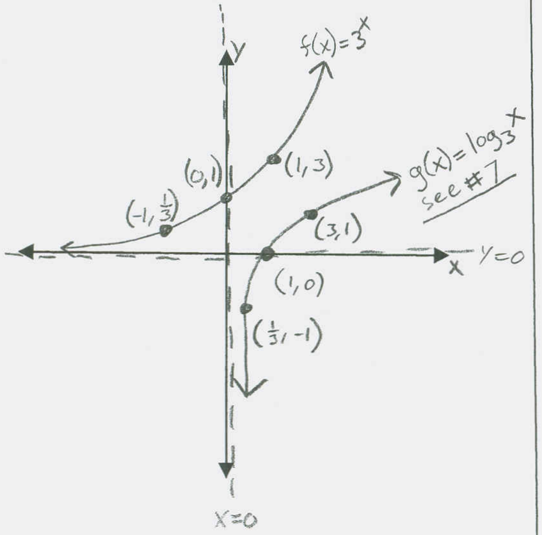
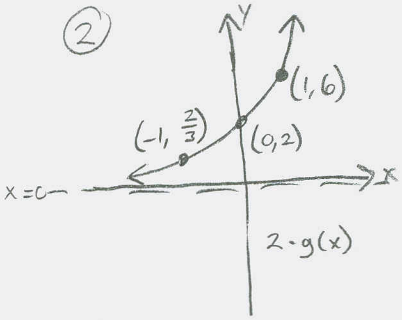
It is really important to know what the basic graphs of exponential and logarithmic functions look like, along with their 3 key points. Also, you need to know how to transform these graphs. ***MAKE SURE YOU KNOW*** your laws of logarithms and exponents!!!!!! It would be wise to review Monday's notes as well.

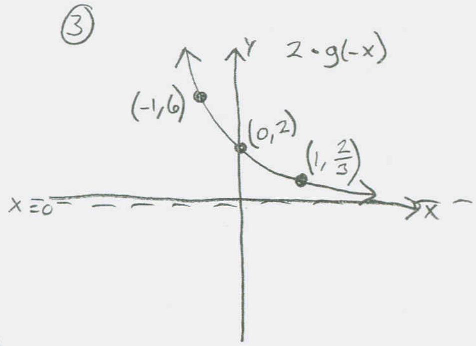
Graph:

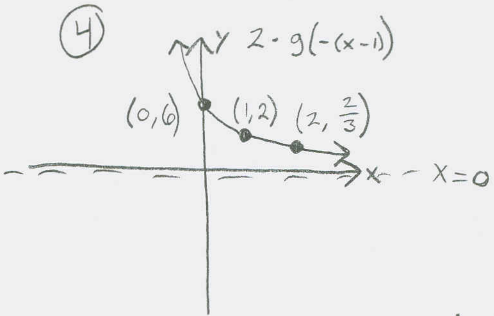
1. $f(x) = 3^x$



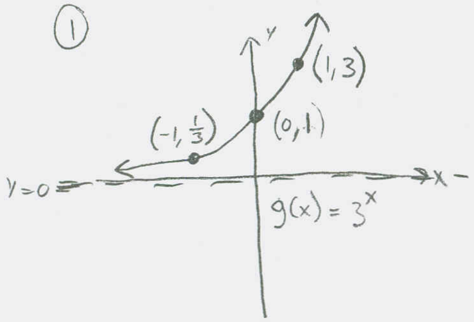
This space for #2:

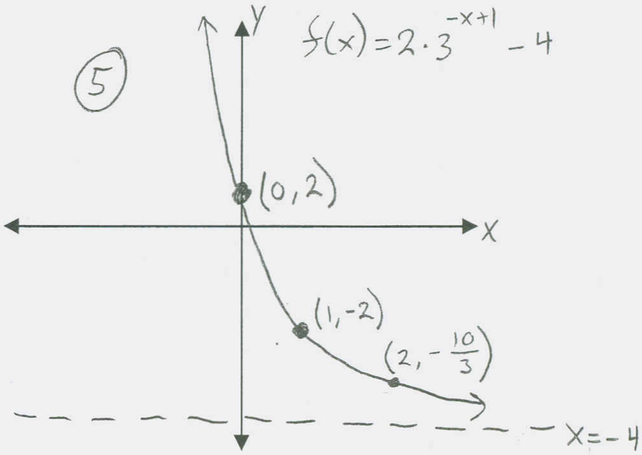






2. $f(x) = 2 \cdot 3^{-x+1} - 4$
(show the transformations)





