3.1 Quadratic Funcs

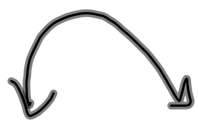
3.2 Polynomial Theory Division

$a>0$

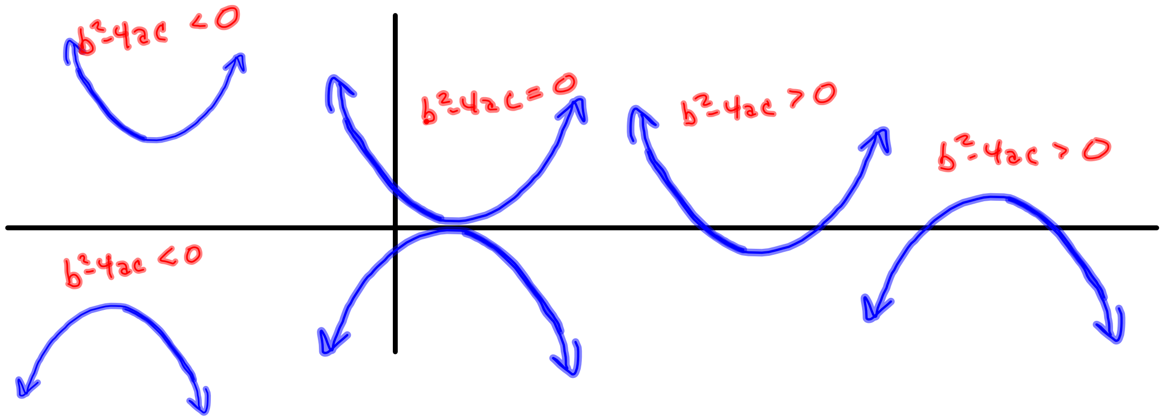
$3x^2+2x-7$

$a<0$

$-2x^2+5x+1123\pi$



$f(x)=ax^2+bx+c$



$f(x) = x^2 - 3x + 2$

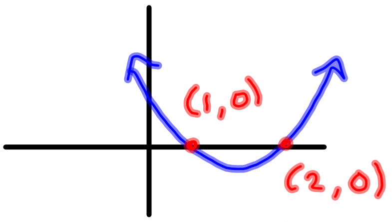
zeros:

$f(x) = 0$

$x^2 - 3x + 2 = 0$

$(x-2)(x-1) = 0$

$x = 2$ or $x = 1$




Rough sketch Just from zeros & basic shape

$x^2 - 3x + 2 = 0$

$x^2 - 3x + \left(\frac{3}{2}\right)^2 = -2 + \frac{9}{4}$

$\left(x - \frac{3}{2}\right)^2 = \frac{1}{4}$

$x - \frac{3}{2} = \pm \frac{1}{2}$

$x = \frac{3}{2} \pm \frac{1}{2}$

$\frac{4}{2} = 2$ $\quad$ $\frac{2}{2} = 1$

$x = 2$ OR $x = 1$

$a = 1, b = -3, c = 2$

$b^2 - 4ac =$

$(-3)^2 - 4(1)(2)$

$= 9 - 8 = 1$

$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$= \frac{3 \pm \sqrt{1}}{2(1)} = \frac{3 \pm 1}{2}$

