

Solve the following equations. For each equation, state whether it is conditional, identity or inconsistent.

1. $\frac{3}{2}x + \frac{1}{3} = \frac{1}{4}x - \frac{1}{6}$ LCD = 24

5pts $24\left(\frac{3}{2}x\right) + 24\left(\frac{1}{3}\right) = 24\left(\frac{1}{4}x\right) - 24\left(\frac{1}{6}\right)$

$12(3x) + 8(1) = 6x - 4(1)$

$36x + 8 = 6x - 4$

Conditional

$30x = -12$

5pts

$x = -\frac{12}{30} = \left[-\frac{2}{5} = x\right]$

$x \in \left\{-\frac{2}{5}\right\}$

2. $\frac{4}{x-1} - \frac{9}{x+1} = \frac{3}{x^2-1}$ LCD = $(x-1)(x+1)$

$(x-1)(x+1)\left(\frac{4}{x-1}\right) - (x-1)(x+1)\left(\frac{9}{x+1}\right) = (x-1)(x+1)\left(\frac{3}{(x-1)(x+1)}\right)$

$(x+1)(4) - (x-1)(9) = 3$

$4x + 4 - 9x + 9 = 3$

$-5x + 13 = 3$

$-5x = -10$

$x = 2$

$x \in \{2\}$

Conditional

3. (5 pts bonus) Give an example of an equation in the variable x that is...

a. ... an identity.

$3x + 5 = 3x + 5$

b. ... inconsistent.

$3x + 5 = 3x - 2$

5 P 13

4. The old combine can harvest the crop in 100 hours, but a new one can do it in 92 hours. How many hours will it take the two of them operating at the same time to harvest the crop?

Let x = the # of hours it takes them to finish if they work together.

Then $\frac{1}{100}x + \frac{1}{92}x = 1$

LCD: $2 \overline{)92}$
 $2 \overline{)46}$
 23

$2300 \left(\frac{1}{100}x \right) + 2300 \left(\frac{1}{92} \right) = 2300$

$2 \overline{)100}$
 $2 \overline{)50}$
 $5 \overline{)25}$
 5

$23x + 25(1) = 2300$

$23x + 25 = 2300$

$23x = 2275 \rightarrow x = \frac{2275}{23} \text{ hrs}$

LCD = $2 \cdot 2 \cdot 5 \cdot 5 \cdot 23 = 2300$

5 P 8

5. How much 60% antifreeze solutions should be added to 4 quarts of 15% antifreeze solution to yield a 50% antifreeze solution?

Let x = the amt of 60% antifreeze soln used (quarts)

Then $.6x + .15(4) = .50(x+4)$

$60x + 15(4) = 50(x+4)$

$(.6)(14) + (.15)(4) = .5(14+4)$

$60x + 60 = 50x + 200$

$10x + 60 = 200$

$9 = 9 \checkmark$

$10x = 140$

$x = \frac{140}{10} = 14 \text{ qts}$

6. Find the distance between (3, 2) and (-3, 5).

5 P 15 $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

$= \sqrt{(-3 - 3)^2 + (5 - 2)^2}$

$= \sqrt{(-6)^2 + 3^2}$

$= \sqrt{36 + 9} = \sqrt{45} = 3\sqrt{5}$

$2 \overline{)90}$
 $5 \overline{)45}$
 $3 \overline{)9}$
 3

1/5

7. Determine the center and radius of the circle given by

SPTS

$$\cancel{x^2 + 6x + 25 + y^2 - 8y - 121} \quad x^2 + 6x + y^2 - 8y = 96$$

$$x^2 + 6x + 9 + y^2 - 8y = 96$$

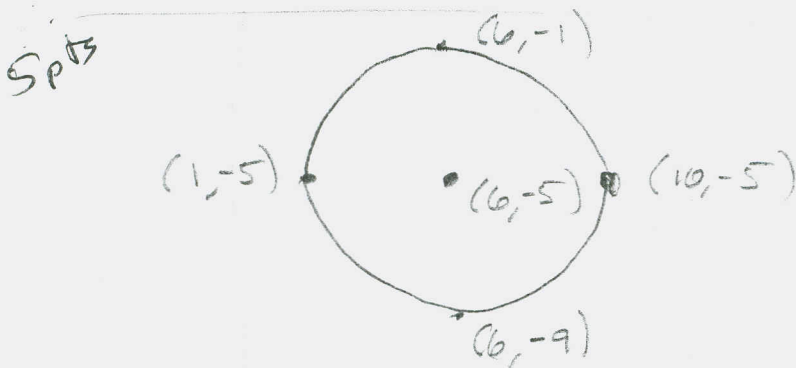
$$x^2 + 6x + 3^2 + y^2 - 8y + 4^2 = 96 + 9 + 16$$

