121 §3.2 #s 5, 11, 15, 19, 25, 29, 37, 41, 47, 53, 55, 58, 65, 77

#s 5-10 Use ordinary division to find quotient & remainder

$(x^2-5x+7) \div (x-2)$

$$
\begin{array}{r l}
 & x-1 \quad r\,5 \\
x-2 \,\big) & x^2-5x+7 \\
 & -(x^2-4x) \\
\hline
 & -x+7 \\
 & -(-x+2) \\
\hline
 & 5
\end{array}
$$

Quotient: $x-1$

Remainder: 5

#s 11-22 Use synthetic division to find quotient and remainder.

11. $(x^2+4x+1) \div (x-2)$

$$
\begin{array}{r|rrr}
2 & 1 & 4 & 1 \\
 & & 2 & 12 \\
\hline
 & 1 & 6 & 13
\end{array}
$$

Quotient: $x+6$

Remainder: 13

15. $(4x^3-5x+2) \div (x-\frac{1}{2})$

$$
\begin{array}{r|rrrr}
\frac{1}{2} & 4 & 0 & -5 & 2 \\
 & & 2 & 1 & -2 \\
\hline
 & 4 & 2 & -4 & 0
\end{array}
$$

Quotient: $4x^2+2x-4$

Remainder: 0

121 §3.2 #s 19, 25, 29, 37, 41, 47, 53, 55, 58, 65, 77

19 $(x^4-3) \div (x-1)$

Quotient: x^3+x^2+x+1

Remainder: -2

$$\begin{array}{r|rrrrr} 1 & 1 & 0 & 0 & 0 & -3 \\ & & 1 & 1 & 1 & 1 \\ \hline & 1 & 1 & 1 & 1 & -2 \end{array}$$

~~#s 25~~ #s 23-34 $f(x) = x^5-1$, $g(x) = x^3-4x^2+8$, $h(x) = 2x^4+x^3-x^2+3x+3$. Find the values by Synthetic Division

25 $f(-2)$:

$$\begin{array}{r|rrrrrl} -2 & 1 & 0 & 0 & 0 & 0 & -1 \\ & & -2 & 4 & -8 & 16 & -32 \\ \hline & 1 & -2 & 4 & -8 & 16 & \boxed{-33 = f(-2)} \end{array}$$

29 $g(-\frac{1}{2})$

$$\begin{array}{r|rrrl} -\frac{1}{2} & 1 & -4 & 0 & 8 \\ & & -\frac{1}{2} & \frac{9}{4} & -\frac{9}{8} \\ \hline & 1 & -\frac{9}{2} & \frac{9}{4} & \boxed{\frac{55}{8} = g(-\frac{1}{2})} \end{array}$$

#s 35-38 Determine if the given binomial is a factor of the polynomial following it. If it is, then factor completely.

121 §3.2 #s 37, 41, 47, 53, 55, 58, 65, 77

37 $x-4$; $x^3+4x^2-17x-60$

4	1	4	-17	-60
		4	32	60
	1	8	15	0

Yes, we have $(x-4)(x^2+8x+15)$

Now to break down $x^2+8x+15$:

$x^2+8x+15 = (x+3)(x+5)$, so final answer is

$\boxed{(x-4)(x+3)(x+5)}$

#s 39-46 Determine if the # is a zero of the function

41 -2; $g(d)= d^3+2d^2+3d+1$

-2	1	2	3	1
		-2	0	-6
	1	0	3	-5

No

#s 47-54 Find all possible rational zeros with rational zeros theorem.

47 $f(x)= x^3-9x^2+26x-24$

$a_n=1$, $a_0=-24$

$\frac{p}{q}$: $\pm1, \pm2, \pm3, \pm4, \pm6, \pm8, \pm12, \pm24$

121 § 3.2 #s 53, 55, 58, 65, 77

53 $M(x) = 18x^3 - 21x^2 + 10x - 2$

$a_n = 18, \quad a_0 = -2$ $\qquad 18 = 2 \cdot 3 \cdot 3$

$\frac{p}{q}$: $\pm 1, \pm \frac{1}{2}, \pm \frac{1}{3}, \pm \frac{1}{6}, \pm \frac{1}{9}, \pm \frac{1}{18}$

$\pm 2, \cancel{\pm \frac{2}{2}}, \pm \frac{2}{3}, \cancel{\pm \frac{2}{6}}, \pm \frac{2}{9}, \cancel{\pm \frac{2}{18}}$

#s 55-78 Find all real and imaginary zeros for each polynomial function.

