KEY

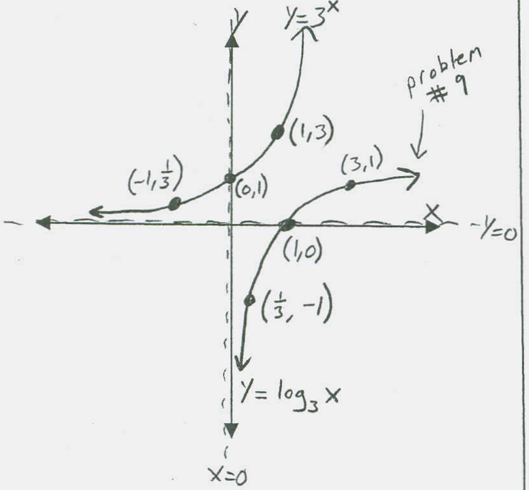
$-(x-1) = -x+1$

$-x \rightarrow -(x-1)$

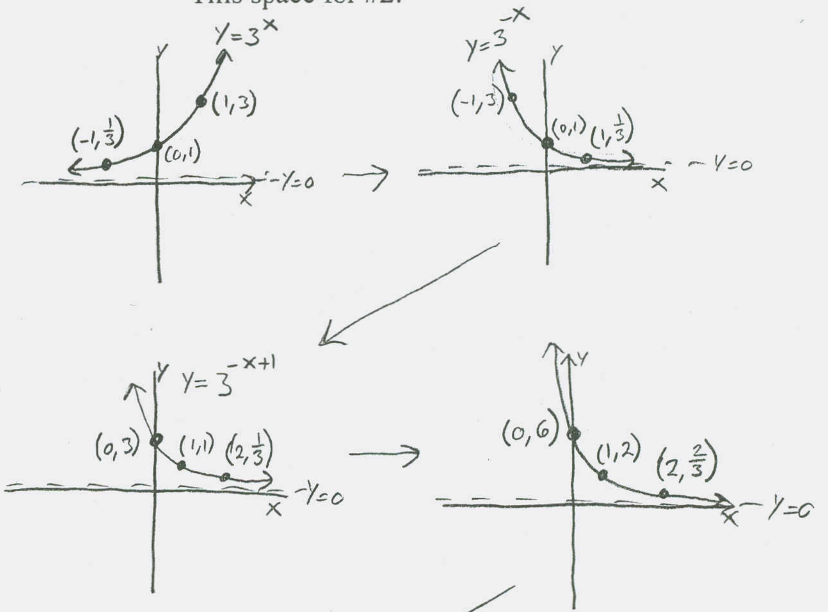
It is really important to know what the basic graphs of exponential and logarithmic functions look like, along with their 3 key points. Also, you need to know how to transform these graphs. Make sure you know your laws of logarithms (s 4.5).

Graph: (sections 1.5 & 4.3)

