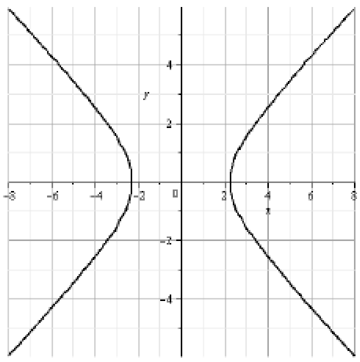
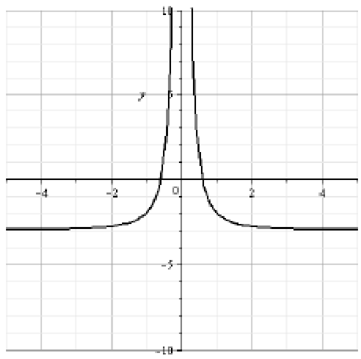
2. (10 pts) For each of the following graphs, determine if the relation is a function. If it is a function, state whether or not it is 1-to-1.



Is it a function? No

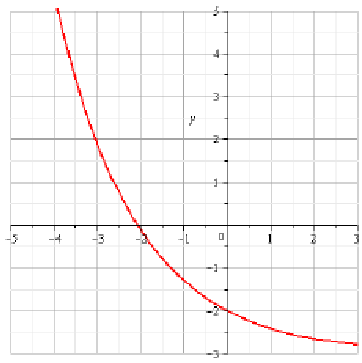
If it is a function, is it 1-to-1?



Is it a function? Yes

If it is a function, is it 1-to-1?

No



Is it a function? Yes

If it is a function, is it 1-to-1?

Yes

3. (5 pts) Determine whether or not $|y+3|-2x=5$ defines y as a function of x. If it does not, show/explain why not. (Solve for y and look at how many solutions you get.)

$|y+3|-2x=5$

$|y+3|=2x+5$

$\boxed{\text{NO}}$

$y+3=2x+5$

$y=2x+2$

OR

$y+3=-(2x+5)$

$y+3=-2x-5$

$y=-2x-8$

Two outcomes for y.

Let $x=1$

$y=2(1)+2=4$

Let $x=1$

$y=-2(1)-8$

$=-10$

So $(1,4)$ and $(1,-10)$ are both pairs in the relation.

4. (10 pts) Let $f(x) = x^2 + 3$. Simplify the difference quotient $\dfrac{f(x+h) - f(x)}{h}$.

$f(x) = x^2 + 3$

$$\frac{f(x+h) - f(x)}{h} = \frac{(x+h)^2 + 3 - (x^2+3)}{h}$$

$$= \frac{x^2 + 2xh + h^2 + 3 - x^2 - 3}{h} = \frac{2xh + h^2}{h}$$

$$= \frac{\cancel{h}(2x+h)}{\cancel{h}} = 2x + h$$

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5. Let $f(x) = \dfrac{x-2}{x-5}$ and $g(x) = \sqrt{x-2}$.

a. (5 pts) What is the domain of f?

$\mathcal{D} = \{x \mid x \neq 5\} = (-\infty, 5) \cup (5, \infty)$

b. (5 pts) What is the domain of g?

Need $x-2 \geq 0 \Rightarrow x \geq 2$ ∴ $\mathcal{D} = \{x \mid x \geq 2\}$
$= [2, \infty)$

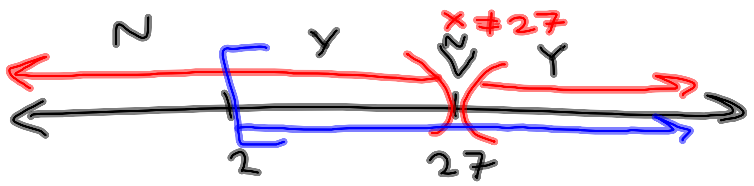
c. (5 pts) Find $(f \circ g)(x)$. (Do not simplify.)

$\boxed{f(g(x))} = \dfrac{g(x)-2}{g(x)-5} = \dfrac{\sqrt{x-2} - 2}{\sqrt{x-2} - 5}$

d. (5 pts) What is the domain of $(f \circ g)(x)$?

