

Solve the following equations. For each equation, state whether it is conditional, identity or inconsistent.

5pts 1. $\frac{3}{2}x + \frac{1}{3} = \frac{1}{4}x - \frac{1}{6}$ LCD = 24

$$24\left(\frac{3}{2}x\right) + 24\left(\frac{1}{3}\right) = 24\left(\frac{1}{4}x\right) - 24\left(\frac{1}{6}\right)$$

$$12(3x) + 8(1) = 6x - 4(1)$$

$$36x + 8 = 6x - 4$$

$$30x = -12 \implies x = -\frac{12}{30} = \boxed{-\frac{2}{5} = x} \quad x \in \left\{-\frac{2}{5}\right\}$$

5pts 2. $\frac{1}{x-1} - \frac{1}{x+1} = \frac{2}{x^2-1}$ LCD = $(x-1)(x+1)$

$$(x-1)(x+1)\left(\frac{1}{x-1}\right) - (x-1)(x+1)\left(\frac{1}{x+1}\right) = \left(\frac{2}{(x-1)(x+1)}\right)(x-1)(x+1)$$

$$x+1 - (x-1) = 2$$

$$2 = 2$$

IDENTITY

$$x \in \left\{x \mid x \neq \pm 1\right\}$$

3. (5 pts bonus) Give an example of an equation in the variable x that is...

a. ... an identity. $3x + 2 = 3x + 2$

b. ... inconsistent. $3x + 2 = 3x - 1$

- 5 pts 4. The old combine can harvest the crop in 96 hours, but a new one can do it in 72 hours. How many hours will it take the two of them operating at the same time to harvest the crop?

Let x = the # of hours it takes them to finish, if they work together.

$$\text{Then } \frac{1}{96}x + \frac{1}{72}x = 1$$

$$288\left(\frac{1}{96}x\right) + 288\left(\frac{1}{72}x\right) = 288(1)$$

$$3x + 4x = 288$$

$$7x = 288$$

$$x = \frac{288}{7} \text{ hrs}$$

$$\begin{array}{r} 2 \overline{)96} \\ 2 \overline{)48} \\ 2 \overline{)24} \\ 2 \overline{)12} \\ 2 \overline{)6} \\ \hline 3 \\ \hline 3 \end{array} \quad \begin{array}{r} 2 \overline{)72} \\ 2 \overline{)36} \\ 2 \overline{)18} \\ 3 \overline{)9} \\ \hline 3 \end{array}$$

$$\text{LCD} = 2^5 \cdot 3^2 = 288$$

- 5 pts 5. How much 75% antifreeze solutions should be added to 3 quarts of 30% antifreeze solution to yield a 50% antifreeze solution?

Let x = the amt of 75% antifreeze sol'n (quarts)

Then

$$.75x + .3(3) = .5(x+3)$$

$$.75x + .9 = .5x + 1.5$$

$$.25x = .6$$

$$x = \frac{.6}{.25} = 2.4 \text{ gts} = x$$

- 5 pts 6. Find the exact distance between $(-2, 5)$ and $(6, 8)$.

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} = \sqrt{(6 - (-2))^2 + (8 - 5)^2}$$