$$(x+3)^2 + (y-4)^2 = 121$$

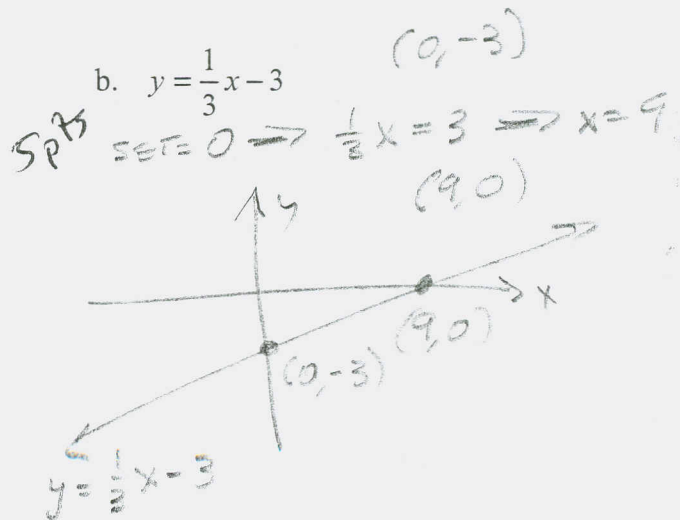
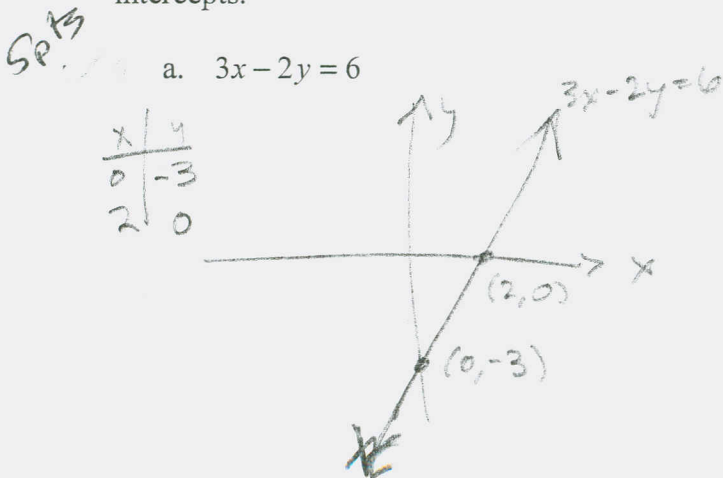
$$(h, k) = (-3, 4)$$

$$r = 11$$

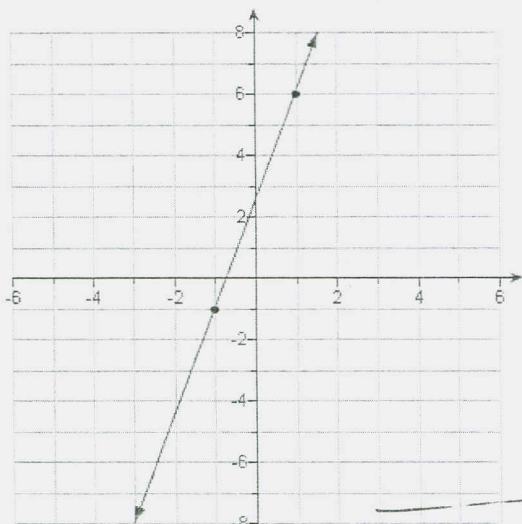
8. Sketch the graph of $(x-6)^2 + (y+5)^2 = 16$.



9. Sketch the graph of each of the following equations. Main points I want to see are the intercepts.



10. Find three forms of the equation of the line in the graph (Hint: Use the two points given.)



$$(x_1, y_1) = (-1, -1)$$

$$(x_2, y_2) = (1, 6)$$

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{6 - (-1)}{1 - (-1)} = \frac{7}{2} \quad 3 \text{pts}$$

$$y = m(x - x_1) + y_1$$

$$y = \frac{7}{2}(x - (-1)) - 1$$

$$y = \frac{7}{2}x + \frac{7}{2} - 1$$

$$y = \frac{7}{2}x + \frac{5}{2}$$

$$2y = 7x + 5$$

$$-7x + 2y = 5$$

3pts a. Point-Slope form:

$$y = \frac{7}{2}(x + 1) - 1$$

3pts b. Slope-Intercept form:

$$y = \frac{7}{2}x + \frac{5}{2}$$

3pts c. Standard form (with integer coefficients):

$$-7x + 2y = 5$$

11. Based on your answer to the previous question, find an equation of the line through (-1, -1) that is perpendicular to the one in the graph. Give your answer in slope-intercept form.

3pts $m_{\perp} = -\frac{2}{7}, (x_1, y_1) = (-1, -1)$

$$y = -\frac{2}{7}(x + 1) - 1$$

Point-Slope

$$y = -\frac{2}{7}x - \frac{9}{7}$$

Final Ans.

