

J is jointly proportional to G and V, and  $J = \sqrt{3}$  when  $G = \sqrt{5}$  and  $V = \sqrt{7}$ .

Find J when  $G = \sqrt{15}$  and  $V = 7$ .

2.6 #15  
#38 from text  
Keith

$y = kx$  y is proportional to x

J =  $3\sqrt{7}$

(Simplify your answer. Type an integer or a fraction.)

Joint Proportionality

$y = kxz$

Inverse Proportionality

$y = \frac{k}{x}$

$J = kGV$

Find constant of proportionality

$\sqrt{3} = k\sqrt{5}\sqrt{7}$

$\frac{\sqrt{3}}{\sqrt{35}} = k$

Square root of the quotient.

Find J:

$J = kGV$

$J = \frac{\sqrt{3}}{\sqrt{35}} \cdot \sqrt{15} \cdot 7$

$= \frac{7\sqrt{3}\sqrt{15}}{\sqrt{7 \cdot 5}}$

$= \frac{7\sqrt{3 \cdot 3 \cdot 5}}{\sqrt{7 \cdot 5}} = 7\sqrt{\frac{3 \cdot 3 \cdot 5}{7 \cdot 5}} = 7 \cdot \frac{3}{\sqrt{7}}$

$= \frac{7 \cdot 3}{\sqrt{7}} = \frac{7 \cdot 3 \sqrt{7}}{\sqrt{7} \sqrt{7}} = \frac{21\sqrt{7}}{7} = 3\sqrt{7}$

$\sqrt{7 \cdot 7} = 7$

Let  $f = \{(-9, -8), (7, 9), (9, -3)\}$  and  $h = \{(-8, 9), (7, -3)\}$ .

Find  $(f \circ h)(x) = f(h(x))$

Nate  
2.4 #11  
#33 in book

$f \circ h = \{\square\}$   
(Type an ordered pair.)

Look for

$$\text{Recall } \mathcal{D}(f \circ h) = \{x \mid x \in \mathcal{D}(h) \text{ and } h(x) \in \mathcal{D}(f)\}$$

$$= \{-8\}$$

$$f(h(-8)) = -3 \rightsquigarrow \{(-8, -3)\}$$

Let  $f(x) = |x|$ ,  $g(x) = x - 7$ , and  $h(x) = x^3$ .

2.4 #16  
#63 in book  
Clint

Write  $F(x) = x^3 - 7$  as a composition of functions.

Which of the following compositions correctly defines  $F(x) = x^3 - 7$ ?  $= (g \circ h)(x)$

cube it, subtract 7.