$\mathcal{D}(f \circ g) = \{x \mid x \in \mathcal{D}(g) \text{ and } g(x) \in \mathcal{D}(f)\}$
$= \{x \mid x \geq 2 \text{ and } \sqrt{x-2} \neq 5\}$

$\sqrt{x-2} = 5$
$x-2 = 25$
$x = 27$



$\{x \mid x \geq 2 \text{ and } x \neq 27\} = [2, 27) \cup (27, \infty)$

5. Let $f(x) = \dfrac{x-2}{x-5}$ and $g(x) = \sqrt{x-2}$.

e. Determine each of the following functions (without simplifying) and state the domain of each in *interval notation*.

i. (5 pts) $(f+g)(x) = \dfrac{x-2}{x-5} + \sqrt{x-2}$

$\mathcal{D}(f+g) = \mathcal{D}(f) \cap \mathcal{D}(g) = \{x \mid x \neq 5 \text{ AND } x \geq 2\}$

$= ((-\infty, 5) \cup (5, \infty)) \cap [2, \infty)$

$= \boxed{[2, 5) \cup (5, \infty)}$

ii. (5 pts) $\left(\dfrac{g}{f}\right)(x) = \dfrac{\sqrt{x-2}}{\left(\dfrac{x-2}{x-5}\right)} = \mathcal{D}(f) \cap \mathcal{D}(g) \cap \{x \mid f(x) \neq 0\}$

$\dfrac{x-2}{x-5} = 0$

$x - 2 = 0$

$x = 2$

$= \{x \mid x \neq 5 \text{ AND } x \geq 2 \text{ AND } x \neq 2\}$

$= \{x \mid x \neq 5 \text{ AND } x > 2\}$

$= (2, 5) \cup (5, \infty)$

6. (5 pts) Answer *one* of the following:

a. Show that $f(x) = \dfrac{x-1}{x+2}$ is 1-to-1, algebraically.

b. Let $f(x) = \dfrac{x-1}{x+2}$. Find $f^{-1}(x)$.

