121 §3.6 II #s 61, 66, 81, 86, 92, 95, 98, 107, 110, 115, 120

#s 61 - 72 Find the oblique asymptote and sketch the graph of each rational function

(61) $f(x) = \frac{x^2+1}{x} = \frac{x^2}{x} + \frac{1}{x} = x + \frac{1}{x}$

O.A. : $\boxed{y = x}$ is oblique asymptote

$\mathcal{D}$: $x \neq 0$: $\{x \mid x \neq 0\} = \boxed{(-\infty, 0) \cup (0, \infty)}$

$\boxed{\text{V.A.}: x = 0}$

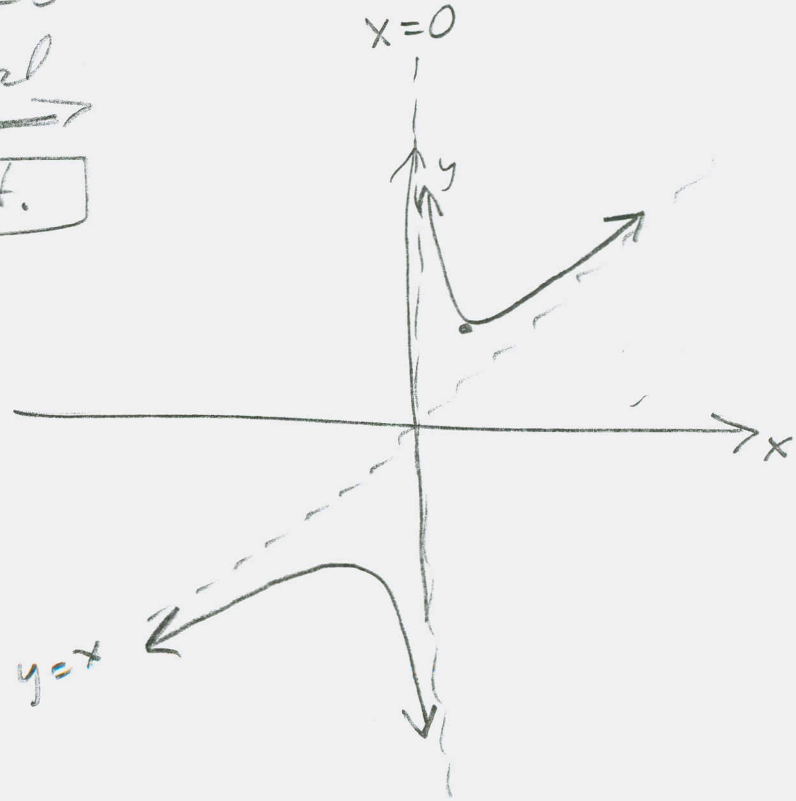
$f(0)$ DNE

$f(x) = 0 \Longrightarrow \frac{x^2+1}{x} = 0$

$\Longrightarrow x^2 + 1 = 0$

$\Longrightarrow$ No real soln $\Longrightarrow$ $\boxed{\text{No x-int.}}$

$f(1) = \frac{1+1}{1} = 2$



§3.6 II #s 66, 81, 86, 92, 95, 98, 107, 110, 115, 120

66 $f(x) = \dfrac{x^2}{x-1}$

$\mathcal{D}$: $\{x \mid x \neq 1\} = (-\infty, 1) \cup (1, \infty)$

$f(0) = \dfrac{0}{-1} = 0 \rightsquigarrow$ $\boxed{(0,0) \text{ is y-int}}$

$f(x) = \dfrac{x^2}{x-1} = 0 \Rightarrow x^2 = 0 \Rightarrow x = 0$

$\Rightarrow$ $\boxed{(0,0) \text{ is x-int}}$

$\boxed{\text{V.A.: } x = 1}$

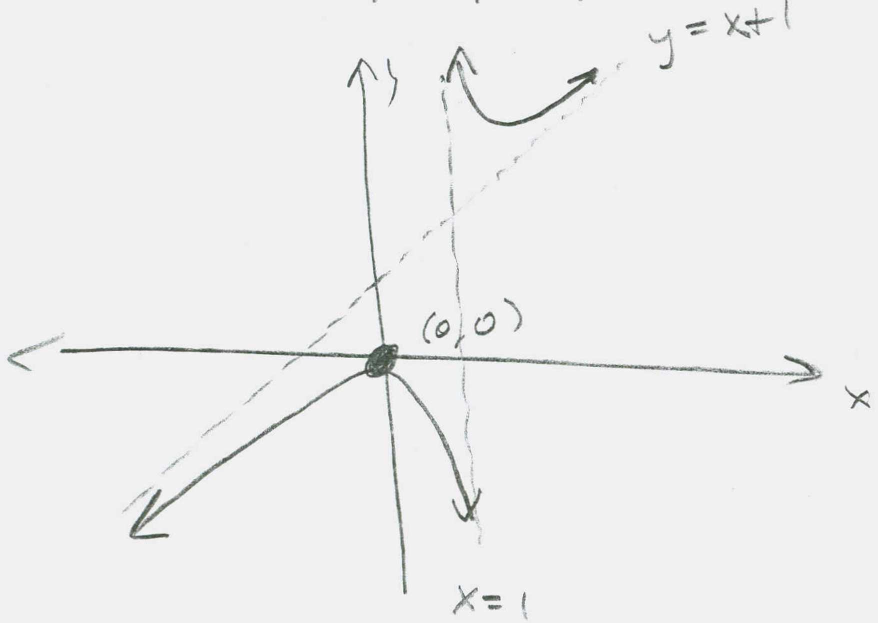
O.A.:

$$\begin{array}{r|rrr} 1 & 1 & 0 & 0 \\ & & 1 & 1 \\ \hline & 1 & 1 & 1 \end{array}$$

This says

$\dfrac{x^2}{x-1} = x+1 + \dfrac{1}{x-1}$

O.A.: $\boxed{y = x+1}$



