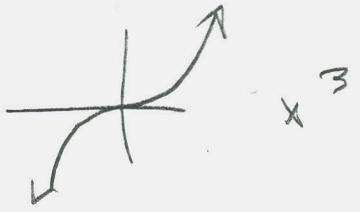
121 §3.5 II #s 45, 50, 53, 54, 69, 70, 72, 75, 78, 84, 92, 98

#s 45-52 Determine whether the limit is $\pm\infty$.

(45) $\lim_{x\to\infty}(x^2-4) = \boxed{+\infty}$

(50) $\lim_{x\to-\infty}(x^3-5) = \boxed{-\infty}$



#s 53-56 Make a rough sketch that shows behavior near x-intercepts and as $x\to\infty$ & $x\to-\infty$

(53) $f(x) = (x-1)^2(x-3)$

As $x\to\infty$, it is handy to ignore the smaller stuff:

$(x-1)^2(x-3) \xrightarrow{x\to\infty} (x)^2(x) = x^3$

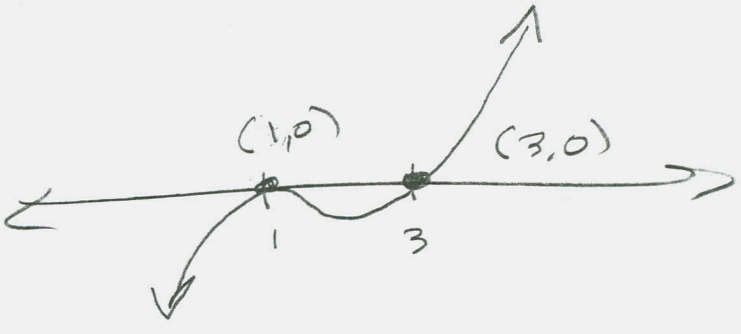
$\lim_{x\to\infty} f(x) = +\infty$

$\lim_{x\to-\infty} f(x) = -\infty$

zeros: $x = 1, 3$

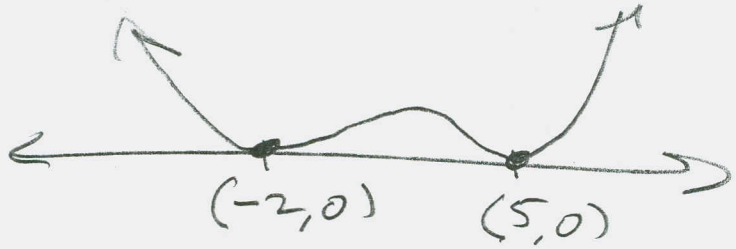
$x=1$ touch

$x=3$ cross



121 §3.5 II #s 54, 69, 70, 72, 75, 78, 84, 92, 98

(54)



$f(x) = (x+2)^2(x-5)^2$

(-2,0) touch

(5,0) touch

As $x \to \infty$, $f(x)$ acts like $(x)^2(x)^2 = x^4$

$\lim_{x\to\infty} f(x) = +\infty$

$\lim_{x\to-\infty} f(x) = -\infty$

#s 65-84 Sketch the graph of each polynomial function.

