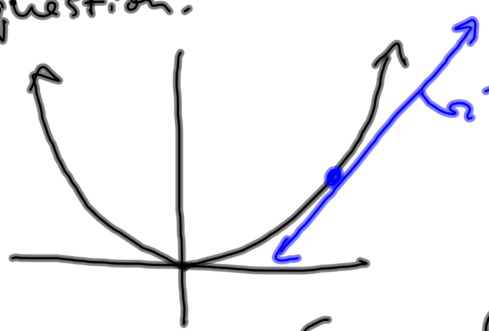
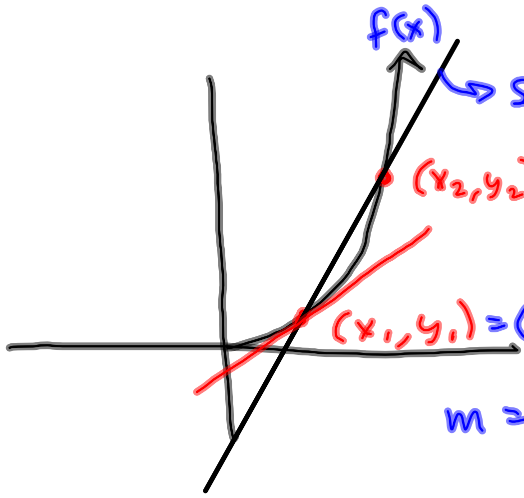
We need a couple difference quotient questions and a quick explanation of what they represent.

In calculus, we generalize slope into something that works for smooth curves. This is the age-old TANGENT LINE question.



Tangent line has same slope as the curve at a single point.

Generalize Slope:



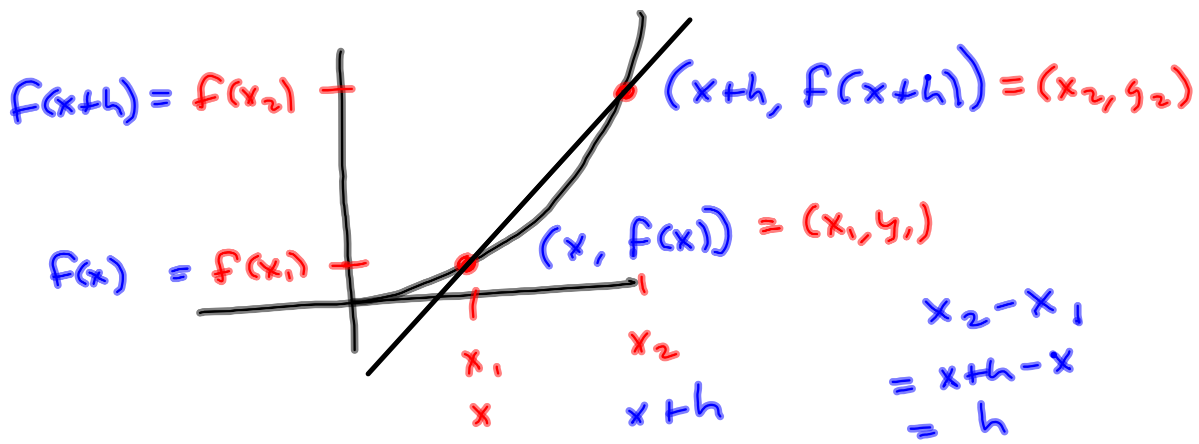
Secant line. Its slope approximates the slope of the curve at $x = x_1$

$(x_2, y_2) = (x_2, f(x_2))$

$(x_1, y_1) = (x_1, f(x_1))$

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{f(x_2) - f(x_1)}{x_2 - x_1}$$

= AVERAGE SLOPE.



$$x_2 - x_1 = x+h-x = h$$

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{f(x+h) - f(x)}{h}$$

$f(x) = x^2$ simplify the $\boxed{\text{difference quotient.}}$

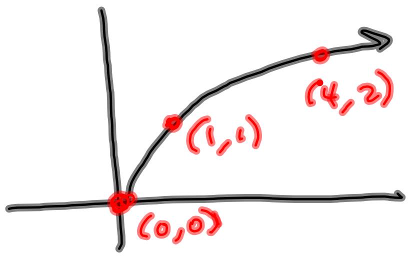
$$\frac{f(x+h) - f(x)}{h} = \frac{(x+h)^2 - x^2}{h} = \frac{x^2 + 2xh + h^2 - x^2}{h}$$

$$= \frac{2xh + h^2}{h} = \frac{h(2x+h)}{h} = \boxed{2x+h}$$

College Algebra Final Answer.