$$= \sqrt{8^2 + 3^2} = \sqrt{64 + 9} = \sqrt{73}$$

7. Determine the center and radius of the circle given by

5 pts

~~$x^2 - 4x + 29 + y^2 + 10y = 49$~~ $x^2 - 4x + y^2 + 10y = 20$

$x^2 - 4x + y^2 + 10y = 20$

$x^2 - 4x + 2^2 + y^2 + 10y + 5^2 = 20 + 4 + 25$

$(x-2)^2 + (y+5)^2 = 49$

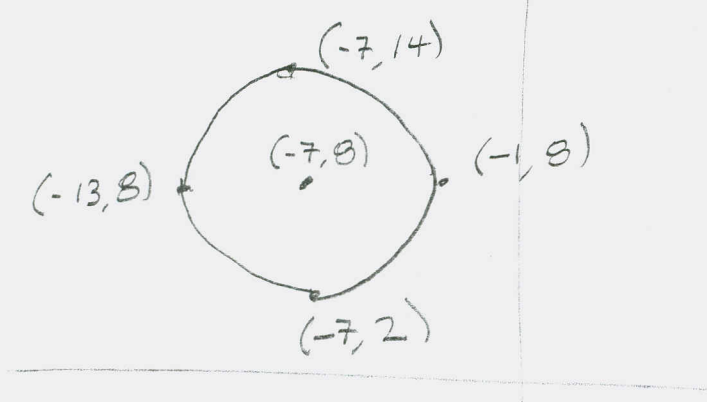
$(h, k) = (2, -5)$
 $r = 7$

8. Sketch the graph of $(x+7)^2 + (y-8)^2 = 36$.

$(h, k) = (-7, 8)$

$r = 6$

5 pts

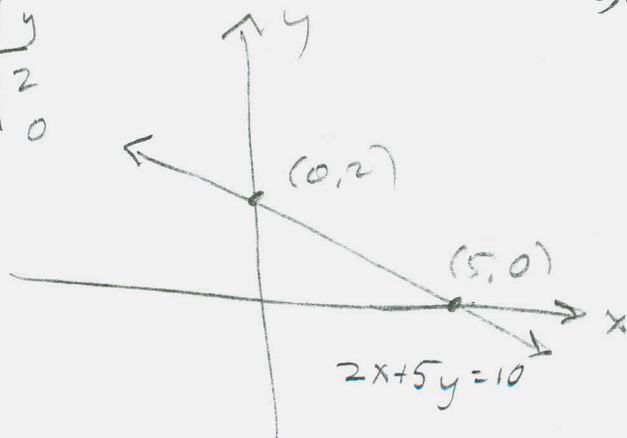


9. Sketch the graph of each of the following equations. Main points I want to see are the intercepts.

5 pts

a. $2x + 5y = 10$

x	y
0	2
5	0

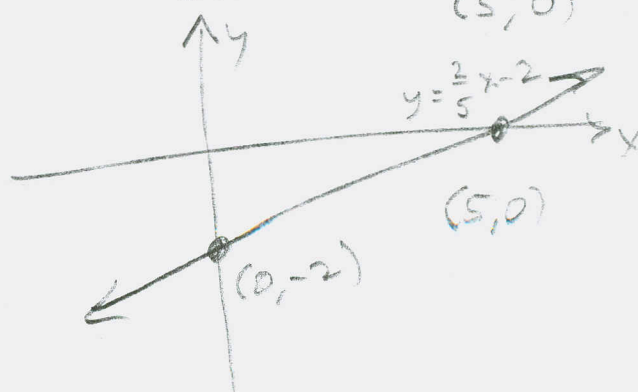


5 pts

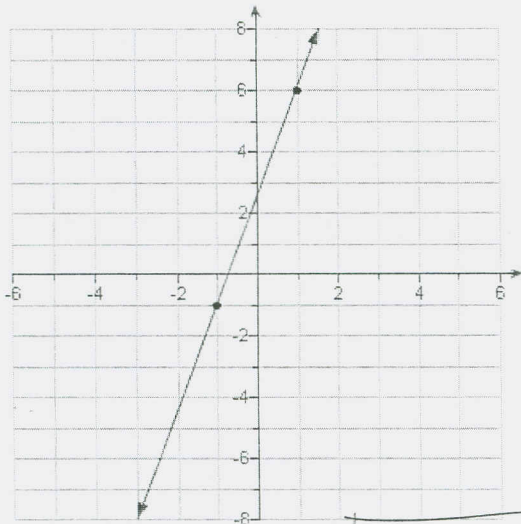
b. $y = \frac{2}{5}x - 2 \Rightarrow (0, -2)$

$y = 0 \Rightarrow \frac{2}{5}x = 2$

$2x = 10 \Rightarrow x = 5$
 $(5, 0)$



10. Find the equation of the line from the graph



$(x_1, y_1) = (-1, -1)$
 $(x_2, y_2) = (1, 6)$

$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{6 - (-1)}{1 - (-1)} = \frac{7}{2}$ 3pts

$y = \frac{7}{2}(x - (-1)) - 1$
 $= \frac{7}{2}x + \frac{7}{2} - 1$

$y = \frac{7}{2}x + \frac{5}{2}$ →

$-\frac{7}{2}x + y = \frac{5}{2}$

$-7x + 2y = 5$

3pts a. Point-Slope form: $y = \frac{7}{2}(x - (-1)) - 1$

3pts b. Slope-Intercept form: $y = \frac{7}{2}x + \frac{5}{2}$

3pts c. Standard form (with integer coefficients): $-7x + 2y = 5$

11. Based on your answer to the previous question, find an equation of the line through $(-1, -1)$ that is perpendicular to the one in the graph. Give your answer in slope-intercept form.