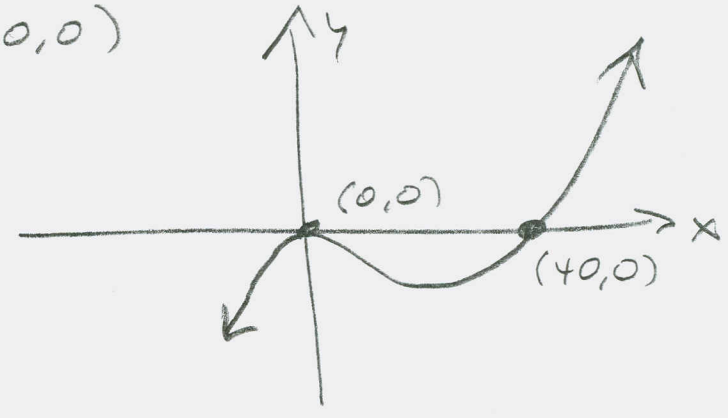
(69) $f(x) = x^3 - 40x^2 = x^2(x-40)$

$f(0) = 0 \rightsquigarrow (0,0)$

$x=0$ touch

$x=40$ cross

x^3 does



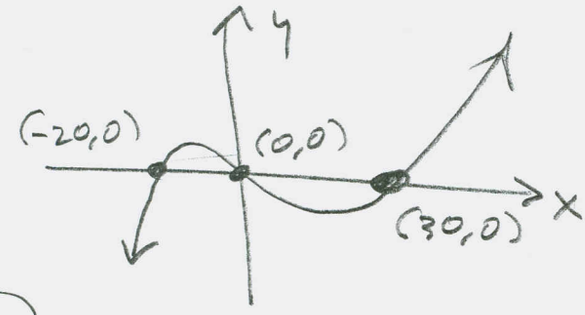
121 §3.5 # #5 70, 72, 75, 78, 84, 92, 98

(70) $f(x) = x^3 - 900x$ Same as #69, only instead of (40,0), it has x-int (900,0)

(75) $f(x) = x^3 - 10x^2 - 600x$

$= x(x^2 - 10x^2 - 600) = x(x-30)(x+20)$

x^3 does this:



2|50
5|25
5

(78) $f(x) = x^3 - 7x^2 - 25x - 50$

1 positive root

$\frac{p}{q}$: $\pm 1, \pm 2, \pm 5, \pm 10, \pm 50$

1| 1 -7 -25 -50
1 -6 NOPE
1 -6 -31

2| 1 -7 -25 -50
2 -10 -70
1 -5 -35 NO

5| 1 -7 -25 -50
5 -10
1 -2 -35

121 §3.5 II #s 78, 84, 92, 98

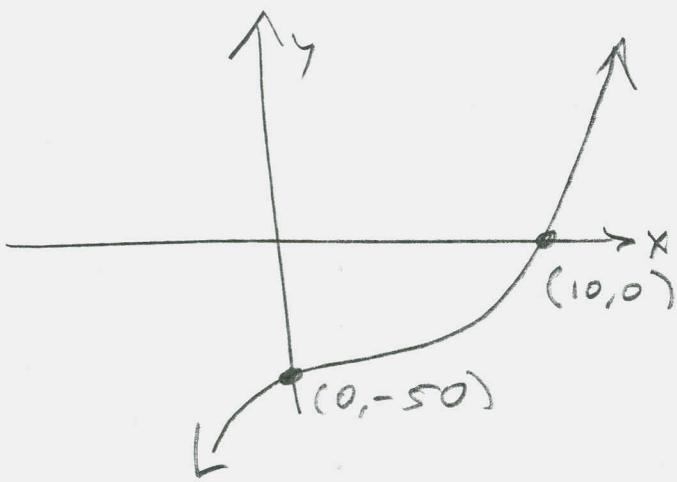
(78) cont'd

10	1	−7	−25	−50
		10	30	50
	1	3	5	0

Finally!

$(x-10)(x^2+3x+5)$ Now break down the quadratic:

$x^2+3x+\left(\frac{3}{2}\right)^2 = -5+\frac{9}{4} = \frac{-20+9}{4} < 0$ No real zeros.



$f(0) = -50$

$(0,-50)$

W/O graphing calculator or calculus, this is about all we can say about $f(x)$.