$\boxed{h \to 0 \quad 2x \text{ is the slope of } x^2 \text{ at } x.}$

Calculus



$\sqrt{x} = f(x)$

Simplify the difference quotient (by rationalizing the NUMERATOR)

$f(x) = \sqrt{x}$

$f(x+h) = \sqrt{x+h}$

$\frac{a}{b} \cdot \frac{c}{d} = \frac{ac}{bd}$

$$\therefore \quad \frac{f(x+h)-f(x)}{h} = \frac{\sqrt{x+h}-\sqrt{x}}{h}$$

$$= \left(\frac{\sqrt{x+h}-\sqrt{x}}{h}\right)\left(\frac{\sqrt{x+h}+\sqrt{x}}{\sqrt{x+h}+\sqrt{x}}\right) =$$

$$= \frac{(\sqrt{x+h}-\sqrt{x})(\sqrt{x+h}+\sqrt{x})}{h(\sqrt{x+h}+\sqrt{x})} = \frac{(\sqrt{x+h})^2-(\sqrt{x})^2}{h(\sqrt{x+h}+\sqrt{x})}$$

$$= \frac{x+h-x}{h(\sqrt{x+h}+\sqrt{x})} = \frac{\cancel{h}}{\cancel{h}(\sqrt{x+h}+\sqrt{x})} = \boxed{\frac{1}{\sqrt{x+h}+\sqrt{x}}}$$

FINAL ANS

Calculus $\xrightarrow{h \to 0}$ $\frac{1}{\sqrt{x}+\sqrt{x}} = \frac{1}{2\sqrt{x}}$

Slope of $\sqrt{x}$ is $\frac{1}{2\sqrt{x}}$

Slope of x^2 is $2x$

Slope of x^5 is $5x^4$

Slope of $\sqrt{x} = x^{\frac{1}{2}}$ is $\frac{1}{2}x^{-\frac{1}{2}} = \frac{1}{2x^{\frac{1}{2}}} = \frac{1}{2\sqrt{x}}$

Sep 14-8:08 AM

$$(a-b)(a+b)=a^2-b^2$$

$$(\square-\Delta)(\square+\Delta)=\square^2-\Delta^2$$

$$(\sqrt{x}-7)(\sqrt{x}+7)=(\sqrt{x})^2-49 \quad = x-49$$

Recall: $\sqrt{x^2}=|x|$, $(\sqrt{x})^2=x$

$$(\sqrt{x+h}-\sqrt{x})(\sqrt{x+h}+\sqrt{x})=(\sqrt{x+h})^2-(\sqrt{x})^2$$

$$=x+h-x=h$$

Sep 14-8:11 AM

$f(x) = x^2$

$f(\text{☺}) = \text{☺}^2$

$f(\boxed{}) = \boxed{}^2$

$f(x+h) = (x+h)^2 = x^2 + 2xh + h^2$

$f(\boxed{x+h}) = \boxed{x+h}^2$

$(x+h)^2 = (x+h)(x+h)$

$= x^2 + xh + xh + h^2$

$= x^2 + 2xh + h^2$

$(a+b)^2 = a^2 + 2ab + b^2$

Difference Quotient (Simplify)

$$f(x) = 3\sqrt{x} \qquad (3\sqrt{x+h})^2 = 3^2(\sqrt{x+h})^2$$

$$\frac{f(x+h)-f(x)}{h} = \left(\frac{3\sqrt{x+h} - 3\sqrt{x}}{h}\right)\left(\frac{3\sqrt{x+h}+3\sqrt{x}}{3\sqrt{x+h}+3\sqrt{x}}\right)$$

$$= \frac{9(x+h) - 9(x)}{h(3\sqrt{x+h}+3\sqrt{x})} = \frac{9x+9h-9x}{h(3\sqrt{x+h}+3\sqrt{x})}$$

$$= \frac{9h}{3h(\sqrt{x+h}+\sqrt{x})} = \boxed{\frac{3}{\sqrt{x+h}+\sqrt{x}}}$$

Sep 14-8:23 AM

Graph the function, and state the $\boxed{\text{domain and range.}}$ 2.2 #12

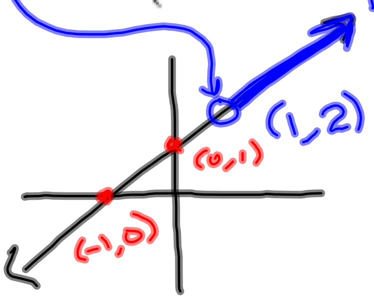
#37 in book.

open

$$f(x) = \begin{cases} x+1 & \text{for } x > 1 \\ x-1 & \text{for } x \le 1 \end{cases}$$

