

Domain

① $\frac{\text{stuff}}{0}$ is bad $\frac{2x+1}{x^2-3x+2}$ D: $x^2-3x+2=0$
Discard Answer

② $\sqrt{\text{negative}}$ is bad $\sqrt{2x-6}$ D: $2x-6 \geq 0$
Keep answer

① $x^2-3x+2=0$
 $(x-2)(x-1)=0$
 $x=1, 2$

② $2x-6 \geq 0$
 $2x \geq 6$
 $x \geq 3$

D = $\{x \mid x \neq 1 \text{ and } x \neq 2\}$
 $= (-\infty, 1) \cup (1, 2) \cup (2, \infty)$
 $= \mathbb{R} \setminus \{1, 2\}$

D = $\{x \mid x \geq 3\}$
 $= [3, \infty)$

Variations Combo Package.

$$\frac{2x+5}{\sqrt{3x-2}}$$

① $\sqrt{3x-2} \neq 0$ ② $3x-2 \geq 0$
 $3x-2 \neq 0$

Combine: $3x-2 > 0$ STRICT
D = $\{x \mid x > \frac{2}{3}\}$

Operations on functions

$$\left. \begin{aligned} (f+g)(x) &= f(x)+g(x) \\ (f-g)(x) &= f(x)-g(x) \\ (fg)(x) &= f(x)g(x) \end{aligned} \right\} \mathcal{D} = \mathcal{D}(f) \cap \mathcal{D}(g)$$

$$\left(\frac{f}{g} \right)(x) = \frac{f(x)}{g(x)} \left. \begin{aligned} \mathcal{D} &= \mathcal{D}(f) \cap \mathcal{D}(g) \\ &\cap \{x \mid g(x) \neq 0\} \end{aligned} \right\}$$

~~$\sqrt{2x+6} \geq 0$~~

$$f(x) = \frac{2x+1}{x-1}$$

$$g(x) = \sqrt{2x+6}$$

$$\mathcal{D}(f): \quad x-1=0$$

$$\quad \quad \quad x=1$$

$$\mathcal{D}(g): \quad 2x+6 \geq 0$$

$$\quad \quad \quad 2x \geq -6$$

$$\quad \quad \quad x \geq -3$$

$$\mathcal{D}(f) = \{x \mid x \neq 1\}$$

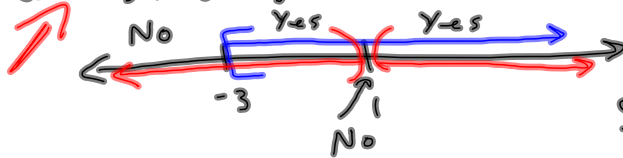
$$\mathcal{D}(g) = \{x \mid x \geq -3\}$$

$$= (-\infty, 1) \cup (1, \infty)$$

$$= [-3, \infty)$$



$$\mathcal{D}(f) \overset{\text{AND}}{\cap} \mathcal{D}(g) = [-3, 1) \cup (1, \infty) = \mathcal{D}^3(f+g) = \mathcal{D}(f-g) = \mathcal{D}(fg)$$



Same Domain for
 $f+g, f-g, fg$

$\frac{f}{g}$ has one little wrinkle:

$$\text{Need } \sqrt{2x+6} \neq 0$$

$$2x+6 \neq 0$$

$$2x \neq -6$$

$$x \neq -3$$

Throw out this
one point

$$\mathcal{D}\left(\frac{f}{g}\right) = (-3, 1) \cup (1, \infty)$$

$$\left(\frac{f}{g}\right)(x) = \frac{f(x)}{g(x)} = \frac{\frac{2x+1}{x-1}}{\sqrt{2x+6}} = \frac{2x+1}{\frac{x-1}{\sqrt{2x+6}}}$$

Stop here
on test

$$= \frac{2x+1}{x-1} \cdot \frac{1}{\sqrt{2x+6}} = \frac{2x+1}{(x-1)\sqrt{2x+6}} = \left(\frac{f}{g}\right)(x)$$

$$\mathcal{D}(f) \cap \mathcal{D}(g) = \{x \mid x \in \mathcal{D}(f) \text{ and } x \in \mathcal{D}(g)\}$$

$$\begin{aligned}(f \circ g)(x) &= f \text{ composed with } g \text{ of } x \\ &= f \text{ of } g \text{ of } x \\ &= f(g(x))\end{aligned}$$

