§8.3 #s 1-3, 5-17 odd, 21, 25, 27, 36, 37, 45, 53, 57, 61, 65*, 69, 71, 73, 75, 78, 79, 87*, 98, 99

(1) Constant ratio r between consecutive terms $\Longrightarrow$ geometric.

(2) The sum of a geometric sequence is a geometric series.

#s 3-8 Find the 1st 4 terms. What's the common ratio?

(3) $a = 3 \cdot 2^{n-1} \longrightarrow \boxed{r = 2}$

$a_1 = 3,\ a_2 = 6,\ a_3 = 12,\ a_4 = 24$

(5) $b_n = (800)\left(\frac{1}{2}\right)^n$ $\boxed{r = \frac{1}{2}}$

$b_1 = 400,\ b_2 = 200,\ b_3 = 100,\ b_4 = 50$

(7) $c_n = \left(-\frac{2}{3}\right)^{n-1}$ $\boxed{r = -\frac{2}{3}}$

$c_1 = 1,\ c_2 = -\frac{2}{3},\ c_3 = \frac{4}{9},\ c_4 = \frac{8}{27}$

#s 9-16 Find common ratio.

(9) $4, 2, 1, \frac{1}{2}, \ldots,$ $\boxed{r = \frac{1}{2}}$ ~~17~~

(11) $10^2, 10^3, 10^4, \ldots$ $\boxed{r = 10}$

(13) $-1, 2, -4, 8, \ldots$ $\boxed{r = -2}$

(15) $1, -1, 1, -1, \ldots$ $\boxed{r = -1}$

121 §8.3 #s 17, 21, 25, 27, 36, 37, 45, 53, 57, 61, 65

69, 71, 73, 75, 78, 79, 87, 98, 99

#s 17-22 write formula for n^{th} term

17 $\frac{1}{6}, \frac{1}{3}, \frac{2}{3}, \frac{4}{3}$

$$\left.\begin{array}{l} r = 2 \\ \frac{1}{6} = a \end{array}\right\} \boxed{a_n = \frac{1}{6} \cdot 2^{n-1}}$$

21 $4, -12, 36, -108, \ldots$

$r = -3,\ a = 4 \Rightarrow \boxed{4 \cdot (-3)^{n-1} = a_n}$

#s 23-34 Geometric or not?

25 $1, 2, 4, 6, 8$ $\boxed{\text{No}}$

27 $2, -4, 8, -16$ $\boxed{\text{Yes}}$ $r = -2$

#s 35-44 Find 1^{st} 4 terms. Geometric?

36 $a_n = 2^n$ $\boxed{2, 4, 8, 16 \quad \text{Yes}}$

37 $a_n = n^2$ $\boxed{1, 4, 9, 16 \quad \text{No}}$

#s 45-52 Find required part of the geometric sequence.

45 How many terms if $a = 3$, $r = \frac{1}{2}$ & last term is $\frac{3}{1024}$

121 §8.3 #s 53, 57, 61, 65, 69, 71, 73, 75, 78, 79, 87, 98, 99

$\boxed{45 \text{ cont'd}}$

$\frac{3}{1024} = ar^{n-1} = 3r^{n-1} \Longrightarrow$

$\frac{1}{1024} = r^{n-1} = \left(\frac{1}{2}\right)^{n-1} = \frac{1}{2^{10}} = \left(\frac{1}{2}\right)^{10}$

$\Longrightarrow 10 = n-1 \Longrightarrow \boxed{11 = n}$

2 | 1024
2 | 512
2 | 256
2 | 128
2 | 64
$32 = 2^5$

#s 53-66 Find the sum. Check by brute force.

53 $1+2+4+8+16 = 31$ ✓

$a=1,\ r=2,\ n=5$

$S_5 = \frac{1(1-2^5)}{1-2} = \frac{1-32}{-1} = \frac{-31}{-1} = 31$ ✓

57 $9+3+1+\frac{1}{3}+\frac{1}{9}+\frac{1}{27} = \frac{364}{27}$ ✓

$a=9,\ r=\frac{1}{3},\ n=6$

$\frac{9\left(1-\left(\frac{1}{3}\right)^6\right)}{1-\frac{1}{3}} = \frac{9\left(1-\frac{1}{729}\right)}{\frac{2}{3}} = \frac{9\left(\frac{728}{729}\right)}{\frac{2}{3}}$

$= (3^2)\left(\frac{728}{3^6}\right)\left(\frac{3}{2}\right) = \frac{364}{3^3} = \frac{364}{27}$ ✓

61 $1.5-3+6-12+24-48+96-192 = -127.5$ ✓

$a=1.5,\ r=-2,\ n=8$

$\frac{1.5(1-(-2)^8)}{1-(-2)} = \frac{1.5(1-256)}{3} = \frac{1.5(-255)}{3} = -127.5$ ✓