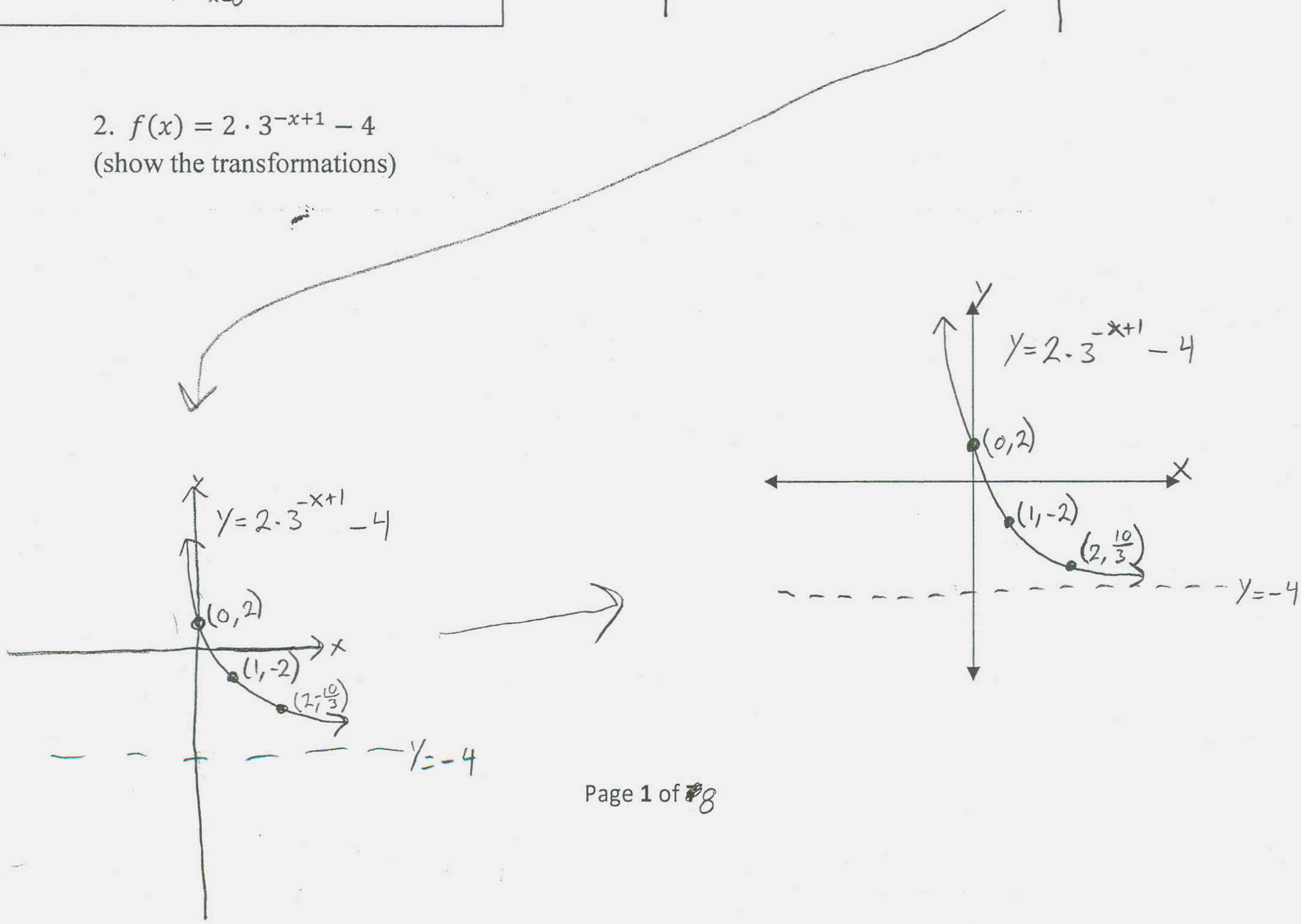
1. $f(x) = 3^x$



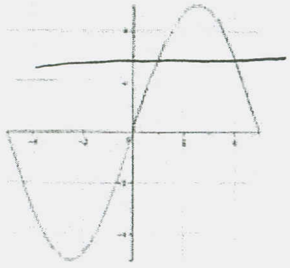
This space for #2:



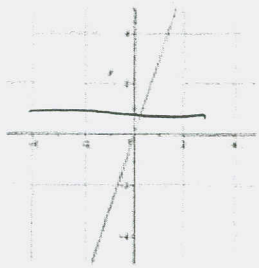
2. $f(x) = 2 \cdot 3^{-x+1} - 4$
(show the transformations)



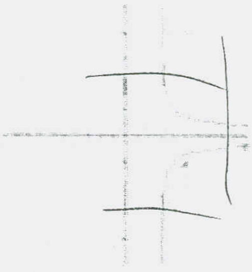
3. Determine which of the following are one-to-one. Indicate by writing "Yes" or "No" on the graphs. State which one isn't a function. (sxn 4.2) use the "Horizontal Line test."



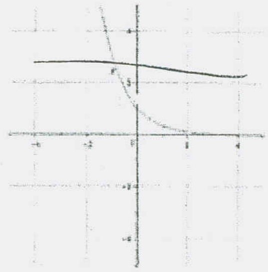
No! Not one-to-one because horizontal line crosses graph more than once.



