121 §3.3 #s 11, 12, 15, 17, 20, 23, 31, 37, 43, 46, 47, 61, 65, 69

#s 5-14 State the degree & find all real & nonreal roots of each eq'n, stating multiplicity when it's greater than one.

(11) $x^2 - 10x + 25 = 0$

$(x-5)^2 = 0$

Degree: 2

Roots: $x = 5$, $m = 2$

(12) $(2x^2 + x)^2 (3x-1)^4 = 0$

$(x(2x+1))^2 (3x-1)^4 = 0$

$x^2 (2x+1)^2 (3x-1)^4 = 0$

$n = 8$

$x = 0$, $m = 2$

$x = -\frac{1}{2}$, $m = 2$

$x = \frac{1}{3}$, $m = 4$

#s 15-22 Find each product.

(15) $(x-3i)(x+3i) = x^2 - (3i)^2 = \boxed{x^2 + 9}$

(17) $(x - (1+\sqrt{2}))(x - (1-\sqrt{2}))$

$= x^2 - (1-\sqrt{2})x - (1+\sqrt{2})x + (1+\sqrt{2})(1-\sqrt{2})$

$= x^2 - x + \sqrt{2}x - x - \sqrt{2}x + 1 - 2$

$= \boxed{x^2 - 2x - 1}$

(20) $(x - (3-i))(x - (3+i))$

$= x^2 - (3+i)x - (3-i)x + (3-i)(3+i)$

$= x^2 - 3x - ix - 3x + ix + 3^2 + 1^2 = \boxed{x^2 - 6x + 10}$

121 § 3.3 #s 23, 31, 37, 43, 46, 47, 61, 65, 69

#s 23 - 42 Find a polynomial with real coefficients that has the given roots.

23 $-3, 5$: $P(x) = (x+3)(x-5) = \boxed{x^2 - 2x - 15}$

31 $0, i\sqrt{3}$ $(x-0)(x-i\sqrt{3})(x-(-i\sqrt{3}))$

$= x(x-i\sqrt{3})(x+i\sqrt{3}) = x(x^2+(\sqrt{3})^2)$

$(a-bi)(a+bi) = a^2+b^2$

$= \boxed{x^3 + 3x}$

37 $1, 2-3i$

$(x-1)(x-(2-3i))(x-(2+3i))$

$= (x-1)(x^2 - (2+3i)x - (2-3i)x + (2-3i)(2+3i))$

$= (x-1)(x^2 - 4x + 2^2 + 3^2)$

$= (x-1)(x^2 - 4x + 13)$

$= x^3 - 4x^2 + 13x$
$\quad - x^2 + 4x - 13$

$\boxed{x^3 - 5x^2 + 17x - 13}$

'21' §3.3 #s 43, 46, 47, 61, 65, 69

#s 43-52 Use Descartes' Rule of Signs to discuss the possibilities for the roots of each eq'n. Do NOT SOLVE THE EQ'N.

43 $P(x) = x^3 + 5x^2 + 7x + 1 = 0$

0 positive roots

$P(-x) = -x^3 + 5x^2 - 7x + 1 = 0$

3 negative roots, or 1 negative & 2 nonreal.

46 $P(x) = -x^4 - 5x^2 - x + 7 = 0$

1 positive root

$P(-x) = -x^4 - 5x^2 + x + 7$

1 negative root

$\therefore$ 2 nonreal roots.

47 $P(y) = y^4 + 5y^2 + 7 = 0$

0 positive roots

$P(-y) = y^4 + 5y^2 + 7$

0 negative roots

$\therefore$ 4 nonreal roots.

121 §3.3 #s 61, 65, 69

#s 61–76 Use Rational Zeros Theorem, Descartes' Rule of Signs and Thm on Bounds as aids in finding all roots of each eq'n.

61 $x^3-4x^2-7x+10=0$

2 or 0 positive

$P(-x)=-x^3-4x^2-7x+10$

1 negative zero.

$a_n=1$ $\quad \frac{p}{q}: \pm1, \pm2, \pm5, \pm10$

$a_0=10$

$x=5$

$$\begin{array}{r|rrrr} 5 & 1 & -4 & -7 & 10 \\ & & 5 & 5 & -10 \\ \hline & 1 & 1 & -2 & 0 \end{array}$$

$(x-5)(x^2+x-2)$

I was just checking to see if $x=5$ was an upper bound! Now that I've reduced it to a quadratic . . .

$x^2+x=2$

$x^2+x+\left(\frac{1}{2}\right)^2=2+\frac{1}{4}=\frac{9}{4}$

$\left(x+\frac{1}{2}\right)^2=\frac{9}{4}$

$x+\frac{1}{2}=\pm\frac{3}{2} \Rightarrow x=\frac{-1\pm3}{2}$ $\begin{cases} -2 \\ +1 \end{cases}$

zeros/roots:

$\boxed{x\in\{-2,1,5\}}$

3.3 #s 65, 69

65 $\quad x^4+2x^3-7x^2+2x-8=0$

3 or 1 positive roots

$x^4-2x^3-7x^2-2x-8=P(-x)$

Exactly one negative root.

$\frac{p}{q}$: $\pm 1, \pm 2, \pm 4, \pm 8$

Bounds:

4	1	2	-7	2	-8
		4	24	68	Pos
	1	6	17	Pos	Pos

4 is upper bd.

-4	1	2	-7	2	-8
		-4	8	-4	8
	1	-2	1	-2	0

$x=4$ is a root!

$(x+4)(x^3-2x^2+x-2)$

Aha! x^3-2x^2+x-2

$= x^2(x-2)+1(x-2)$

$= (x^2+1)(x-2) = (x-i)(x+i)(x-2)$

$\Rightarrow$ roots are $\boxed{x \in \{-4, 2, \pm i\}}$

121 §3.3 #69

69 $P(x) = x^4 + 2x^3 - 3x^2 - 4x + 4 = 0$

2 or 0 positive roots.

$P(-x) = x^4 - 2x^3 - 3x^2 + 4x + 4$

2 or 0 negative

$\frac{p}{q}$: $\pm 1, \pm 2, \pm 4$

$$\begin{array}{r|rrrrr} 1 & 1 & 2 & -3 & -4 & 4 \\ & & 1 & 3 & 0 & -4 \\ \hline & 1 & 3 & 0 & -4 & 0 \end{array}$$

$(x-1)(x^3+3x^2-4)$

Nice

$$\begin{array}{r|rrrr} 1 & 1 & 3 & 0 & -4 \\ & & 1 & 4 & 4 \\ \hline & 1 & 4 & 4 & 0 \end{array}$$

Nice!

$(x-1)^2(x^2+4x+4)$

$= (x-1)^2(x+2)^2$

$x \in \{1, -2\}$

1 is root of multiplicity 2 and

2 " " " " " .

I skipped Bounds on zeros, because the rational "possibles" weren't very big.