a. Assume $f(x_1) = f(x_2)$

solve for x_1

$$\frac{x_1-1}{x_1+2} = \frac{x_2-1}{x_2+2}$$

$$(x_2+2)(x_1-1) = (x_2-1)(x_1+2)$$

$$x_2x_1 - x_2 + 2x_1 - 2 = x_2x_1 + 2x_2 - x_1 - 2$$

$$-x_2 + 2x_1 = 2x_2 - x_1$$

$$2x_1 + x_1 = 2x_2 + x_2$$

$$3x_1 = 3x_2$$

$$x_1 = x_2$$

Solve for $y = f^{-1}(x)$

b. $$x = \frac{y-1}{y+2}$$

$$x(y+2) = y-1$$

$$xy + 2x = y - 1$$

$$xy - y = -2x - 1$$

$$y(x-1) = -2x-1$$

$$y = \boxed{\frac{-2x-1}{x-1} = f^{-1}(x)}$$

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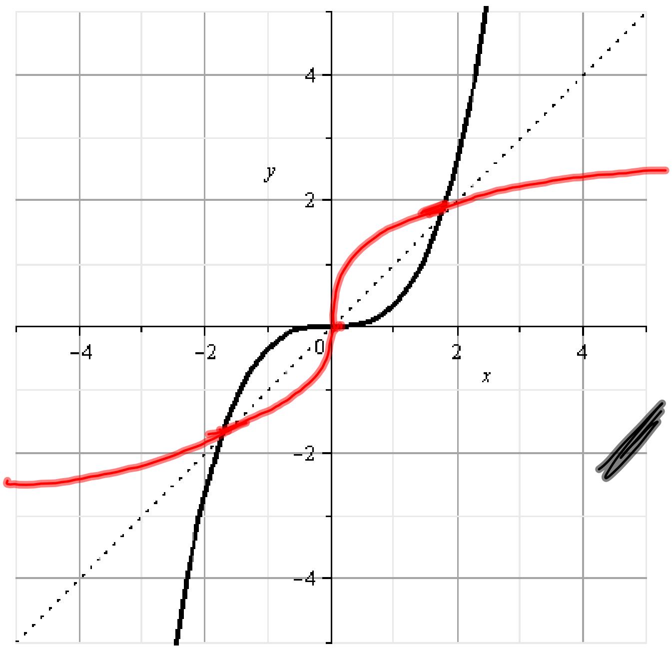
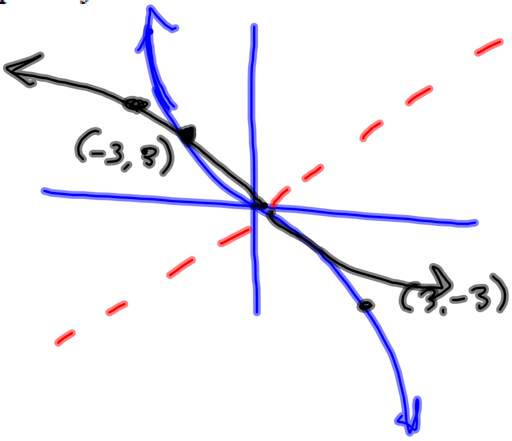
7. (5 pts) The graph of f is given. Sketch the graph of f^{-1}.





8. (5 pts) If f varies jointly as m_1 and m_2 and inversely with the square of r, write the equation describing this relationship.

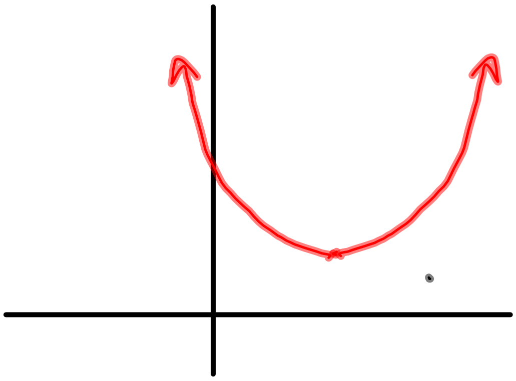
$$f = k\frac{m_1 m_2}{r^2}$$

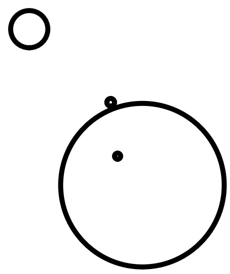
$$F = G\frac{m_1 m_2}{r^2}$$ Force of Gravity between planets.

9. Graph each of the following functions using techniques of shifting, compressing, stretching, and/or reflecting. Start with the graph of the basic function and show all stages in separate sketches. Track 3 key points through the transformations.

a. (5 pts) $h(x) = 2(x-3)^2 + 3$

$x^2 \longrightarrow (x-3)^2 \longrightarrow 2(x-3)^2 \longrightarrow 2(x-3)^2+3$





b. (5 pts) $g(x)=\sqrt{3-x}+5$ (~~Hint: $3-x=-x+3$ is one way.~~ $3-x=-(x-3)$ is another.)

$$=\sqrt{-(x-3)}+5$$

$$\sqrt{x} \longrightarrow \sqrt{-x} \longrightarrow \sqrt{-(x-3)}$$

$$\longrightarrow \sqrt{-(x-3)}+5$$

$$\cancel{\sqrt{3-x}+5=\sqrt{-x+3}+5}$$

$$\boxed{\sqrt{-x} \longrightarrow \sqrt{-x+3} \longrightarrow \sqrt{-x+3}+5}$$

No!!!!!!

$$\sqrt{x} \longrightarrow \sqrt{x+3} \longrightarrow \sqrt{-x+3}$$

$$\longrightarrow \sqrt{-x+3}+5$$ is OK.

$g(x) = -3\sqrt{-2x+4} - 3 = -3\sqrt{-2(x-2)} - 3$

$f(x) = \sqrt{x}$ $\quad = \boxed{-3f(-2(x-2)) - 3}$

$h(x) = -3(-2x+4)^2 - 3 = -3(-2(x-2))^2 - 3$

$f(x) = x^2$ $\quad = -3f(-2(x-2)) - 3$

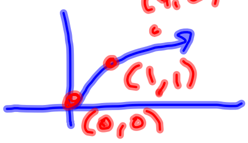
$k(x) = -3|-2x+4|^2 - 3 = -3|-2(x-2)| - 3$

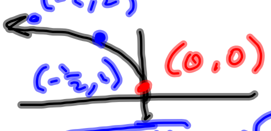
$f(x) = |x|$ $\quad = -3f(-2(x-2)) - 3$

Let's do $g(x)$!