$m_{\perp} = -\frac{2}{7}$ $y = -\frac{2}{7}(x - (-1)) - 1$
 $= -\frac{2}{7}x - \frac{2}{7} - 1$

3pts $y = -\frac{2}{7}x - \frac{9}{7}$

12. Compute the discriminant for each of the following quadratic equations, and determine the nature of the solutions (How many, and whether it/they is/are real or non-real):

3pts a. $5x^2 - 38x + 21$ $b^2 - 4ac = (-38)^2 - 4(5)(21) = 1444 - 420$
 $= 1024 \Rightarrow 2 \text{ real solns}$

3pts b. $9x^2 + 12x + 4$ $b^2 - 4ac = 12^2 - 4(9)(4) = 144 - 144 = 0$
 $\Rightarrow 1 \text{ real sol'n}$

3pts c. $x^2 + 12x + 4$ $b^2 - 4ac = 12^2 - 4(1)(4) = 144 - 4 = 140 \rightarrow$
 2 real solns

13. Find all real or non-real solutions of the following quadratic equations by completing the square AND by quadratic formula. Write in the lowest terms possible, e.g. $\sqrt{12} = 2\sqrt{3}$. Do not use decimal approximations via calculator.

3p/3

$$a. x^2 - x + 1 = 0$$

$$b^2 - 4ac = (-1)^2 - 4(1)(1) \\ = 1 - 4 = -3$$

$$x = \frac{1 \pm \sqrt{-3}}{2} = \boxed{\frac{1 \pm i\sqrt{3}}{2} = x}$$

3p/3

$$x^2 - x = -1$$

$$x^2 - x + \left(\frac{1}{2}\right)^2 = -1 + \frac{1}{4} = -\frac{3}{4}$$

$$\left(x - \frac{1}{2}\right)^2 = -\frac{3}{4}$$

$$x - \frac{1}{2} = \pm \sqrt{-\frac{3}{4}} = \pm i \frac{\sqrt{3}}{2}$$

$$\boxed{x = \frac{1 \pm i\sqrt{3}}{2}}$$

3p/3

$$b. x^2 - 2x - 2 = 0$$

$$b^2 - 4ac = (-2)^2 - 4(1)(-2) \\ = 4 + 8 = 12$$

$$x = \frac{2 \pm \sqrt{12}}{2} = \frac{2 \pm 2\sqrt{3}}{2}$$

$$= \frac{2(1 \pm \sqrt{3})}{2} = \boxed{1 \pm \sqrt{3} = x}$$

3p/3

$$x^2 - 2x = 2$$

$$x^2 - 2x + 1^2 = 2 + 1^2$$

$$(x-1)^2 = 3$$

$$x-1 = \pm \sqrt{3}$$

$$\boxed{x = 1 \pm \sqrt{3}}$$

14. Solve the following inequalities:

3pts a. $3 - 5x < 6$

$$-5x < 3$$

$$x > -\frac{3}{5}$$

3pts b. $\frac{x}{5} - 7 > \frac{2}{3}$

LCD = 15

$$15\left(\frac{x}{5}\right) - 15(7) > 15\left(\frac{2}{3}\right)$$

$$3x - 105 > 10$$

$$3x > 115$$

$$x > \frac{115}{3}$$

$$\left(\frac{115}{3}, \infty\right)$$

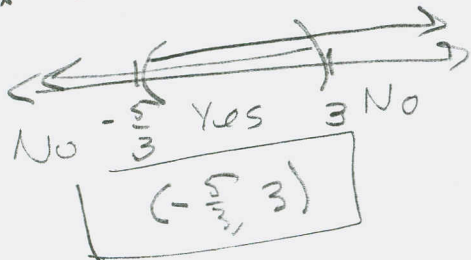
3pts c. $|3x - 2| < 7$

$$3x - 2 < 7 \text{ and } 3x - 2 > -7$$

$$3x < 9$$

$$3x > -5$$

$$x < 3 \text{ and } x > -\frac{5}{3}$$



3pts d. $|3x - 2| \leq -7$

Never!

$$\emptyset$$

3pts e. $|3x - 2| > -7$

Always!

$$(-\infty, \infty)$$

4pts f. $|3x - 2| - 10 \geq -7$

$$|3x - 2| \geq 3$$

$$3x - 2 \geq 3 \text{ OR } 3x - 2 \leq -3$$

$$3x \geq 5$$

$$3x \leq -1$$

$$x \geq \frac{5}{3} \text{ OR } x \leq -\frac{1}{3}$$

$$\left(-\infty, -\frac{1}{3}\right] \cup \left[\frac{5}{3}, \infty\right)$$