Yes



Not a function



Yes

4. (sxn 4.1) For $f(x) = 3x$, $g(x) = 2x^2 - 1$, find :

a. $(f \circ f)(2)$

$f(f(2)) = f(3 \cdot 2) = f(6) = 3 \cdot 6 = 18$

b. $(f \circ g)(2)$

$f(g(2)) = f(2(2)^2 - 1) = f(7) = 3 \cdot 7 = 21$

c. $(g \circ g)(2)$

$g(g(x)) = g(2 \cdot 2^2 - 1) = g(7) = 2(7)^2 - 1 = 2(49) - 1 = 98 - 1 = 97$

5. (s 4.1) For $f(x) = \frac{1}{x+3}$ *and* $g(x) = \frac{2}{x} + 3$, find $(f \circ g)(x)$ and its domain.

this answer is ok.

$$f(g(x)) = f\left(\frac{2}{x} + 3\right) = \frac{1}{\left(\frac{2}{x}+3\right) + 3} = \frac{1}{\frac{2}{x} + 6} = \frac{1}{\frac{2}{x} + \frac{6x}{x}} = \frac{1}{\frac{2+6x}{x}} = \frac{x}{2+6x}$$

$\mathcal{D}(g): \mathbb{R} \setminus \{0\}$

$\mathcal{D}(f \circ g)(x)$: Need $g(x) \neq -3$

$\frac{2}{x} + 3 = -3$

$\frac{2}{x} = -6$

$2 = -6x$

$\frac{2}{-6} = x$

$x = -\frac{1}{3}$

$\mathcal{D} = \mathbb{R} \setminus \{-\frac{1}{3}, 0\}$

6. (s 4.2) Let $f(x) = x^3$. Find $f^{-1}(x)$, and graph both f and f^{-1} on the same coordinate axes.

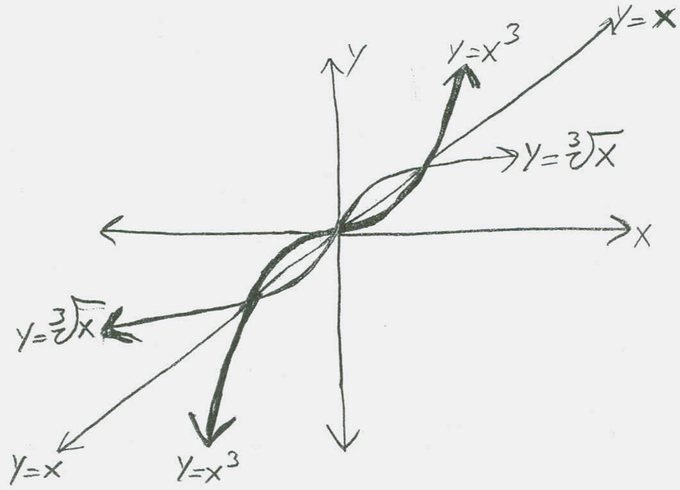
$y = x^3$

$x = y^3$

$\sqrt[3]{x} = \sqrt[3]{y^3}$

$y = \sqrt[3]{x}$

$f^{-1}(x) = \sqrt[3]{x}$



(graphs are symmetric about the line $y=x$)

7. (s 4.1) Find functions f and g so that $(f \circ g)(x) = H$, given that $H = (2x - 3)^4$.

Let $f(x) = x^4$
$g(x) = (2x-3)$

$f(g(x)) = f(2x-3) = (2x-3)^4 = H(x)$

8. (s 4.2) Let $f(x) = \frac{2x-3}{x+4}$. Find $f^{-1}(x)$.

$y = \frac{2x-3}{x+4}$

$\Rightarrow x = \frac{2y-3}{y+4}$

$\Rightarrow x(y+4) = 2y-3$

$\Rightarrow xy + 4x = 2y-3$

$\Rightarrow xy - 2y = -4x-3$

$\Rightarrow y(x-2) = -4x-3$

$\Rightarrow y = \frac{-4x-3}{x-2} = f^{-1}(x)$

9. (s 4.4) In the space for the graph #1, sketch the graph of $g(x) = \log_3 x$ on the same set of axes as your graph of $f(x) = 3^x$.

10. (s 4.4 &4.6) Evaluate $\log_4 256$ without a calculator.

$256 = 4^4$

$\log_4 256 = \log_4((4)^4) = 4\log_4 4 = 4(1) = 4$

$4\,|\,256$
$4\,|\,64$
$4\,|\,16$
4

11. (s 4.5) Express $\log_3 \sqrt[7]{\dfrac{10x^7y^{-4}}{z^{-3}}}$ in terms of logarithms x, y, and z.

$$\log_3\left(\frac{10x^7y^{-4}}{z^{-3}}\right)^{1/7} = \log_3\left(\frac{10x^7z^3}{y^4}\right)^{1/7} = \frac{1}{7}\left(\log_3 10 + \log_3 x^7 + \log_3 z^3 - \log_3 y^4\right)$$

$$= \frac{1}{7}\left(\log_3 10 + 7\log_3 x + 3\log_3 z - 4\log_3 y\right)$$

12. (s 4.6) Solve correct to four decimal places: $2^{3x} = 3^{2x-4}$. You will need to have a calculator to find the approximate final answer.