$$h(x) = x^3 \quad g(x) = x - 7$$

$$F(x) = g(h(x)) = g(x^3) = x^3 - 7 = F(x)$$

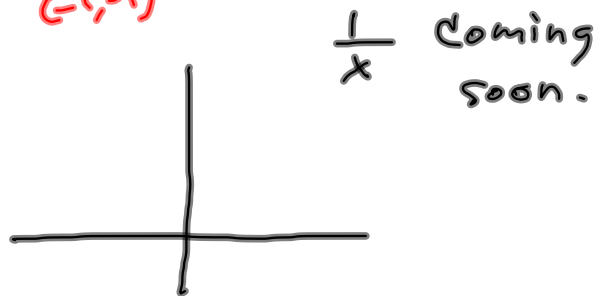
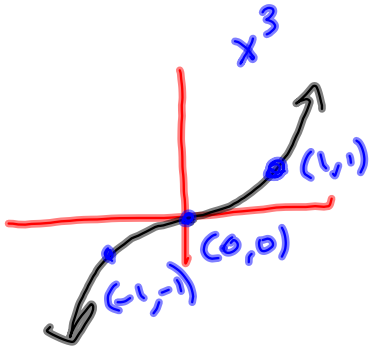
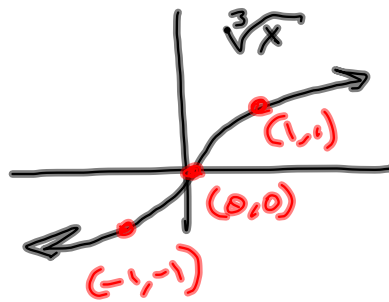
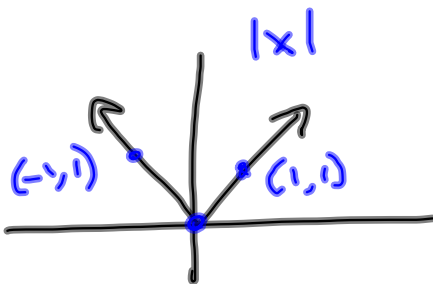
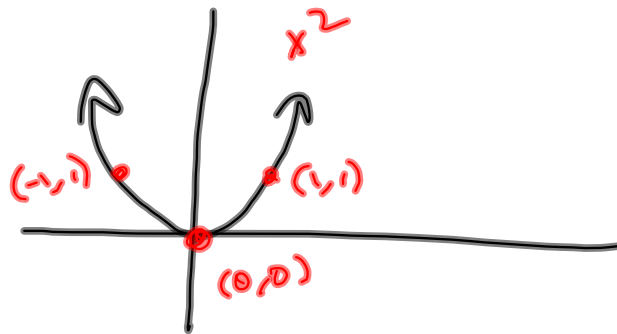
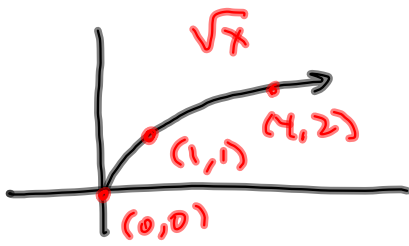
See functions inside other functions.

$f(x) = (2x-3)^3$  write as a composition  
of two simpler functions

$$g(x) = 2x-3$$

$$h(x) = x^3$$

$$h(g(x)) = h(2x-3) = (2x-3)^3$$



Let  $f(x) = x - 5$ .

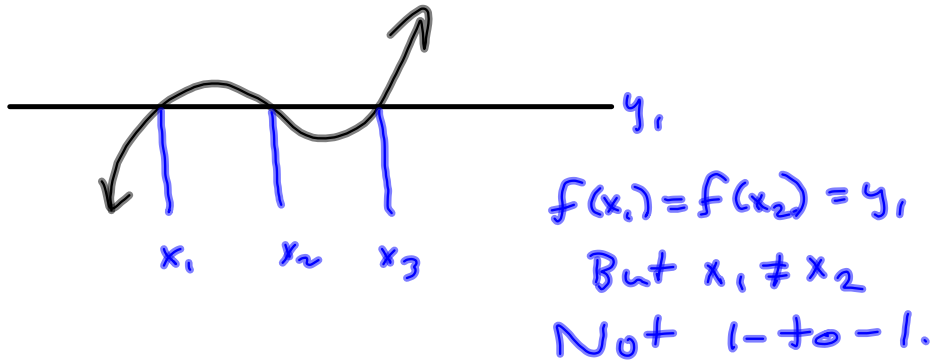
2.4 #15  
#57 in book

Find  $(f \circ f)(x)$  and its domain.

$$\begin{aligned}(f \circ f)(x) &= f(f(x)) = f(x-5) = (x-5) - 5 \\ &= x - 10\end{aligned}$$

§2.5 Function: A relation in which each  $x$  in the domain corresponds with one  $y$  in the range. **Vertical Line Test**

1-to-1 Function: A function in which each  $y$  in the range corresponds with one  $x$  in the domain. **Horizontal Line Test.**



1-to-1 is important:  
we want the INVERSE relation to be a function.

$F: \{(1,2), (3,5), (4,2)\}$  is a function.  
Not 1-to-1.

$F^{-1}: \{(2,1), (5,3), (2,4)\}$  is NOT a function  
b/c  $F$  isn't 1-to-1.

The inverse of  $f$ , or "f-inverse" is written  $f^{-1}$ , the inverse with respect to the operation of function composition.