x = 0 is root of m = 2 <u>touches</u>.

x = 1 is root of m = 1 <u>crosses</u>.

121 3.6 II #s 81, 86, 92, 95, 98, 107, 110, 115, 120

#s 81–88 Sketch the graph of each rational function. Note that the functions are not in lowest terms.

FIND THE DOMAIN FIRST. WHEN YOU CANCEL, LATER, YOU'LL DISCOVER WHICH discarded x-values WILL CORRESPOND TO VERTICAL ASYMPTOTES & WHICH WILL CORRESPOND TO HOLES → NEW!

81 $f(x) = \dfrac{x+1}{x^2-1}$

$\mathcal{D} = \{x \mid x^2-1 \neq 0\} = \{x \mid x \neq \pm 1\} =$

$= (-\infty, -1) \cup (-1, 1) \cup (1, \infty)$

Lowest terms: $\dfrac{x+1}{x^2-1} = \dfrac{x+1}{(x+1)(x-1)}$

$= \dfrac{1}{x-1}$. This function looks just like $\dfrac{1}{x-1}$, EXCEPT it has a HOLE @ $x=-1$: @ $x=-1$, we have hole @

$\dfrac{1}{-1-1} = \dfrac{1}{-2} \rightsquigarrow \boxed{(-1, -\tfrac{1}{2}) = \text{HOLE}}$

121 §3.6 II #s 81, 86, 92, 95, 98, 107, 110, 115, 120

(81) cnt'd

HOLE: $(-1, -\frac{1}{2})$

V.A.: $x = 1$

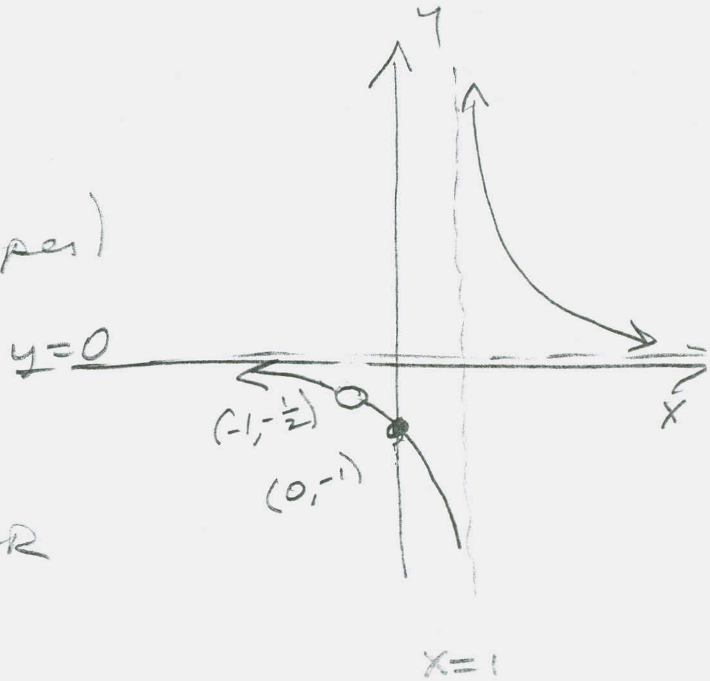
H.A.: $y = 0$ (f is proper)

y-int: $f(0) = -1 \rightsquigarrow$

$(0, -1)$ is y-int

x-int: $\frac{1}{x-1} = 0$ NEVER

No x-int



(86) $f(x) = \dfrac{-x^5 + x^3}{x^3 - x}$

$\mathcal{D} = \{x \mid x^3 - x \neq 0\} = \{x \mid x \neq 0 \text{ and } x \neq \pm 1\}$