$2^{3x} = 3^{2x-4}$

$\Rightarrow \ln 2^{3x} = \ln 3^{2x-4}$

$\Rightarrow 3x(\ln 2) = (2x-4)(\ln 3)$

$\Rightarrow 3x(\ln 2) = 2x(\ln 3) - 4(\ln 3)$

$\Rightarrow (x\cdot 3)(\ln 2) - (x\cdot 2)(\ln 3) = -4(\ln 3)$

$\Rightarrow x\big(3(\ln 2) - 2(\ln 3)\big) = -4(\ln 3)$

$\Rightarrow x = \dfrac{-4(\ln 3)}{3(\ln 2) - 2(\ln 3)}$

$\Rightarrow x \approx 37.30969515$

$\boxed{x \approx 37.3097}$

13. (s 4.5 & 4.6) Solve: $\log_5(x-4) + \log_5(x+2) = \log_5 7$.

$\Rightarrow \log_5[(x-4)(x+2)] = \log_5 7$

$\Rightarrow 5^{[\log_5(x-4)(x+2)]} = 5^{(\log_5 7)}$

$\Rightarrow (x-4)(x+2) = 7$

$\Rightarrow x^2 + 2x - 4x - 8 = 7$

$\Rightarrow x^2 - 2x - 15 = 0$

$\Rightarrow (x-5)(x+3) = 0$

$x - 5 = 0$ $\quad x + 3 = 0$

$x = 5$ $\quad x = -3$

$\boxed{x \in \{5\}}$

check:

$x = 5$:

$\log_5(5-4) = \log_5(1)$ this is > 0 ✓

$\log_5(5+2) = \log_5(7)$ ✓

$x = -3$:

$\log_5(-3-4) = \log_5(-7)$ ✗

this must be greater than zero. throw away $x = -3$

14. (s 4.5) Write the following as the logarithm of a single expression. Assume that variables represent positive numbers. $3\log_5(x+2) - 4\log_5(x-7) + \log_5 9$.

$\Rightarrow \log_5(x+2)^3 + \log_5 9 - \log_5(x-7)^4 = \boxed{\log_5\left(\frac{9\cdot(x+2)^3}{(x-7)^4}\right)}$

15. Solve $\frac{e^x+e^{-x}}{2} = 1$.

$\Rightarrow e^x + e^{-x} = 2$

$\Rightarrow e^x(e^x + e^{-x} - 2) = 0 \cdot e^x$

$\Rightarrow e^{2x} - 2e^x + e^0 = 0$ $\quad$ let $u = e^x$

$\Rightarrow u^2 - 2u + 1 = 0$

$\Rightarrow (u-1)^2 = 0$

$\Rightarrow u - 1 = 0$

$\Rightarrow u = 1$

$\Rightarrow e^x = 1$

$\Rightarrow \ln e^x = \ln 1$

$x(\ln e) = \ln 1$

$x(1) = \ln 1$

$x = \ln(1)$

$\boxed{x = 0}$

check: $\frac{e^0 + e^{-0}}{2} = 1$

$\Rightarrow \frac{1+1}{2} = 1$

$\Rightarrow \frac{2}{2} = 1$

$\Rightarrow 1 = 1$ ✓

16. (4.7) Find the amount that results from each investment.

a) \$100 invested at 11% compounded daily after a period of 3 years.

$P = \$100,\ r = .11,\ t = 3,\ m = 365$

$A = P\left(1+\frac{r}{m}\right)^{mt} = 100\left(1+\frac{.11}{365}\right)^{(365\cdot 3)} \approx 139.0898977 \approx \$139.10$

b) compounded continuously?