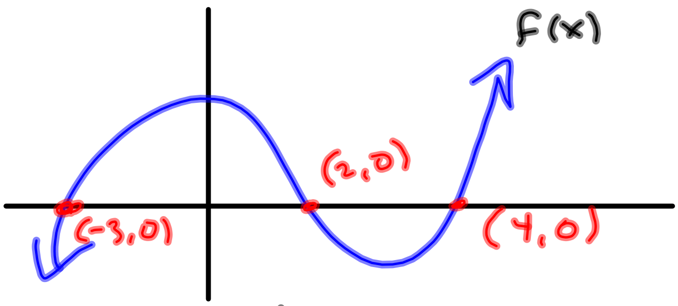
$x = 2$ OR $x = 1$

Solve $f(x) = 0$

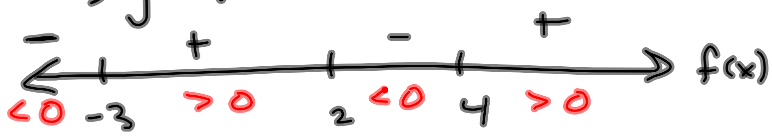
$\Rightarrow x \in \{-3, 2, 0\}$

Solve $f(x) > 0$

$\Rightarrow x \in (-3, 2) \cup (4, \infty)$

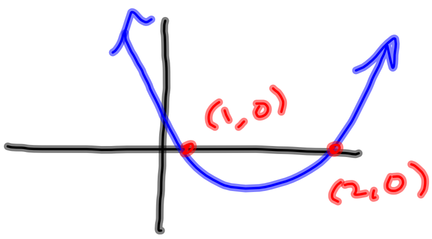


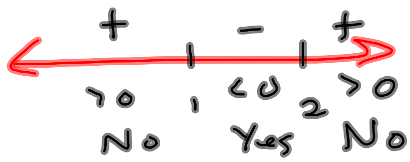

Sign pattern:



$f(x) = x^2 - 3x + 2 \leq 0$

critical: $x = 1, 2$



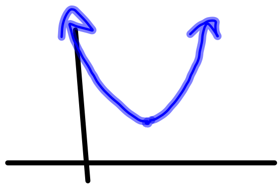

$x \in [1, 2]$

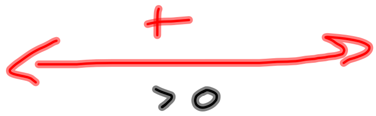
$f(x) = x^2 - 3x + 9 \leq 0$

$b^2 - 4ac = 0$

$(-3)^2 - 4(1)(9)$

$= 9 - 36 = -27 < 0$






$f(x) \leq 0$

Nevah!

$\emptyset$

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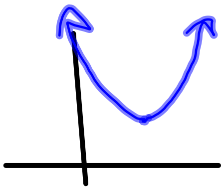
$f(x) = x^2 - 3x + 9$



2, 3, 5, 7, 11, 13

Solve $x^2 - 3x + 9 \geq 0$
w/o picture!

$b^2 - 4ac = \dots = \underline{-27}$

No real zeros

opens up

$\longrightarrow x$

$+$

$\longleftrightarrow f(x)$

$> 0$

$x \in (-\infty, \infty) = \mathbb{R}$

$x^2 - 3x - 9 \geq 0$

$a = 1, b = -3, c = -9$

$b^2 - 4ac = (-3)^2 - 4(1)(-9)$

$= 9 + 36 = 45$

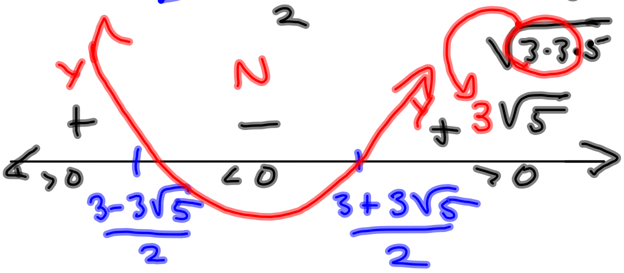
$x = \dfrac{3 \pm \sqrt{45}}{2(1)}$

$= \dfrac{3 \pm 3\sqrt{5}}{2}$

3 | 45
3 | 15
5

$\sqrt{3 \cdot 3 \cdot 5}$

$3\sqrt{5}$




$<$ want $\geq 0$

$\left(-\infty, \dfrac{3-3\sqrt{5}}{2}\right] \cup \left[\dfrac{3+3\sqrt{5}}{2}, \infty\right)$

Division of Polynomials $x^1-1$ is linear

$(x^3+7x^2-5x+11) \div (x-1)$

$\frac{x^3}{x}=x^2$

$$
\begin{array}{r|l}
 & x^2 + 8x + 3 \ r\ 14 \\
\hline
x-1 & x^3+7x^2-5x+11 \\
 & -(x^3-x^2) \\
\hline
 & 8x^2-5x+11 \\
 & -(8x^2-8x) \\
\hline
 & 3x+11 \\
 & -(3x-3) \\
\hline
 & 14
\end{array}
$$