55 $f(x) = x^3 - 9x^2 + 26x - 24$

$\pm 1, \pm 2, \pm 3, \pm 4, \pm 8, \pm 12, \pm 24$

$$\begin{array}{r|rrrr} 1 & 1 & -9 & 26 & -24 \\ & & 1 & -8 & 18 \\ \hline & 1 & -8 & 18 & \text{No} \end{array} \qquad \begin{array}{r|rrrr} -1 & 1 & -9 & 26 & -24 \\ & & -1 & 10 & \\ \hline & 1 & -10 & 36 & \text{No} \end{array}$$

$$\begin{array}{r|rrrr} 2 & 1 & -9 & 26 & -24 \\ & & 2 & -14 & 24 \\ \hline & 1 & -7 & 12 & 0 \text{ Yes!} \end{array}$$

This gives $f(x) = (x-2)(x^2 - 7x + 12)$

Now break down $x^2 - 7x + 12$:

$(x-4)(x-3)$ does it. So

$f(x) = (x-2)(x-4)(x-3)$ & its zeros are

$\boxed{\{2, 3, 4\}}$

121 §3.2 #s 58, 65, 77

58 $m(x) = x^3 + 4x^2 + 4x + 3$

$\pm 1, \pm 3$

$$\begin{array}{r|rrrr} 1 & 1 & 4 & 4 & 3 \\ & & 1 & 5 & \\ \hline & 1 & 5 & 9 & \text{No} \end{array} \qquad \begin{array}{r|rrrr} -1 & 1 & 4 & 4 & 3 \\ & & -1 & -3 & -1 \\ \hline & 1 & 3 & 1 & \text{No} \end{array}$$

$$\begin{array}{r|rrrr} 3 & 1 & 4 & 4 & 3 \\ & & 3 & 21 & \\ \hline & 1 & 7 & 25 & \text{No} \end{array} \qquad \begin{array}{r|rrrr} -3 & 1 & 4 & 4 & 3 \\ & & -3 & -3 & -3 \\ \hline & 1 & 1 & 1 & 0 \end{array} \text{ Yes.}$$

This gives $m(x) = (x+3)(x^2+x+1)$

Now to solve $x^2+x+1 = 0$

$$x^2 + x = -1$$

$$x^2 + x + \left(\tfrac{1}{2}\right)^2 = -1 + \tfrac{1}{4}$$

$$\left(x+\tfrac{1}{2}\right)^2 = -\tfrac{3}{4}$$

$$x + \tfrac{1}{2} = \pm\sqrt{-\tfrac{3}{4}} = \pm i\frac{\sqrt{3}}{\sqrt{4}} = \pm\frac{i\sqrt{3}}{2}$$

$$\boxed{\begin{array}{c} x = \dfrac{-1 \pm i\sqrt{3}}{2} \\ \text{OR} \\ x = -3 \text{, by above work} \end{array}}$$

Factored ~~Form~~:

$$(x+3)\left(x - \left(\frac{-1+i\sqrt{3}}{2}\right)\right)\left(x - \left(\frac{-1-i\sqrt{3}}{2}\right)\right)$$

121 §3.2 #s 65, 77

65 $S(w) = w^4 + w^3 - w^2 + w - 2$

$\pm 1, \pm 2$

$$\begin{array}{r|rrrrr} 1 & 1 & 1 & -1 & 1 & -2 \\ & & 1 & 2 & 1 & 2 \\ \hline & 1 & 2 & 1 & 2 & 0 \end{array}$$ Yes

This gives $(w-1)(\underline{w^3+2w+w+2})$

See if 1 works again:

$$\begin{array}{r|rrrr} 1 & 1 & 2 & 1 & 2 \\ & & 1 & 3 & 4 \\ \hline & 1 & 3 & 4 & \end{array}$$ No

$$\begin{array}{r|rrrr} -1 & 1 & 2 & 1 & 2 \\ & & -1 & -1 & 0 \\ \hline & 1 & 1 & 0 & \end{array}$$ No

Notice I'm working with $w^3+2w+w+2$, now that I've split of a factor of $w-1$.

$$\begin{array}{r|rrrr} -2 & 1 & 2 & 1 & 2 \\ & & -2 & 0 & -2 \\ \hline & 1 & 0 & 1 & \end{array}$$ Yes

This gives us

$(w-1)(w+2)(w^2+1)$

$w^2+1=0$

$w^2=-1$

$w = \pm\sqrt{-1} = \pm i$

$\boxed{\text{Zeros: } w = -2, 1, \pm i}$

Factored Form:

$(w-1)(w+2)(w-i)(w+i)$

#77 Factor the two pieces separately!

$x = 1, 3, 5, 2 \pm \sqrt{3}$