$A = Pe^{rt} = 100 \cdot e^{(.11\cdot 3)} \approx 139.0968128 \approx \$139.10$

17. (4.7) If a sum of money is worth \$2000 five years from now, find the present value if the interest rate is 6% APR compounded weekly.

present-value Formula $\Rightarrow P = \frac{A}{(1+i)^n} = A(1+i)^{-n} = A\left(1+\frac{r}{m}\right)^{-mt}$

$A = \$2{,}000$

$r = .06$

$t = 5$ yrs

$m = 52$

$P = \frac{2{,}000}{\left(1+\frac{.06}{52}\right)^{(52\cdot 5)}} \approx \$1{,}481.89$

18. (4.7) Find the future value of \$120 invested at 12% for 12 years, compounded monthly.

$A = P(1+i)^n = P\left(1+\frac{r}{m}\right)^{mt}$

$P = \$120$

$m = 12$

$t = 12$ years

$r = .12$

$A = 120\left(1+\frac{.12}{12}\right)^{(12\cdot 12)} = 120(1+.01)^{(144)} \approx \$502.87$

19. (4.7) If Joe Monty with SI publications wants to make an investment of $20 at 4% interest compounded daily, how long will he have to wait until his investment triples?? (i.e. find the tripling time).

$P = 20 \quad t = ?$ Solve for t

$r = .04 \quad n = 365$

$A = P(1+\frac{r}{n})^{nt} = 3P$

$\Rightarrow \cancel{P}(1+\frac{r}{n})^{nt} = 3\cancel{P}$

$\Rightarrow (1+\frac{r}{n})^{nt} = 3$

$\Rightarrow \ln(1+\frac{r}{n})^{nt} = \ln 3$

$\Rightarrow nt(\ln(1+\frac{r}{n})) = \ln 3$

$\longrightarrow t = \dfrac{\ln 3}{n[\ln(1+\frac{r}{n})]} = \dfrac{\ln 3}{365(\ln(1+\frac{.04}{365}))} = 27.46681214 \text{ yrs}$

tripling time is $\approx$ ~~27.5~~ years
27.5

20. (s 3.4) What is the domain of $\sqrt{\dfrac{x-2}{(x+3)^2(x-7)^3}}$? Need $\dfrac{x-2}{(x+3)^2(x-7)^3} \geq 0$

And Need $(x+3)^2(x-7) \neq 0$

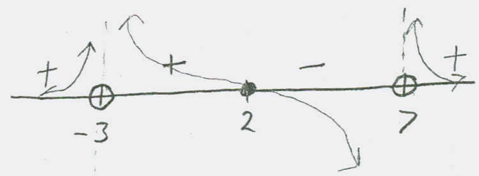
E.B.

$\dfrac{x}{x^5} = \dfrac{1}{x^4}$

$x = 2 \quad m = 1$ cross

$x = -3 \quad m = 2$ not cross

$x = 7 \quad m = 3$ cross



$\mathcal{D}: x \in (-\infty, -3) \cup (-3, 2] \cup (7, \infty)$

see practice test 3. #s 6a & 11

Think about how this relates to $\ln \dfrac{x-2}{(x+3)^2(x-7)^3}$

$\mathcal{D}$ of $\sqrt{x}$ is $(0, \infty)$

$\mathcal{D}$ of $\ln x$ is also $(0, \infty)$

i.e. $\mathcal{D}$ of $\sqrt{\dfrac{x-2}{(x+3)^2(x-7)^3}} = \mathcal{D}$ of $\ln\left(\dfrac{x-2}{(x+3)^2(x-7)^3}\right)$

21. (s 4.7) What is r_{eff} ? Derive it from $A = P\left(1+\frac{r}{m}\right)^{mt}$

r_{eff} is the simple interest rate that gives same return after one year as some compound rate

$A = P(1+r_{eff}) = P(1+\frac{r}{m})^{mt}$ $t = 1$ year

$\Rightarrow \cancel{P}(1+r_{eff}) = \cancel{P}(1+\frac{r}{m})^{m}$

$\Rightarrow 1+r_{eff} = (1+\frac{r}{m})^{m}$

$\Rightarrow r_{eff} = (1+\frac{r}{m})^{m} - 1$

22. You should spend some time looking over the online notes, especially w-dnesday's noted regarding domain questions and inverse relationships (i.e. if $f(x) = b^x$, then $(f)^{-1} \log_b x)$. Try writing down Steve's question from the notes and doing it by yourself. Be sure you know your formulas and properties! Also, Steve said there will probably be a 4.8 bonus question (exponential growth and decay).