This says: (This is called the Division Algorithm)

$x^3+7x^2-5x+11 = (x-1)(x^2+8x+3)+14$

OR (Dividing by $x-1$):

$$\frac{x^3+7x^2-5x+11}{x-1} = x^2+8x+3+\frac{14}{x-1}$$

$f(x) = x^3 + 7x^2 - 5x + 11 = (x-1)(x^2+8x+3) + 14$

what's $f(1)$? $f(1) = 14$

REMAINDER THEOREM

$f(c)$ is the remainder when $f(x)$ is divided by $x - c$.

To find $f(1)$, I divided by $x - 1$.

FACTOR THEOREM is the Remainder Theorem, when the remainder happens to be zero.

$f(x) = x^2 - 3x + 2 = (x-1)(x-2)$

$f(2) = 0$ because $(x-2)$ is a factor.

Is $x-1$ a factor of $x^3+1$?

Let $f(x) = x^3 + 1$

$f(1) = 1^3 + 1 = 2 \neq 0$

So $x-1$ is Not a factor?

Is $x+1$ a factor? Yes.

$f(-1) = (-1)^3 + 1 = -1 + 1 = 0 = 0$

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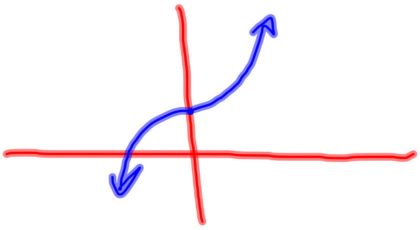
Factor $f(x) = x^3 + 1 = (x+1)(\underline{x^2 - x + 1})$

$$
\begin{array}{r|l}
 & x^2 - x + 1 \quad r\, 0 \\
\hline
x+1 & x^3 + 0x^2 + 0x + 1 \\
 & -(x^3 + x^2) \\
\hline
 & -x^2 + 0x + 1 \\
 & -(-x^2 - x) \\
\hline
 & x + 1 \\
 & -(x+1) \\
\hline
 & 0
\end{array}
$$



Does $x^2 - x + 1$ break down? Not according to the graph of $x^3 + 1$. Algebraically:

$x^2 - x + 1$

$a = 1, b = -1, c = 1$

$b^2 - 4ac = (-1)^2 - 4(1)(1)$

$= 1 - 4$

$= -3 < 0$

No real zeros.

No factor.

$f(x) = x^2 - 3x - 9$

$\frac{3 \pm 3\sqrt{5}}{2}$ are its zeros

$\therefore \quad f(x) = \left(x - \left(\frac{3-3\sqrt{5}}{2}\right)\right)\left(x - \left(\frac{3+3\sqrt{5}}{2}\right)\right)$

is how it factors.

Synthetic Division

$\left(x - \left(\frac{3-3\sqrt{5}}{2}\right)\right)\left(x - \left(\frac{3+3\sqrt{5}}{2}\right)\right)$

$= x^2 - x\left(\frac{3+3\sqrt{5}}{2}\right) - \left(\frac{3-3\sqrt{5}}{2}\right)x + \left(\frac{3-3\sqrt{5}}{2}\right)\left(\frac{3+3\sqrt{5}}{2}\right)$

$= x^2 - \frac{3x + 3\sqrt{5}x}{2} - \frac{3x - 3\sqrt{5}x}{2} + \frac{(3-3\sqrt{5})(3+3\sqrt{5})}{(2)(2)}$

$= x^2 - \frac{3x}{2} - \frac{3\sqrt{5}x}{2} - \frac{3x}{2} + \frac{3\sqrt{5}x}{2} + \frac{9 + 9\sqrt{5} - 9\sqrt{5} - 9 \cdot 5}{4}$

$= x^2 - \frac{6x}{2} + \frac{9 - 45}{4} = x^2 - 3x - \frac{36}{4} = x^2 - 3x - 9$