(84) $f(x) = x(x+6)^2(x^2-x-12)$

$= x(x+6)^2(x-4)(x+3)$

$(0,0)$, $(-6,0)$, $(4,0)$, $(-3,0)$

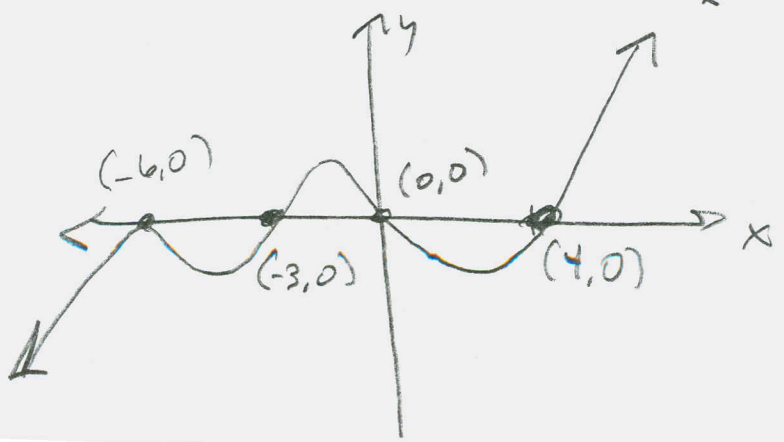
cross, touch, cross, cross

$f(0) = 0$

$\lim_{x\to\infty} f(x) = \lim_{x\to\infty} x^5 = +\infty$

$\lim_{x\to-\infty} f(x) = \lim_{x\to-\infty} x^5$

$= -\infty$



121 §3.5 II #s 92, 98

#s 85–96 Solve each polynomial inequality with test-point method

(92) $x^3 + 7x^2 - 36 \le 0$

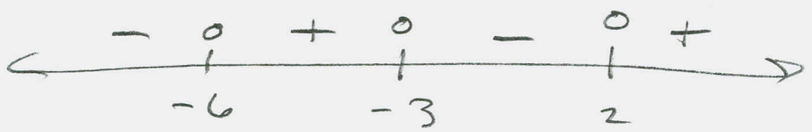
$\pm 1, \pm 2, \pm 3, \pm 4, \pm 6, \pm 9, \pm 12, \pm 18, \pm 36$

1 | 1 7 0 −36
1 8
1 8 8

2 | 1 7 0 −36
2 18 36
1 9 18 0

$(x-2)(x^2+9x+18)$

$= (x-2)(x+3)(x+6) \le 0$



$(-\infty, -6)$ $x = -7$ $f(-7) = -36 < 0$ −

$(-6, -3)$ $x = -4$ $f(-4) = 12 > 0$ +

$(-3, 2)$ $x = 0$ $f(0) = -36 < 0$ −

$(2, \infty)$ $x = 3$ $f(3) = 54 > 0$ +

Want ≤ 0, so take the "−" intervals

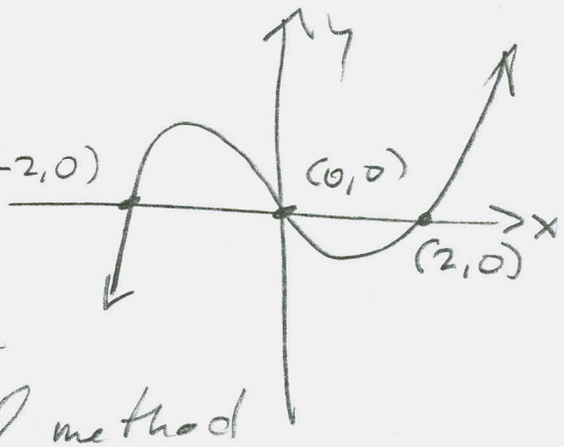
$$\boxed{x \in (-\infty, -6] \cup [-3, 2]}$$

121 §3.5 II #98

#s 97-104 state Soln Sets by reading the graph.

(98) $x^3 - 4x \leq 0$

$x \in (-\infty, -2] \cup [0, 2]$



Compare this visual method to #92 Test Point method. The Test Point method is a sure way, but the better you're visualizing these, the faster & more efficiently you're going to work them.

The sign pattern informs the graph informs the sign pattern