121 § 4.4 #s 1, 7, 15, 17, 19, 21, 26, 30, 35, 39, 41, 51, 53, 65, 67, 69, 75

#s 1–14 Solve each eq'n

① $\log_2(x) = 3$

$2^{\log_2(x)} = 2^3$

$\boxed{x = 2^3 = 8}$ $x \in \{8\}$

⑦ $\log_x(9) = 2$

$x^{\log_x(9)} = x^2$

$9 = x^2$

$x = \pm 3 \Longrightarrow \boxed{x = 3}$ $x \in \{3\}$

-3 can't be a base.

#s 15–32

Solve. Give exact sol'ns.

⑮ $\log_2(x+2) + \log_2(x-2) = 5$

$\log_2((x+2)(x-2)) = 5$

$2^{\log_2(x^2-4)} = 2^5$

$x^2 - 4 = 32$

$x^2 = 36$

$x = \pm 6$

$-6 \notin \mathcal{D} \Longrightarrow \boxed{x = 6}$ $x \in \{6\}$

⑰ $\log\left(\frac{x-3}{2}\right) + \log\left(\frac{x+2}{7}\right) = 0$

$10^{\log\left(\frac{x-3}{2}\right) + \log\left(\frac{x+2}{7}\right)} = 10^0$

$10^{\log\left(\frac{x-3}{2}\right)} \, 10^{\log\left(\frac{x+2}{7}\right)} = 1$

$\left(\frac{x-3}{2}\right)\left(\frac{x+2}{7}\right) = 1$

$x^2 - x - 6 = 14$

$x^2 - x - 20 = 0$

Usually, I'll combine the logs, 1st. This time, I used $x^{a+b} = x^a x^b$.

$(x-5)(x+4) = 0$

$x \in \{-4, 5\}$

$x = -4 \notin \mathcal{D}$

$\boxed{x \in \{5\}}$

121 § 4.4 #s 19, 21, 26, 30, 35, 39, 41, 51, 53, 65, 67, 69, 75

(19) $\log(x+1) - \log(x) = 3$

$\log\left(\frac{x+1}{x}\right) = 3$

$\frac{x+1}{x} = 10^3$

$x+1 = 1000x$

$-999x = -1$

$\boxed{x = \frac{1}{999}}$ $x \in \left\{\frac{1}{999}\right\}$

(21) $\log_4(x) - \log_4(x+2) = 2$

$\frac{x}{x+2} = 4^2 = 16$

$x = 16(x+2)$

$x = 16x + 32$

$-15x = 32$

$x = -\frac{32}{15} \notin \mathcal{D}$

$\boxed{\emptyset}$

(26) $\log_3(x) = \log_3(2) - \log_3(x-2)$

$\log_3(x(x-2)) = \log_3(2)$

$x^2 - 2x = 2$

$x^2 - 2x - 2 = 0$

Quad. or Complete Square..

$x = 1 \pm \sqrt{3}$, but $x = 1 - \sqrt{3} \notin \mathcal{D}$, so

$\boxed{x \in \{1+\sqrt{3}\}}$

121 §4.4 #s 30, 35, 39, 41, 51, 53, 65, 67, 69, 75

30 $\log_3(x) + \log_3(1/x) = 0$

$\log_3\left(x \cdot \frac{1}{x}\right) = 0$

$\log_3(1) = 0$

$3^{\log_3(1)} = 3^0$

$1 = 1$

Always true for

$x \in \mathcal{D} = \{ x \mid x > 0 \;\&\; \frac{1}{x} > 0 \}$

$(0,\infty)$ and $(0,\infty)$

$= \boxed{(0,\infty)}$!

#s 33–58 Solve. Round to 4 decimal places (WAIT UNTIL FINAL ANSWER TO ROUND!!!)

35 $(1.09)^{4x} = 3.4$

$\ln\left((1.09)^{4x}\right) = \ln(3.4)$

$4x \ln(1.09) = \ln(3.4)$

$x = \frac{\ln(3.4)}{4\ln(1.09)} \approx 3.550151272 \approx \boxed{3.5502} \approx x$

121 §4.4 #s 39, 41, 51, 53, 65, 67, 69, 75

(39) $e^{-3x^2} = 9$

$-3x^2 = \ln(9)$

" ≤ 0 " = " > 0 " ?!

Impossible!

$\ln(9) > 0$, b/c $9 > e$

$-3x^2 \le 0$ b/c $x^2 \ge 0$

$\boxed{\emptyset}$

(41) $6^x = 3^{x+1}$

$\ln(6^x) = \ln(3^{x+1})$

$x \ln(6) = (x+1)\ln(3)$

$\to x\ln(6) = x\ln(3) + \ln(3)$

let $\ln(6) = a$, $\ln(3) = b$.