121 §8.3 #s 65, 69, 71, 73, 75, 78, 79, 87, 98, 99

65 $\sum_{i=0}^{7} 200(1.01)^i = 200(1.01)^0 + 200(1.01)^1 + 200(1.01)^2$

$+ 200(1.01)^3 + 200(1.01)^4 + 200(1.01)^5 +$

$+ 200(1.01)^6 + 200(1.01)^7 \approx \boxed{1657.134113}$

$a = 200,\ r = 1.01,\ n = 8$ (starts @ 0, ends @ 7)

$$\frac{200(1-1.01^8)}{1-1.01} \approx \frac{200(-.0828567056)}{-.01} \approx \boxed{1657.134113}$$

#s 67 - 72 Write the series in Σ-notation

69 $.6 + .06 + .006 + \cdots$

$a = .6 \quad r = .1 \implies \boxed{\sum_{k=0}^{\infty} .6(.1)^{k-1}}$

71 $-4.5 + 1.5 - .5 + \frac{1}{6} - \cdots$

$a = -4.5,\ r = -\frac{1}{3}$ $\boxed{\sum_{k=0}^{\infty} (-4.5)\left(-\frac{1}{3}\right)^{k-1}}$

#s 73 - 86 Find sum, where possible.

73 $3 - 1 + \frac{1}{3} - \frac{1}{9} + \frac{1}{27} - \cdots$

$a = 3,\ r = -\frac{1}{3},\ \sum_{k=0}^{\infty} 3\left(-\frac{1}{3}\right)^{k-1} = \frac{a}{1-r} = \frac{3}{1-(-\frac{1}{3})} = \frac{3}{\frac{4}{3}} =$

$= (3)\left(\frac{3}{4}\right) = \boxed{\frac{9}{4}}$

121 §8.3 #s 75, 78, 79, 87, 98, 99

75 $.9 + .09 + .009 + \ldots$

$a = .9,\ r = .1 \Longrightarrow S = \dfrac{.9}{1-.1} = \boxed{1}$

78 $1.2 - 2.4 + 4.8 - 9.6 + \ldots$

$a = 1.2,\ r = -2 \Longrightarrow$ $\boxed{\text{No sum}}$ $(|r| > 1)$

79 $\sum_{k=1}^{\infty} 34(0.01)^k$ This is a trick.

$= \sum_{k=1}^{\infty} (34)(0.01)(0.01)^{k-1} = \sum_{k=1}^{\infty} .34(.01)^{k-1}$

$a = .34,\ r = .01 \qquad \dfrac{a}{1-r} = \dfrac{.34}{1-.01} = \dfrac{.34}{.99} = \boxed{\dfrac{34}{99}} = .\overline{34}$

#s 87-90

87 Write each repeating decimal as series.

$.04444\ldots = .04 + .004 + .0004 + \ldots$

$a = .04,\ r = .1 \qquad \sum_{k=0}^{\infty} (.04)(.1)^{k-1} \qquad \dfrac{.04}{1-.1} = \dfrac{.04}{.9}$

$= \dfrac{4}{90} = \boxed{\dfrac{2}{45}}$

121 §8.3 #s 98, 99

98 a) End of 40th yr, Payments @ beginning of year

Beginning $9000 / yr, 8% annually

$$\underbrace{9000(1.08)}_{1^{st}\ \text{term} = a} + 9000(1.08)^2 + \cdots + 9000(1.08)^{40}$$

$$\overset{ar}{=} \underbrace{(9000)(1.08)}_{a}\underbrace{(1.08)^1}_{r}$$

$a = 9000(1.08)$

$r = 1.08$

$n = 40$

$$\frac{a(1-r^n)}{1-r} = \frac{9000(1.08)(1-1.08^{40})}{1-1.08}$$

$$\approx \boxed{\$2518029.36}$$

99 $100 @ end of each month for 30 yrs.
Account earns 9% apr compounded monthly
What's value of annuity @ end?

$$n = \left(\frac{1\ \text{payment}}{1\ \text{month}}\right)\left(\frac{12\ \text{months}}{1\ \text{yr}}\right)(30\ \text{years}) = 360\ \text{payments}$$

$$i = \frac{r}{m} = \frac{.09}{12} = \text{interest rate per period.}$$

$$S = \underset{\text{Last pmt}}{100} + 100\left(1+\frac{.09}{12}\right) + 100\left(1+\frac{.09}{12}\right)^2 + \cdots + \underset{1^{st}\ \text{pmt}}{100\left(1+\frac{.09}{12}\right)^{359}}$$

$a = 100$, $r = 1 + \frac{.09}{12}$, $n = 360 \implies S = \frac{a(1-r^n)}{1-r}$

$$= \frac{100\left(1-\left(1+\frac{.09}{12}\right)^{360}\right)}{1-\left(1+\frac{.09}{12}\right)} \approx \boxed{\$183,074.35}$$