Arithmetic:

Addition:  $3 \quad -3 \quad 3 + -3 = 0 = \text{Identity}$

Multiplication:  $3 \quad \frac{1}{3} \quad 3 \cdot \frac{1}{3} = 1 = \text{Identity}$

Composition:  $f(x) \quad f^{-1}(x) \quad f \circ f^{-1} = x = \text{Identity}$

$f^{-1} \neq \frac{1}{f}$ , although it kind of acts that way, if you view composition as multiplication, but DON'T.



$$\text{Let } f(x) = 3x - 7, \quad g(x) = \frac{1}{3}x + \frac{7}{3} = \frac{x+7}{3}$$

$$\text{Claim: } g(x) = f^{-1}(x)$$

$$\text{Proof: } (f \circ g)(x) = f(g(x)) = f\left(\frac{x+7}{3}\right)$$

$$= 3\left(\frac{x+7}{3}\right) - 7 = x + 7 - 7 = x \quad \text{Sweet. } \square$$

$f(x)$  is 1-to-1:

If  $x_1 \neq x_2$ , then  $f(x_1) \neq f(x_2)$

If  $f(x_1) = f(x_2)$ , then  $x_1 = x_2$

↳ we use this.

$$f(x) = 3x - 7 \text{ is } 1\text{-to-1}$$

Proof! Suppose  $f(x) = f(x_2)$ .

$$3x_1 - 7 = 3x_2 - 7$$

$$3x_1 = 3x_2$$

$$x_1 = x_2 \quad \square$$

$f(x) = x^2$  is not 1-to-1.

METHOD 1! Proof  $f(x_1) = f(x_2)$

$$x_1^2 = x_2^2$$

$$\sqrt{x_1^2} = \sqrt{x_2^2}$$

$$|x_1| = |x_2|$$

$$x_1 = x_2 \text{ OR } x_1 = -x_2$$

Two possibilities

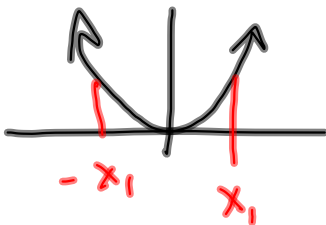
METHOD 2! Counterexample

Find two  $x$ -values

$\ni f(x)$  is the same.

$$x_1 = -1, x_2 = +1$$

$$f(x_1) = (-1)^2 = 1 = 1^2 = f(x_2)$$



To find  $f^{-1}(x)$ , swap  $x$  &  $y$ , solve for  $y$

$$f(x) = 3x - 7$$

$$y = 3x - 7$$

$$x = 3y - 7$$

$$3y - 7 = x$$

$$3y = x + 7$$

$$y = \frac{x+7}{3} = f^{-1}(x)$$

Find  $f^{-1}(x)$ 

$$f(x) = y = \frac{x+1}{2x-1}$$

$$x = \frac{y+1}{2y-1}$$

$$(2y-1)x = \frac{y+1}{\cancel{2y-1}} \cdot \cancel{(2y-1)}$$

$$2yx - x = y + 1$$

$$-y = -y$$

$$2yx - x - y = 1$$

$$+x = +x$$

$$2yx - y = x + 1$$

$$y(2x-1) = x+1$$

$$y = \boxed{\frac{x+1}{2x-1} = f^{-1}(x)}$$

$$f^{-1}(x) = \frac{x+1}{2x-1} = g(x)$$

$$f(g(x)) = f\left(\frac{x+1}{2x-1}\right)$$

$$= \frac{\frac{x+1}{2x-1} + 1}{2\left(\frac{x+1}{2x-1}\right) - 1}$$

$$= \frac{\frac{x+1}{2x-1} + \frac{2x-1}{2x-1}}{\frac{2x+2}{2x-1} - \frac{2x-1}{2x-1}}$$

$$= \frac{\frac{x+1+2x-1}{2x-1}}{\frac{2x+2-(2x-1)}{2x-1}}$$

$$\frac{\frac{3x}{2x-1}}{\frac{3}{2x-1}} =$$

$$\frac{\cancel{3x}}{\cancel{2x-1}} \cdot \frac{\cancel{2x-1}}{3} = \frac{3x}{3} = x$$