$4x^2+28x+49 \leq 0$

$\{-\frac{7}{2}\}$

Synthetic Division

$$(3x^3 - 4x^2 + 5x - 6) \div (x-5)$$

$$\begin{array}{r|rrrr} 5 & 3 & -4 & 5 & -6 \\ & & 15 & 55 & 300 \\ \hline & 3 & 11 & 60 & 294 \\ & x^2 & x & c & r \end{array}$$

$$f(x) = 3x^3 - 4x^2 + 5x - 6 = (x-5)(3x^2 + 11x + 60) + 294$$

$$f(5) = 294$$

Build a question:

$(2+\sqrt{2})(2-\sqrt{2})$
$= 4-2$
$= 2$

$(x-2)(x+1)(x-(2+\sqrt{2}))(x-(2-\sqrt{2}))$

$(x-2)(x+1)(x^2-2x+\sqrt{2}x-2x-\sqrt{2}x+2)$

$= (x-2)(x+1)(x^2-4x+2)$

$= (x-2)(x^3-4x^2+2x+x^2-4x+2)$

$= (x-2)(x^3-3x^2-2x+2)$

$= x^4-3x^3-2x^2+2x-2x^3+6x^2+4x-4$

$= x^4-5x^3+4x^2+6x-4$

Is $x=2$ a zero of
x^4-5x^3+6x-4 ?

2	1	−5	4	6	−4
		2	−6	−4	4
	1	−3	−2	2	0

Yes! S.T.

This says
$x^4-5x^3+4x^2+6x-4$
$= (x-2)(x^3-3x^2-2x+2)$

$x=2$ IS a zero