$f(x) = \sqrt{x} \longrightarrow f(-2x) = \sqrt{-2x} \longrightarrow$

$(1, 1) \longrightarrow (-\frac{1}{2}, 1)$



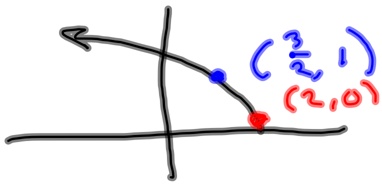


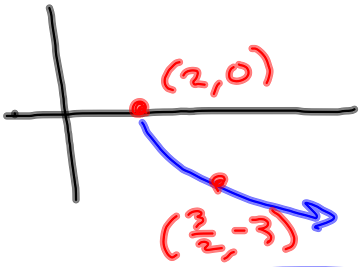
$\sqrt{-2(-\frac{1}{2})} = \sqrt{1} = 1$

$\sqrt{-2(x-2)}$

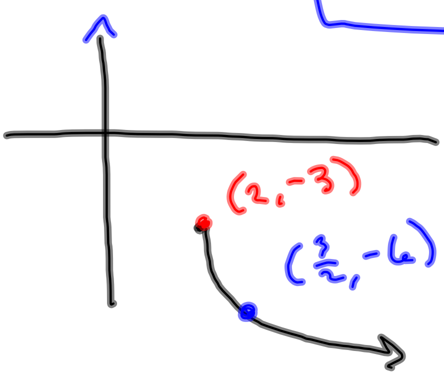
$f(-2(x-2)) \longrightarrow -3f(-2(x-2)) = -3\sqrt{-2(x-2)}$

$(\frac{3}{2}, 1)$ $\quad (\frac{3}{2}, -3)$





$-3f(-2(x-2)) - 3 = \boxed{-3\sqrt{-2(x-2)} - 3 = g(x)}$



$D = [2, \infty)$

$R = (-\infty, -3]$

A sequence of moves that will keep you on the right track:

1. Horizontal stretch/compress/reflect	$f(ax)$	Multiply x's by $1/a$
2. Horizontal shift	$f(x+a)$	Left a (Subtract a from x's)
3. Vertical stretch/compress/reflect	$a\,f(x)$	Multiply y's by a.
4. Vertical shift.	$f(x)+a$	Up a units.

If you stick to this sequence of moves, you will be OK. If you mix up #3 and #4 you will NOT be OK. If you mix up #1 and #2 you will NOT be OK.

You could do the vertical stuff first and then the horizontal, but why not just stick with what we have?

$$a f(bx+c) + d = a f\left(b\left(x+\frac{c}{a}\right)\right) + d$$

$$2\sqrt{-3x+6} + 5 = 2\sqrt{-3(x-2)} + 5$$

$$\sqrt{x} \longrightarrow \overset{1}{\sqrt{-3x}} \longrightarrow \overset{2}{\sqrt{-3(x-2)}} \longrightarrow \overset{3}{2\sqrt{-3(x-2)}}$$

$$\longrightarrow \overset{4}{2\sqrt{-3(x-2)} + 5}$$

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$|2x-3| \leq 5$

$2x-3 \leq 5$ and $2x-3 \geq -5$

$2x \leq 8$ $\quad$ $2x \geq -2$

$x \leq 4$ $\quad$ $x \geq -1$

$\{x \mid x \leq 4 \text{ AND } x \geq -1\}$

$= [-1, 4]$

$|2x-3| > 5$

$2x-3 > 5$ OR $2x-3 < -5$

$2x > 8$ $\quad$ $2x < -2$

$x > 4$ $\quad$ $x < -1$

$\{x \mid x > 4 \text{ OR } x < -1\}$

$= \boxed{(-\infty, -1) \cup (4, \infty)}$