$= (-\infty, -1) \cup (-1, 0) \cup (0, 1) \cup (1, \infty)$

$x^3 - x = 0$

$x(x^2 - 1) = 0$

$x(x-1)(x+1) = 0$

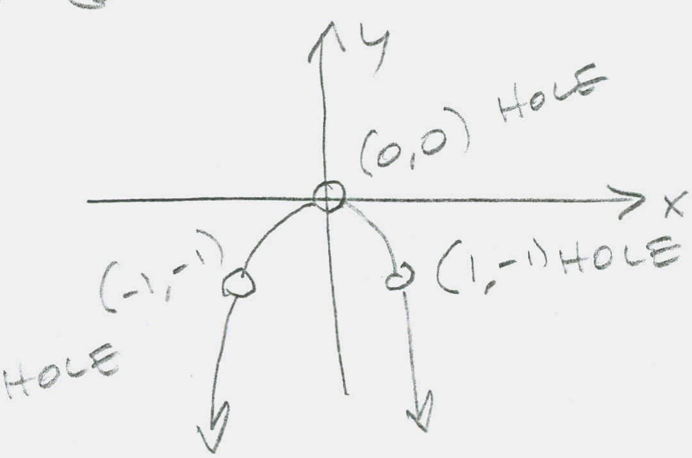
Lowest terms: $f(x) = \dfrac{-x^3(x^2-1)}{x(x^2-1)} = -x^2$

So $f(x) = -x^2$, except @ the holes:

$x = 0 \rightsquigarrow (0, 0)$

$x = -1 \rightsquigarrow (-1, -1)$

$x = 1 \rightsquigarrow (1, -1)$



121 §3.6 II #s 92, 95, 98, 107, 110, 115, 120

#s 89-94 Sketch the graph of each rational function.

(92) $f(x) = \dfrac{x^2+1}{x^3-4x}$ is in lowest terms $\Rightarrow$ (No holes)

$\mathcal{D} = \{x \mid x^3-4x \neq 0\} = \{x \mid x \neq 0 \text{ and } x \neq \pm 2$

$x^3-4x = x(x^2-4) = x(x-2)(x+2) = 0 \Rightarrow \ldots$

V.A.: $x=-2$, $x=0$, $x=2$ No HOLES

H.A.: $y=0$ (f is proper)

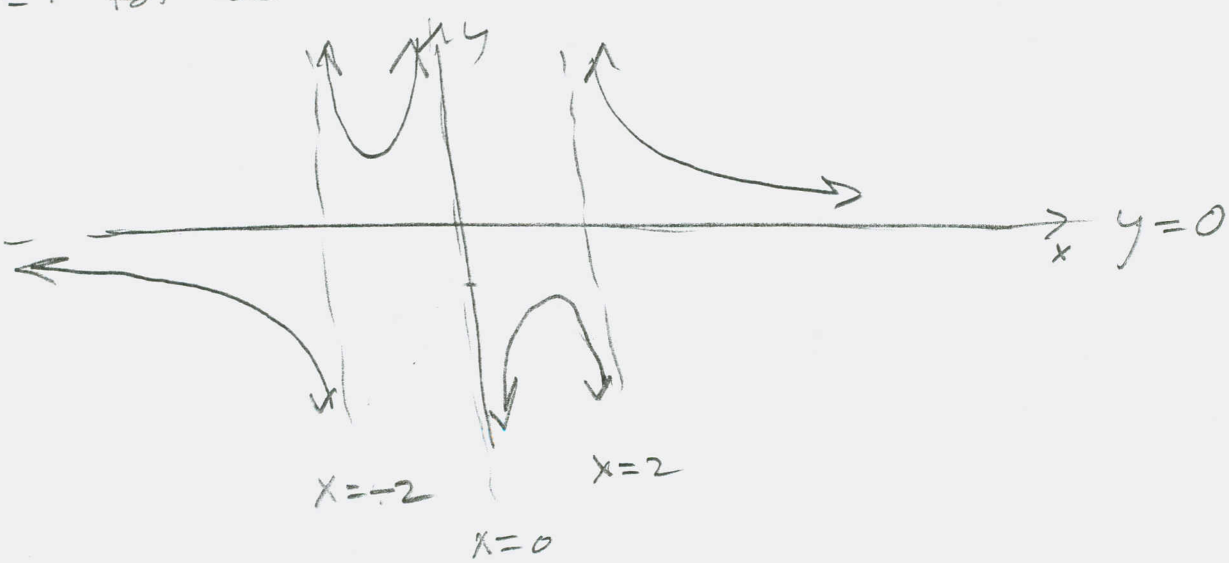
y-int: None. $0 \notin \mathcal{D}(f)$

x-int: None. $x^2+1=0$ has no real roots.

