121 §4.2 #s 9, 13, 17, 21, 25, 29, 31, 34, 38, 47, 49, 51, 59, 63, 67, 69, 73, 79, 83, 87, 89, 93, 97, 99, 101, 107, 113, 117, 121, 127, 129

#s 9–16 Make the statement true by replacing the question mark w/ a real #

9 $2^{?} = 64$

$$\boxed{\begin{matrix} ? = 6 \\ 2^6 = 64 \end{matrix}}$$

$2\underline{|64}$
$2\underline{|32}$
$2\underline{|16}$
$2\underline{|8}$
$2\underline{|4}$
2

13 $16^{?} = 2$

$(2^4)^{?} = 2$

$2^{4?} = 2^1$

$4? = 1$

$\boxed{? = \frac{1}{4}}$

$\boxed{16^{1/4} = 2}$

#s 17–32 Find the indicated value of the logarithmic function.

17 $\log_2(64) =$

$= \log_2(2^6) = \boxed{6}$

21 $\log_{16}(2)$

$= \log_{16}(16^{\frac{1}{4}})$

$= \boxed{\frac{1}{4}}$ See #13!

25 $\log(0.1)$

$= \log(10^{-1})$

$= \boxed{-1}$

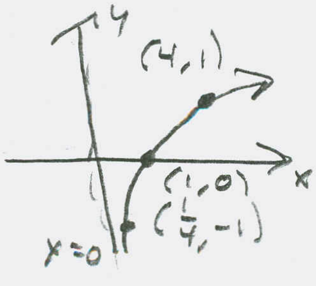
29 $\boxed{\ln(e) = 1}$

31 $\ln(e^{-5})$

$= \boxed{-5}$

#s 33–46 Sketch the graph of each func. and state domain & range.

34 $y = \log_4 x$

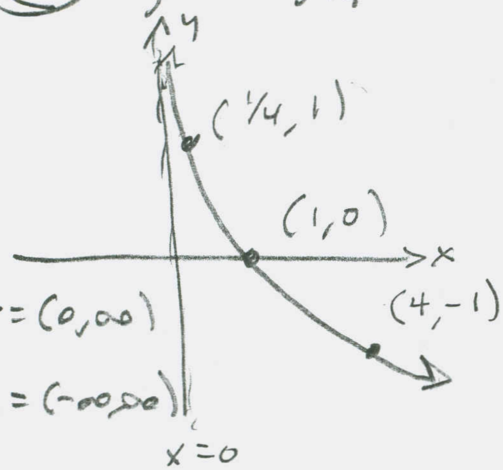


$\mathcal{D} = (0, \infty)$

$\mathcal{R} = (-\infty, \infty)$

121 §4.2 #s 38, 47, 49, 51, 59, 63, 67, 69, 73, 79, 83, 87, 89, 93, 97, 99, 101, 107, 113, 117, 121, 127, 129

38 $y = \log_{1/4}(x)$



$D = (0, \infty)$

$R = (-\infty, \infty)$

#s 47-54 Use graph or table to find each limit.

47 $\lim_{x \to \infty} \log_3(x) = \infty$

49 $\lim_{x \to 0^+} \log_{\frac{1}{2}}(x) = \infty$

51 $\lim_{x \to 0^+} \ln(x) = -\infty$

#s 59-68 Write each eq'n as an equivalent exponential eq'n

59 $\log_2(32) = 5$

$2^{\log_2(32)} = 2^5$

$\boxed{32 = 2^5}$

I always do these with inverse func. idea

63 $\log(1000) = z$

$10^{\log(1000)} = 10^z$

$\boxed{1000 = 10^z}$

121 §4.2 #s 67, 69, 73, 79, 83, 87, 89, 93, 97, 99, 101, 107, 113, 117, 121, 127, 129

67 $\log_a(x) = m$

$a^{\log_a(x)} = a^m$

$\boxed{x = a^m}$

#s 69-78 Write each eq'n as an equivalent logarithmic expression.