12. Compute the discriminant for each of the following quadratic equations, and determine the nature of the solutions (How many, and whether it/they is/are real or non-real):

3pts a. $3x^2 - 22x + 35$

$$b^2 - 4ac = (-22)^2 - 4(3)(35)$$

$$= 484 - 420 = 64 \quad \boxed{2 \text{ real solns}}$$

3pts b. $4x^2 + 8x + 13$

$$b^2 - 4ac = 8^2 - 4(4)(13) = 64 - 208$$

$$\boxed{2 \text{ nonreal solns}}$$

3pts c. $4x^2 + 12x + 9$

$$b^2 - 4ac = 12^2 - 4(4)(9) = 144 - 144 = 0 \quad \boxed{1 \text{ real soln}}$$

3pts

13. Find all real or non-real solutions of the following quadratic equations by completing the square AND by quadratic formula. Write in the lowest terms possible, e.g. $\sqrt{12} = 2\sqrt{3}$. Do not use decimal approximations via calculator.

3pts a. $x^2 - 2x - 1 = 0$
 $a = 1, b = -2, c = -1$
 $b^2 - 4ac = (-2)^2 - 4(1)(-1)$
 $= 4 + 4 = 8$

$$x = \frac{2 \pm \sqrt{8}}{2} = \frac{2 \pm 2\sqrt{2}}{2}$$

$$= \frac{2(1 \pm \sqrt{2})}{2} = \boxed{1 \pm \sqrt{2} = x}$$

3pts $x^2 - 2x = 1$
 $x^2 - 2x + 1 = 1 + 1$

$$(x-1)^2 = 2$$

$$x-1 = \pm\sqrt{2}$$

$$\boxed{x = 1 \pm \sqrt{2}}$$

$$x^2 - x + \frac{3}{4} = 0$$

b. ~~$x^2 - \frac{2}{3}x + \frac{1}{3} = 0$~~

$$b^2 - 4ac = (-1)^2 - 4(1)(\frac{3}{4})$$

$$= 1 - 3 = -2$$

$$x = \frac{1 \pm \sqrt{-2}}{2} = \boxed{\frac{1 \pm i\sqrt{2}}{2}}$$

3pts

$$x = \frac{1}{2} \pm i \frac{\sqrt{2}}{2}$$

$$= \frac{1}{2} \left(\frac{1}{2} \pm i \frac{\sqrt{2}}{2} \right)$$

$$= \frac{1}{3} \pm i \frac{\sqrt{3}}{3}$$

$$x^2 - x = -\frac{3}{4}$$

$$x^2 - x + (\frac{1}{2})^2 = -\frac{3}{4} + \frac{1}{4}$$

$$(x - \frac{1}{2})^2 = -\frac{2}{4} = -\frac{1}{2}$$

$$x - \frac{1}{2} = \pm \sqrt{-\frac{1}{2}} = \pm i \frac{\sqrt{2}}{2}$$

$$x = \frac{1}{2} \pm i \frac{\sqrt{2}}{2}$$

$$\boxed{x = \frac{1 \pm i\sqrt{2}}{2}}$$

3pts

14. Solve the following inequalities

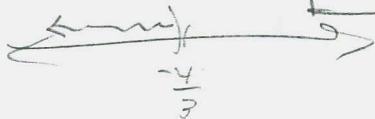
write soln set in interval notation.

a. $2 - 3x > 6$

3pts $-3x > 4$

$x < -\frac{4}{3}$

$(-\infty, -\frac{4}{3})$



b. $\frac{x}{2} - 7 > \frac{2}{3}$

LCD = 6

3pts

$6(\frac{x}{2}) - 6(7) = 6(\frac{2}{3})$

$3x - 42 = 4$

c. $|2x - 3| \geq 7$

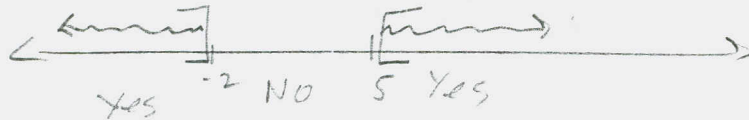
3pts $2x - 3 \geq 7$ OR $2x - 3 \leq -7$

$2x \geq 10$

$2x \leq -4$

$x \geq 5$

$x \leq -2$



$x \in (-\infty, -2] \cup [5, \infty)$

d. $|2x - 3| > -7$

3pts

$(-\infty, \infty)$

e. $|2x - 3| \leq -7$

\emptyset

3pts

f. $|2x - 3| - 8 \leq -7$

4pts

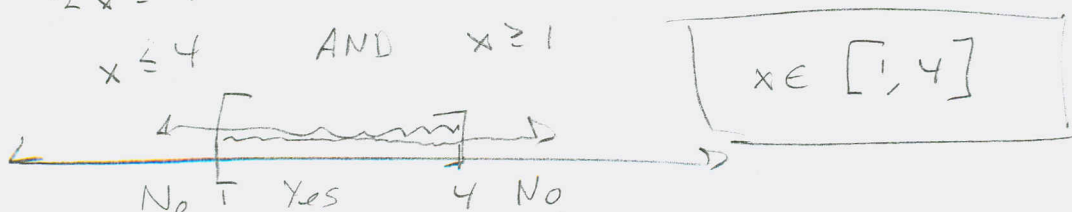
$|2x - 3| \leq 1$

$2x - 3 \leq 1$ AND $2x - 3 \geq -1$

$2x \leq 4$

$2x \geq 2$

$x \leq 4$ AND $x \geq 1$



$x \in [1, 4]$