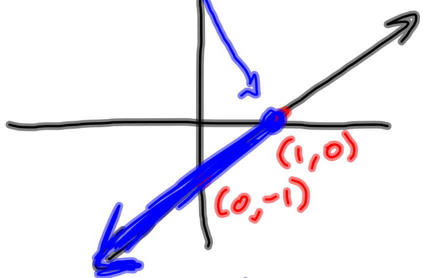
x=1 is suture point

closed



x+1

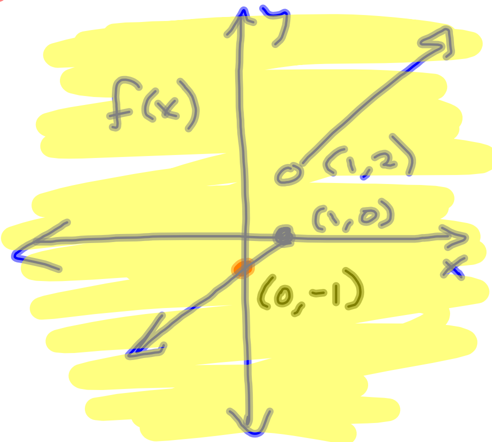
For x>1



x-1

For x≤1

FINAL



Needs f(x)

$D = (-\infty, 1] \cup (1, \infty) = (-\infty, \infty)$

$R = (-\infty, 0] \cup (2, \infty)$ Needs picture.

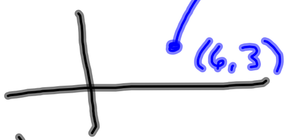
Find the domain and range of the function.

2.2 #24

#101 in book

$g(x) = \sqrt{x-6} + 3$

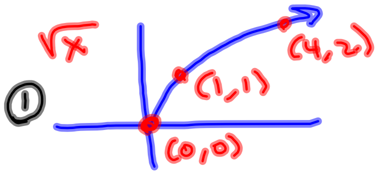
$f(x-6)$ stuff happens 6 units later to this guy. Shift Right 6 unit



$f(x) = \sqrt{x}$

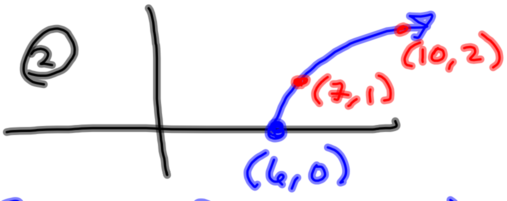
$f(x-6) = \sqrt{x-6}$ is right 6

$\sqrt{x-6} = f(x-6)$



$D = [0, \infty)$

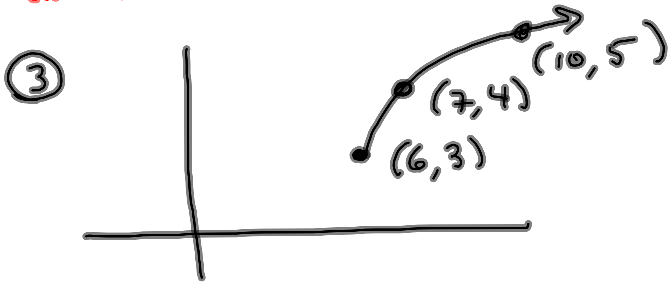
$R = [0, \infty)$



$D = [6, \infty)$

$R = [0, \infty)$

$f(x-6) + 3 = \sqrt{x-6} + 3$



$D = [6, \infty)$

$R = [3, \infty)$

Graph the function. Determine the domain and range of the graph.

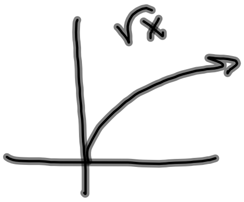
2.2 #13

#39 in book.

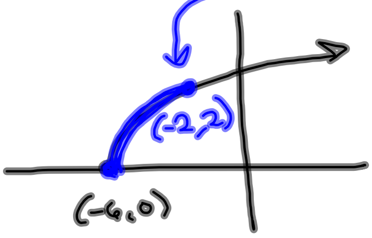
$$f(x)=\begin{cases}\sqrt{x+6} & \text{for } -6\le x\le -2\\ -x & \text{for } x>-2\end{cases}$$