69 $5^3 = 125$

$\log_5(5^3) = \log_5(125)$

$\boxed{3 = \log_5(125)}$

73 $y = 10^m$

$\log(y) = \log(10^m)$

$\boxed{\log(y) = m}$

#s 79-88 Find f^{-1}

79 $f(x) = 2^x$

$y = 2^x$

$x = 2^y$

$\log_2(x) = \log_2(2^y)$

$\boxed{\log_2(x) = y = f^{-1}(x)}$

83 $f(x) = \ln(x-1)$

$y = \ln(x-1)$

$x = \ln(y-1)$

$e^x = e^{\ln(y-1)}$

$e^x = y-1$

$\boxed{e^x + 1 = y = f^{-1}(x)}$

87 $f(x) = \frac{1}{2}\cdot 10^{x-1} + 5$

$x = \frac{1}{2}\cdot 10^{y-1} + 5$

$\frac{1}{2}\cdot 10^{y-1} + 5 = x$

$\frac{1}{2}\cdot 10^{y-1} = x-5$

$10^{y-1} = \frac{1}{2}(x-5)$

$\log(10^{y-1}) = \log(\frac{1}{2}(x-5))$

121 §4.2 #s 87, 89, 93, 97, 99, 101, 107, 113, 117, 121, 127, 129

87 cnt'd

$$\log(10^{y-1}) = \log(\tfrac{1}{2}(x-5))$$

$$y-1 = \log(\tfrac{1}{2}(x-5))$$

$$\boxed{f^{-1}(x) = y = \log(\tfrac{1}{2}(x-5)) + 1}$$

#s 89-112 Solve each equation EXACTLY

89 $\log_2(x) = 8$

$$2^{\log_2(x)} = 2^8$$

$$\boxed{x = 2^8} = 256$$

93 $\log_x(16) = 2$

$$x^{\log_x(16)} = x^2$$

$$16 = x^2$$

$$x^2 = 16$$

$$x = \pm 4$$

$$\boxed{x = 4} \quad (b > 0)$$

97 $\ln(x-3) = \ln(2x-9)$

$$e^{\ln(x-3)} = e^{\ln(2x-9)}$$

$$x-3 = 2x-9$$

$$-x = -6$$

$$\boxed{x = 6}$$

99 $\log_x(18) = 2$

$$x^{\log_x(18)} = x^2$$

$$18 = x^2$$

$$x^2 = 18$$

$$x = \pm\sqrt{18} = \pm 3\sqrt{2}$$

$$\boxed{x = 3\sqrt{2}} \quad (b > 0)$$

121 §4.2 #s 101, 107, 113, 117, 121, 127, 129

101 $3^{x+1} = 7$

$\log_3(3^{x+1}) = \log_3(7)$

$x+1 = \log_3(7)$

$\boxed{x = \log_3(7) - 1}$

107 $4^{2x-1} = \frac{1}{2}$

$\log_4(4^{2x-1}) = \log_4(\frac{1}{2})$

$2x-1 = \log_4(\frac{1}{2})$

$2x-1 = -\frac{1}{2}$

$2x = \frac{1}{2}$

$\boxed{x = \frac{1}{4}}$

#s 113–120 Find the solution approximately (to 3 places)

113 $10^x = 25$

$\log(10^x) = \log(25)$

$x = \log(25)$

$\boxed{x \approx 1.398}$

117 $5e^x = 4$

$e^x = \frac{4}{5}$

$\ln(e^x) = \ln(\frac{4}{5})$

$x = \ln(\frac{4}{5})$

$\boxed{x \approx -.223}$

121 Find the amount of time, to the nearest $10^{\underline{th}}$ of a year, that it would take for \$10 to grow to \$20 at each of the following rates, compounded continuously.

121 §4.2 #s 121, 127, 129

121 cont'd This is "doubling time" question

(a) 2% $Pe^{rt} = 2P$

$10e^{.02t} = 20$

$e^{.02t} = \frac{20}{10}$

$\ln(e^{.02t}) = \ln(2)$

$.02t = \ln(2)$

$t = \frac{\ln(2)}{.02}$

$\boxed{t \approx 34.7 \text{ yrs}}$

34.65735903

(b) 4%

$10e^{.04t} = 20$

$e^{.04t} = 2$

⋮

$t = \frac{\ln(2)}{.04}$

$\approx \boxed{17.3 \text{ yrs}}$

17.32867951

(c) 8%

⋮

$t = \frac{\ln(2)}{.08}$

$\approx \boxed{8.7 \text{ yrs}}$

8.664339757

(d) 16%

⋮

$t = \frac{\ln(2)}{.16}$

$\approx \boxed{4.3 \text{ yrs}}$

4.332169878

121 §4.2 #s 127, 129

127 Solve $A = Pe^{rt}$ for r. Then find the rate @ which a deposit of \$1000 would double in 3 yrs. (The \$1000 is arbitrary. Setting $A = 2P$ accomplishes the same purpose)

$$A = Pe^{rt}$$

$$\frac{A}{P} = e^{rt}$$

$$e^{rt} = \frac{A}{P}$$

$$\ln(e^{rt}) = \ln\left(\frac{A}{P}\right)$$

$$rt = \ln\left(\frac{A}{P}\right)$$

$$\boxed{r = \frac{1}{t}\ln\left(\frac{A}{P}\right)}$$

Now, let $t = 3$, $A = 2000$, $P = 1000$

$$r = \frac{1}{3}\ln\left(\frac{2000}{1000}\right)$$

$$r = \frac{1}{3}\ln(2)$$

$$r \approx .2310490602$$

$$\approx \boxed{23.1\%}$$

129 Rule of 70

(a) Find the time it takes for an investment to double @ 10% compounded cnt'sly.

$$A = Pe^{rt} = 2P = \text{Double}$$

$$Pe^{.1t} = 2P$$

$$e^{.1t} = 2$$

$$.1t = \ln(2)$$

$$t = \frac{\ln(2)}{.1} \approx 6.93147180$$

$$\approx \boxed{6.9 \text{ yrs}}$$

121 § 4.2 # 129

129 cont'd

(b) The time that it takes an investment to double is approximately 70 divided by r (r given as a %, here). So, @ 10%, an investment doubles about every 7 yrs. Explain why this works.

The reason it works is because $\ln(2) \approx .6931471806$. When r is reported as a decimal (not a %), solving $Pe^{rt} = 2P$ for t amounts to $\frac{\ln(2)}{r} \approx \frac{.7}{r}$

$$\frac{.7}{.1} = \frac{70}{10}$$

$$\frac{.7}{.05} = \frac{70}{5}$$

etc.