$$\begin{array}{ccccccc} - & u & + & u & - & u & + \\ & -2 & & 0 & & 2 & \end{array}$$

$f(3) = \dfrac{9+1}{27-12} = \dfrac{10}{15}$ is $+$

m=1 for all zeros of denom & numerator.



121 §3.6 II #s 95, 98, 107, 110, 115, 120

#s 95-114 Solve w/ test point method.

(95) $\dfrac{x-4}{x+2} \le 0$ Find zeros of numerator & denominator.

$x-4=0 \Rightarrow x=4$, $x+2=0 \Rightarrow x=-2$

This breaks up $(-\infty,\infty)$ into 3 pieces:

	Test		
$(-\infty,-2)$	$x=-3$	$f(-3)=\frac{-3-4}{-3+2}=\frac{-7}{-1}=7$	+
$(-2,4)$	$x=0$	$f(0)=-\frac{4}{2}=-2$	−
$(4,\infty)$	$x=5$	$f(5)=\frac{5-4}{5+2}=\frac{1}{7}$	+

Now, analyze with respect to the inequality:

Want $\dfrac{x-4}{x+2} \le 0$ So we want the <0 & the $=0$ stuff

− 0

$\boxed{x \in (-\infty,-2)\cup[4,\infty)}$

Don't include $x=-2$. It's not defined at $x=-2$. That's the "u" over the -2 on the sign pattern.

121 §3.6 II #s 98, 107, 110, 115, 120

(98) $\frac{p+1}{2p-1} \geq 1$ Get everything on one side.

$$\frac{p+1}{2p-1} - 1 \geq 0$$

$$\frac{p+1}{2p-1} - 1 \cdot \frac{2p-1}{2p-1} \geq 0$$

$$\frac{p+1-(2p-1)}{2p-1} \geq 0$$

$$\frac{p+1-2p+1}{2p-1} \geq 0$$

$$f(p) = \frac{-p+2}{2p-1} \geq 0$$

Want ≥ 0:

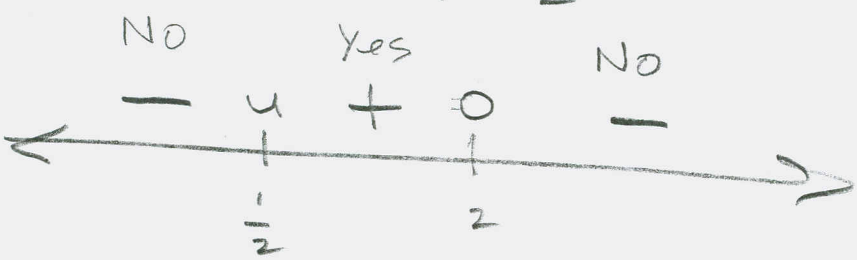
$$\boxed{\left(\tfrac{1}{2}, 2\right]}$$

Again, $x = \frac{1}{2} \notin \mathcal{D}$.

$-p+2=0$ $\qquad$ $2p-1=0$

$p=2$ $\qquad$ $2p=1$

$p=\frac{1}{2}$



$\left(-\infty, \frac{1}{2}\right)$ $\quad x=0 \quad f(0) = \frac{-0+2}{2(0)-1} = -2 < 0$ $\quad -$

$\left(\frac{1}{2}, 2\right)$ $\quad x=1 \quad f(1) = \frac{-1+2}{2(1)-1} = \frac{1}{1} = 1 > 0$ $\quad +$

$(2, \infty)$ $\quad x=3 \quad f(3) = \frac{-3+2}{2(3)-1} = \frac{-1}{5} < 0$ $\quad -$