Car goes 20 miles per gallon of gas
Gas costs \$3.50 per gallon

Let x = the amount of money I have.
Then I can write distance travelled
as a function of my money.

$$f(x) = \frac{2x+1}{x-1}, \quad g(x) = 3x+6$$

$$(f \circ g)(x) = f(g(x)) = \frac{2(3x+6)+1}{(3x+6)-1}$$

$$\mathcal{D}(f) = \mathbb{R} \setminus \{1\}$$

$$\mathcal{D}(g) = \mathbb{R}$$

$$= \{x \mid x \neq 1\}$$

$$\mathcal{D}(f \circ g) = \{x \mid x \in \mathcal{D}(g) \text{ AND } \underline{g(x) \in \mathcal{D}(f)}\}$$

$$= \{x \mid \underbrace{x \in \mathbb{R}}_{\text{No restriction}} \text{ and } \underline{3x+6 \neq 1}\} = \{x \mid x \neq -\frac{5}{3}\}$$

g(x) chews on x. f(g(x)) chews on g(x)

$$3x+6=1$$

$$3x = -5$$

$$x = -\frac{5}{3}$$

$$f(x) = \frac{2x+1}{x-1}, \quad g(x) = \sqrt{2x+8}$$

$$\mathcal{D}(f) = \mathbb{R} \setminus \{1\} = \{x \mid x \neq 1\}$$

$$\mathcal{D}(g) = [-4, \infty) = \{x \mid x \geq -4\}$$

$$(f \circ g)(x) = \frac{2\sqrt{2x+8} + 1}{\sqrt{2x+8} - 1}$$

$$\mathcal{D}(f \circ g) = \{x \mid x \in \mathcal{D}(g) \text{ and } g(x) \in \mathcal{D}(f)\}$$

$$= \{x \mid x \geq -4 \text{ and } \sqrt{2x+8} \neq 1\}$$

$$= \{x \mid x \in [-4, \infty) \text{ and } x \neq -\frac{7}{2}\}$$

$$\sqrt{2x+8} \neq 1$$

$$\sqrt{2x+8}^2 \neq 1^2$$

$$2x+8 \neq 1$$

$$2x \neq -7$$

$$x \neq -\frac{7}{2}$$

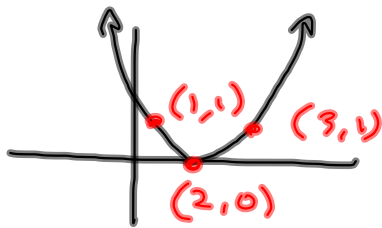
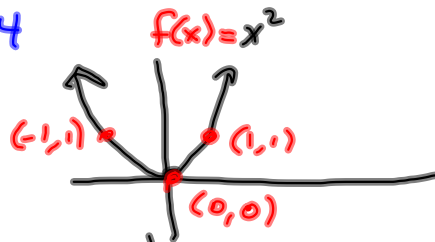
Completing the square

Graph $g(x) = x^2 - 4x - 7$

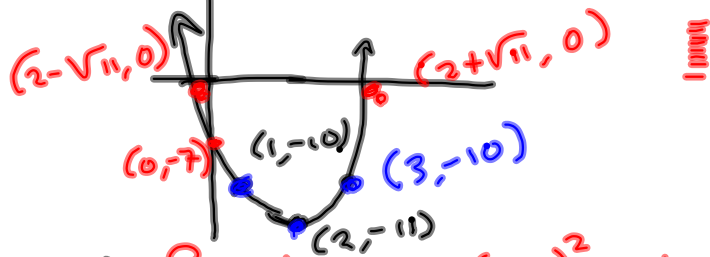
$$= x^2 - 4x + 2^2 - 7 - 4$$

$$= (x-2)^2 - 11$$

Right 2 Down 11



$$f(x-2) = (x-2)^2$$



$$g(x) = f(x-2) - 11 = (x-2)^2 - 11$$

Complete Graph: Show x- & y-intercepts

y-int: $g(0) = -7 \rightsquigarrow (0, -7)$

x-int: $g(x) = 0$

$$x^2 - 4x - 7 = 0$$

$$(x-2)^2 - 11 = 0$$

$$(x-2)^2 = 11$$

$$|x-2| = \sqrt{11}$$

$$x-2 = \pm \sqrt{11}$$

$$x = 2 \pm \sqrt{11}$$

$$\sqrt{\text{scribble}^2} = |\text{scribble}|$$

$$\sqrt{\text{scribble}^2} = \text{scribble}$$