Then

$x \cdot b = x \cdot a + a$

$bx = ax + a$

$bx - ax = a$

$x(b-a) = a$

$x = \frac{a}{b-a} =$

$x\ln(6) = x\ln(3) + 1\ln(3)$

$x\ln(6) - x\ln(3) = \ln(3)$

$x(\ln(6) - \ln(3)) = \ln(3)$

$x = \frac{\ln(3)}{\ln(6) - \ln(3)}$

≈ 1.584962501

$\approx \boxed{1.5850 \approx x}$

(51) $(\log(z))^2 = \log(z^2)$

$(\log(z))^2 = 2\log(z)$

$u^2 = 2u$

$u^2 - 2u = 0$

$u(u-2) = 0$

$u = 0$ OR $u = 2$

$\log(z) = 0$ OR $\log(z) = 2$

$z = 1$ OR $z = 10^2 = 100$

$\boxed{z \in \{1, 100\}}$

Square inside-vs-outside the log is tricky!

121 §4.4 #s 53, 65, 67, 69, 75

53 $4(1.02)^x = 3(1.03)^x$

$$\frac{4}{3}(1.02)^x = (1.03)^x$$

$$\ln\left(\frac{4}{3}(1.02)^x\right) = \ln\left((1.03)^x\right)$$

$$\ln\left(\frac{4}{3}\right) + \ln\left((1.02)^x\right) = x\ln(1.03)$$

$$\ln\left(\frac{4}{3}\right) + x\ln(1.02) = x\ln(1.03)$$

Let $a = \ln\left(\frac{4}{3}\right)$, $b = \ln(1.02)$, $c = \ln(1.03)$

$$a + bx = cx$$

$$bx - cx = -a$$

$$x(b-c) = -a$$

$$x = \frac{-a}{b-c} = \frac{a}{c-b} = \frac{\ln\left(\frac{4}{3}\right)}{\ln(1.03) - \ln(1.02)}$$

$$\approx 29.48717854$$

$$\approx \boxed{29.4872 \approx x}$$

65 The half-life of a radioactive substance is 10,000 yrs. What's the decay rate?

$t = 10000$

$A = \frac{1}{2}P$

$$A = Pe^{-kt}$$

$$Pe^{-10000k} = \frac{1}{2}P$$

$$e^{-10000k} = \frac{1}{2}$$

$$-10000k = \ln\left(\frac{1}{2}\right)$$

PERFECT $k = \dfrac{\ln\left(\frac{1}{2}\right)}{-10000}$ OR $\dfrac{\ln 2}{10000}$

$\approx 6.931471806 \times 10^{-5}$

OR $\approx .00006931471806$

OR $\approx .006931471806\,\%$

121 §4.4 #s 67, 69, 75

(67) A bone contains 10% of C-14 it had when critter was alive. <u>ASSUME</u> $\frac{1}{2}$-life of C-14 is 5730 yrs. How old is the bone?

(1) Build Model

(2) Use Model

(1) $\frac{1}{2}$-life is 5730 yrs:

$$Pe^{-5730K} = \frac{1}{2}P$$

$$e^{-5730K} = \frac{1}{2}$$

$$\ln\left(e^{-5730K}\right) = \ln\left(\tfrac{1}{2}\right)$$

$$-5730K = \ln\left(\tfrac{1}{2}\right)$$

$$K = \frac{\ln\left(\frac{1}{2}\right)}{-5730}$$

(2) 10% is left. Find age:

$$Pe^{-Kt} = .10P$$

$$e^{-Kt} = .10$$

$$-Kt = \ln(.10)$$

$$t = \frac{\ln(.1)}{-K}$$

$$= \frac{\ln(.1)}{-\left(\frac{\ln(\frac{1}{2})}{-5730}\right)} = \frac{\ln(.1)}{-\ln(\frac{1}{2})}(-5730)$$

≈ 19034.64798 yrs

$\approx$ $\boxed{19{,}035 \text{ yrs old}}$

REMARKS

Notice how I didn't <u>TOUCH</u> my calculator until the very end?

ALSO, @ the end,

$$\frac{A}{\frac{B}{C}} = A \cdot \frac{C}{B} \text{ OR } \frac{A}{B} \cdot C$$

121 § 4.4 #s 69, 75

69 How long for 12 g of C-14 to decay to 10 g of C-14?

Find t:

$$Pe^{-kt} = 12e^{-kt} \overset{\text{SET}}{=} 10$$

$$e^{-kt} = \frac{10}{12} = \frac{5}{6}$$

$$-kt = \ln\left(\frac{5}{6}\right)$$

$$t = \frac{\ln\left(\frac{5}{6}\right)}{-k}$$

$$= \frac{\ln\left(\frac{5}{6}\right)}{-\left(\frac{\ln\left(\frac{1}{2}\right)}{-5730}\right)}$$

$$= \ln\left(\frac{5}{6}\right)\left(\frac{5730}{\ln\left(\frac{1}{2}\right)}\right)$$

$$\approx 1507.187145$$

$$\approx \boxed{1507 \text{ yrs old}}$$

Found & Dated in 1951.

75 How old are the scrolls, if there's 79.3% of the original C-14 present?

$$Pe^{-kt} = .793P$$

$$e^{-kt} = .793$$

$$-kt = \ln(.793)$$

$$t = \frac{\ln(.793)}{-k}$$

$$= \ln(.793)\left(\frac{5730}{\ln\left(\frac{1}{2}\right)}\right)$$

$$\approx 1917.299422$$

(1951 − 1917 = $\boxed{34 \text{ A.D.!}}$