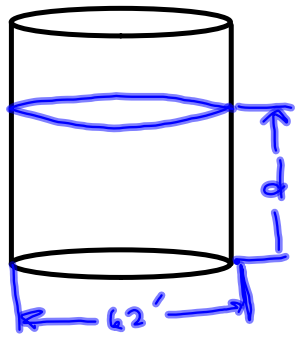


Contains 498,000 gallons
What's the depth?
1 cubic foot ≈ 7.5 gallons.

Volume = 498,000 gallons

$V = \pi r^2 d = 498{,}000$ gal

$= \pi (31)^2 d = 498{,}000$ gal

$31^2 \pi d = \frac{498{,}000}{7.5}$ ft^3

$d = \frac{498{,}000}{(31)^2 \pi \cdot 7.5}$ ft

≈ 21.9935 ft

$\approx$ 22 ft

$\pi (\text{ft})^2 \text{ft} = \text{gal}$?

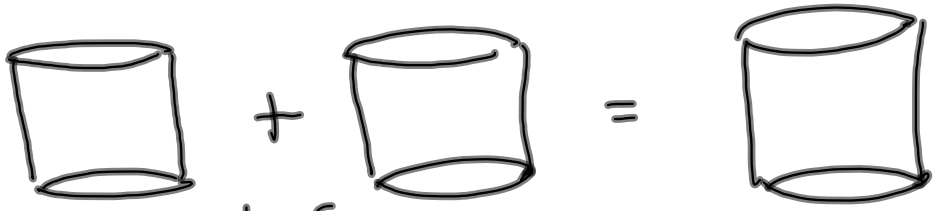
ft^3

Scratch

$(498{,}000 \text{ gal})\left(\frac{1 \text{ ft}^3}{7.5 \text{ gal}}\right)$

A tank holds 175 liters of a chemical solution. Currently, the solution has a strength of 40%. How much of this should be drained and replaced with 90% solution to get a final strength of 60%?

How many liters should be drained and replaced with 90% solution? ☐ liters



Amt of pure chemical

40% .4y

90% .9x

60% .6(175)

Assign Variables.

Let x = amt of 40% that's drained

= amt of 90%

y = amt of 40% that remains.

Units: Liters

$.9x + .4y = .6(175)$

$x + y = 175$

Not enough info.

$y = 175 - x$

Ahhh!

So, $.9x + .4(175 - x) = .6(175)$, which we can solve for x!

$i = \sqrt{-1}$ $\qquad i^2 = -1$

$\sqrt{4} = 2$

$\sqrt{-4} = 2i$

$\sqrt{-4} = \sqrt{(-1)(4)} = \sqrt{-1}\sqrt{4} = i\sqrt{4} = i \cdot 2 = 2i$

Solve

$x^2 = -5$

SRP: $x = \pm\sqrt{-5} = \pm i\sqrt{5}$

$3x^2 + 7 = 0$

$3x^2 = -7$

$x^2 = -\frac{7}{3}$

$x = \pm\sqrt{-\frac{7}{3}} = \pm i\sqrt{\frac{7}{3}}$

MyLab accepts $-i\cdot\sqrt{\frac{7}{3}}$, $i\cdot\sqrt{\frac{7}{3}}$

Down the road, we may not like it this way:

$\sqrt{\frac{7}{3}} = \sqrt{\frac{7}{3}}\sqrt{\frac{3}{3}} = \frac{\sqrt{7\cdot 3}}{\sqrt{3\cdot 3}} = \frac{\sqrt{21}}{3}$

Rationalizing the denominator is a good skill.

Using the discriminant

$x^2 - 5x - 11$

How many & what flavor?

2 real, irrational

$a = 1, b = -5, c = -11$

$b^2 - 4ac = 25 - 4(1)(-11) = 69 > 0$ ∴ 69 is not a perfect square.

$x^2 - 5x + \left(\frac{5}{2}\right)^2 = 11 + \frac{25}{4} = \frac{69}{4}$

$\left(x - \frac{5}{2}\right)^2 = \frac{69}{4}$

$x - \frac{5}{2} = \pm\sqrt{\frac{69}{4}} \Longrightarrow x = \frac{5}{2} \pm \frac{\sqrt{69}}{2}$

$x^2 - 10x + 25 = (x-5)^2 = 0 \Rightarrow x = 5$

$b^2 - 4ac = 100 - 4(1)(25) = 100 - 100 = 0$

One, real, repeated root.

$x^2 + 5x + 20$

$b^2 - 4ac = 5^2 - 4(1)(20)$

$= 25 - 80$

$= -55 < 0$

Two, nonreal solutions

$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$= \frac{-5 \pm \sqrt{-55}}{2(1)}$

$= \frac{-5 \pm i\sqrt{55}}{2}$

$= -\frac{5}{2} \pm i\frac{\sqrt{55}}{2}$

$-3x+5>2$

$-3x>-3$

Flip it! $\frac{-3x}{-3} < \frac{-3}{-3}$

$x<1$

$\{x \mid x<1\}$

$=(-\infty, 1)$

Student gets burned

$-3x+5>2$

FALSE! $\frac{-3x}{-3} > \frac{-3}{-3}$

$x<1$

$\{x \mid x<1\}$

$=(-\infty, 1)$

Slickest way that's legal:

$-3x+5>2$

$-3x>-3$ Flip it!

$\{x \mid x<1\} =$

$(-\infty, 1)$

Write in interval notation

$\{x \mid x < 5 \text{ and } x > 6\} = \varnothing = \{\}$

$\{x \mid x < 5 \text{ OR } x > 6\} = (-\infty, 5) \cup (6, \infty)$

Find an equation of the line thru $(-7,-8)$, $(-4,7)$

(x_1,y_1) (x_2,y_2)

$y = m(x-x_1)+y_1$

$$m = \frac{y_2-y_1}{x_2-x_1} = \frac{7-(-8)}{-4-(-7)} = \frac{15}{3} = 5$$

(i) Point-Slope Form

$$y = 5(x-(-7)) - 8$$

(ii) Standard Form

$Ax+By=C$, A,B,C all integers.

Clear fracs, if any

$$-5x + y = 27$$

(iii) Slope-Intercept form

$y=mx+b$

$$y = 5(x+7) - 8 = 5x + 35 - 8$$

$$y = 5x + 27$$

$y = 5x + 27$

Find a line parallel to this, that passes thru $(-1, 7\pi)$

$$y = m(x - x_1) + y_1$$

$y = 5(x+1) + 7\pi$ Done!

Perpendicular, this time

$m = 5 \Rightarrow m_\perp = -\frac{1}{5}$

$y = -\frac{1}{5}(x+1) + 7\pi$

Aug 31-8:48 AM

Solve by completing the square:

$x^2 + 14x = -45$

$x^2 + 14x + 7^2 = -45 + 49$

$(x+7)^2 = 4$

$x + 7 = \pm\sqrt{4} = \pm 2$

$x = -7 \pm 2$

$-7+2 = -5$

$-7-2 = -9$

$\{-9, -5\}$

Solve by factoring

$x^2 + 14x = -45$

$1 \cdot x^2 + 14x + 45 = 0$

Factors of $1 \cdot 45$ that add up to 14

$9 \cdot 5 = 45$ $\quad 9 + 5 = 14$

$x^2 + 9x + 5x + 45$

$x(x+9) + 5(x+9)$

$(x+9)(x+5)$

Different / novel approach.

$(x+5)(x+9) = 0$

$x + 5 = 0$ OR $x + 9 = 0$

$x = -5$ OR $x = -9$

$\{-9, -5\}$