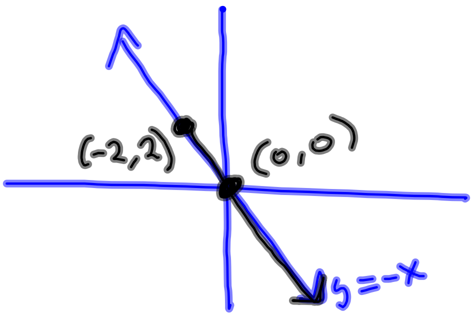
Sutures:
$x=-6,\ x=-2$

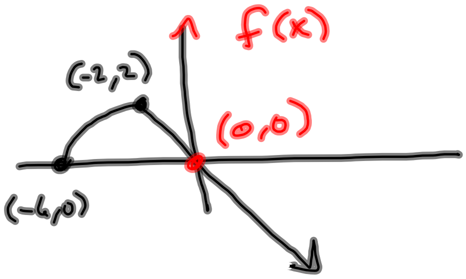
$\sqrt{x+6}$



$\sqrt{-2+6}=\sqrt{4}=2$







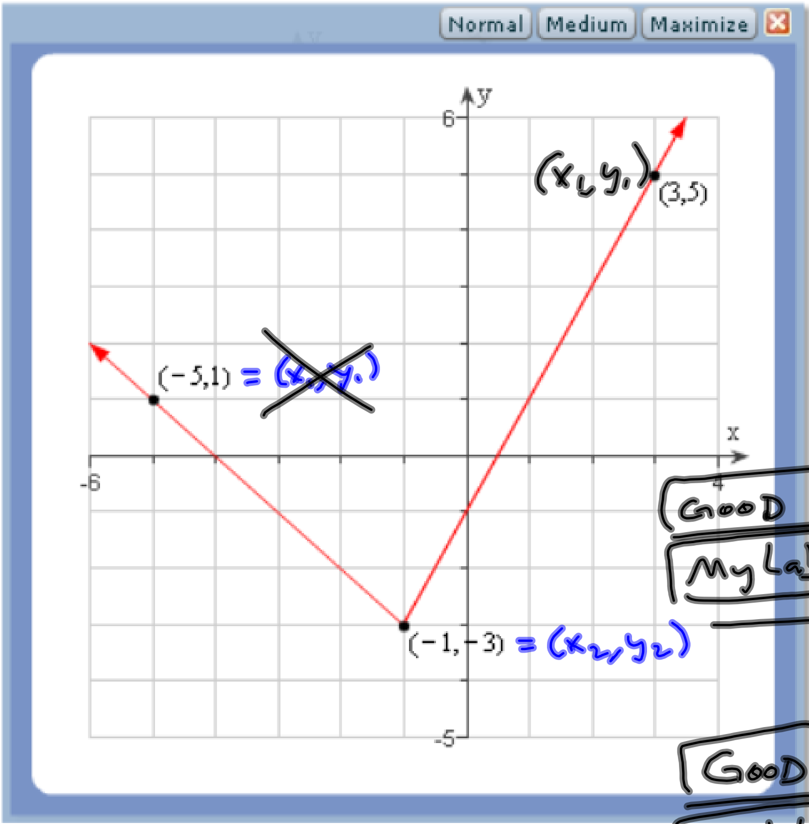
$\mathcal{D}=[-6,\infty)$

$\mathcal{R}=(-\infty,2]$

2.2 #21

Write a piecewise function for the graph given below.

73 from the book.



(x_1, y_1) at $(3,5)$

$(-5,1) = \cancel{(x_1, y_1)}$

$(-1,-3) = (x_2, y_2)$

$x = -1$ is suture value.

$x \le -1$

$m = \dfrac{y_2 - y_1}{x_2 - x_1} = \dfrac{-3-1}{-1-(-5)} = -1$

$y = m(x - x_1) + y_1$

$\boxed{\text{GOOD}}$ $= -1(x+5) + 1$

$\boxed{\text{MyLab}}$ $= -x - 4$

$x > -1$

$m = \dfrac{-3-5}{-1-3} = 2$

$\boxed{\text{GOOD}}$ $y = 2(x-3) + 5$

$\boxed{\text{Mylab}}$ $y = 2x - 1$

$$f(x) = \begin{cases} -(x+5)+1 & \text{for } x \le -1 \\ 2(x-3)+5 & \text{for } x > -1 \end{cases}$$ Test version

$$f(x) = \begin{cases} -x-4 & \text{for } x \le -1 \\ 2x-1 & \text{for } x > -1 \end{cases}$$ Mylab version.

§2.3 Should be up in minutes.
Whole Durn chapter by this afternoon.
§2.1 Deadline extended to Thursday.

Next Time:
Shift, Stretch, Compress, Reflect
the basic functions
ON the board.

$$\boxed{g(x) = -2\sqrt{-3x+6} - 11}$$

$$g(x) = 3(x-2)^2 + 5$$
$$g(x) = 5\sqrt{x-6} + 4$$

Sep 14-8:23 AM