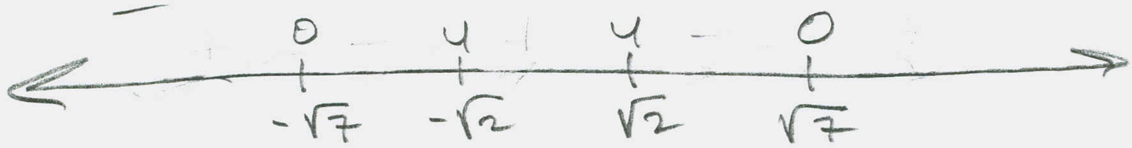
121 §3.6 II #s 107, 110, 115, 120

107 $\dfrac{x^2-7}{2-x^2} \le 0$

$x^2-7=0$ $\qquad$ $2-x^2=0$

$x=\pm\sqrt{7}$ $\qquad$ $x=\pm\sqrt{2}$

$\underline{m=1}$ $\qquad$ $\underline{m=1}$



If you can use H.A., here, you save LOTS of time.

$y = \dfrac{x^2}{-x^2} = -1$

$y=-1$ is H.A. This means negative to far right & far left.

Want ≤ 0. Don't lose sight of $\mathcal{D}$.

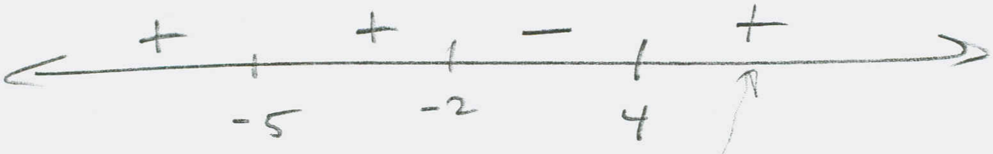
$x \ne \pm\sqrt{2}$

$\boxed{(-\infty,-\sqrt{7}] \cup (-\sqrt{2},\sqrt{2}) \cup (\sqrt{7},\infty)}$

How'd I know it alternated in sign?

121 §3.6 II #s 110, 115, 120

(110) $$\frac{x^2-2x-8}{x^2+10x+25} = \frac{(x-4)(x+2)}{(x+5)^2} \le 0$$

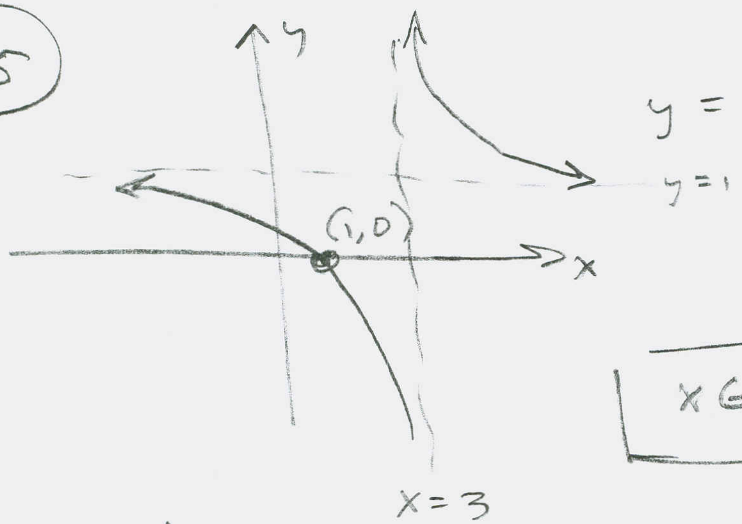


H.A. $y = \frac{x^2}{x^2} = 1$, $y = 1$

Why didn't sign change as we cross $x = 5$?

$\boxed{x \in [-2, 4]}$

(115)

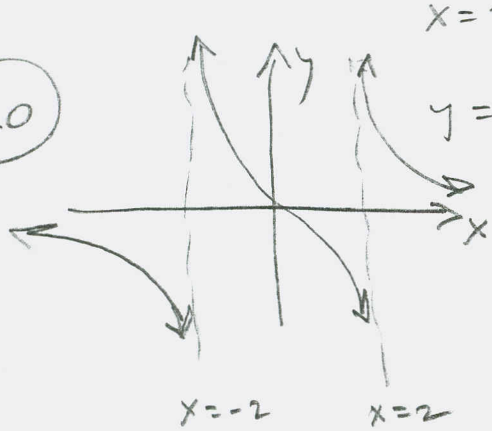


$y = \frac{x-1}{x-3}$

$\frac{x-1}{x-3} > 0 \Longrightarrow$

$\boxed{x \in (-\infty, 1) \cup (3, \infty)}$

(120)



$y = \frac{x}{x^2-4}$

$\frac{x}{x^2-4} > 0 \Longrightarrow$

$\boxed{x \in (-2, 0) \cup (2, \infty)}$