① $g(x) = 3^x$

② $2 \cdot g(x) = 2 \cdot 3^x$

③ $2 \cdot g(-x) = 2 \cdot 3^{-x}$ $\quad -x+1 = -(x-1)$

④ $2 \cdot g(-(x-1)) = 2 \cdot 3^{-(x-1)}$

⑤ $2 \cdot g(1-x) - 4 = 2 \cdot 3^{-(x-1)} - 4 = f(x)$

3. Evaluate the following expressions without using a calculator.

i. $8^{\left(\frac{2}{3}\right)} = \left(8^{\frac{1}{3}}\right)^2 = \left(\sqrt[3]{8}\right)^2 = \left(\sqrt[3]{2\cdot 2\cdot 2}\right)^2 = (2)^2 = \boxed{4}$

ii. $\left(\frac{1}{2}\right)^{-4} = \frac{1^{-4}}{2^{-4}} = (1)(2)^4 = \boxed{16}$

since base is 4, divide by 4 $\rightarrow$ $4\,|\,256$, $4\,|\,64$, $4\,|\,16$, 4 $= 4^4$

iii. $\log_4 256 = \log_4 (4)^4 = 4\log_4(4) = \boxed{4}$

4. Express $\log_3 \sqrt[7]{\frac{10x^7y^{-4}}{z^{-3}}}$ in terms of logarithms x, y, and z.

$\log_3\left[\left(\frac{10x^7y^{-4}}{z^{-3}}\right)^{1/7}\right] = \frac{1}{7}\log_3\left(\frac{10x^7z^3}{y^4}\right) = \boxed{\frac{1}{7}\left[\log_3(10) + \log_3(x^7) + \log_3(z^3) - \log_3(y^4)\right]}$

this is prob far enough

$= \frac{1}{7}\log_3(10) + \frac{1}{7}(7)\log_3(x) + \frac{1}{7}(3)\log_3(z) - \frac{1}{7}(4)\log_3(y)$

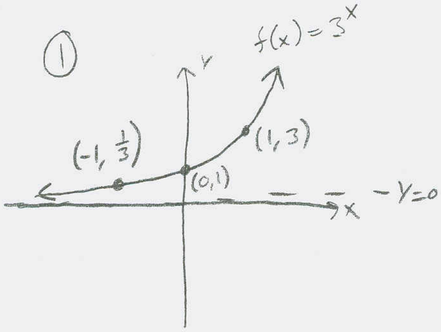
$= \boxed{\frac{1}{7}\log_3(10) + \log_3(x) + \frac{3}{7}\log_3(z) - \frac{4}{7}\log_3(y)}$ ← this a good answer

5. Use transformations to graph the following functions

i. $h(x) = 2 \cdot 3^{x-2} + 5$

1. $f(x) = 3^x$
2. $2f(x) = 2 \cdot 3^x$
3. $2f(x-2) = 2 \cdot 3^{x-2}$
4. $2f(x-2) + 5 = 2 \cdot 3^{x-2} + 5 = h(x)$

1



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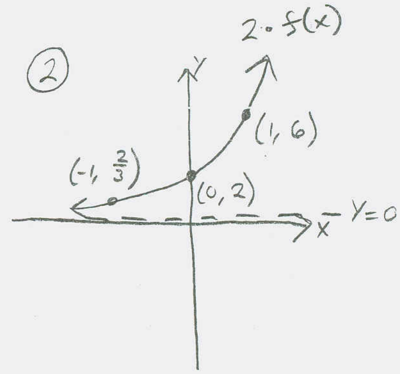

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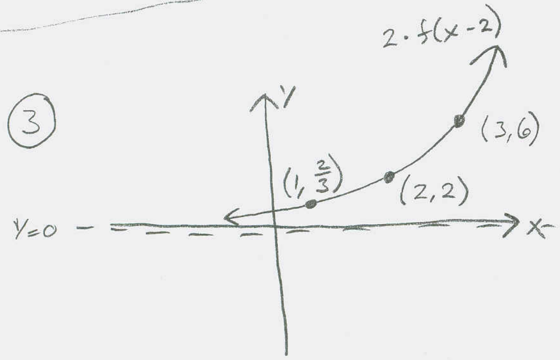

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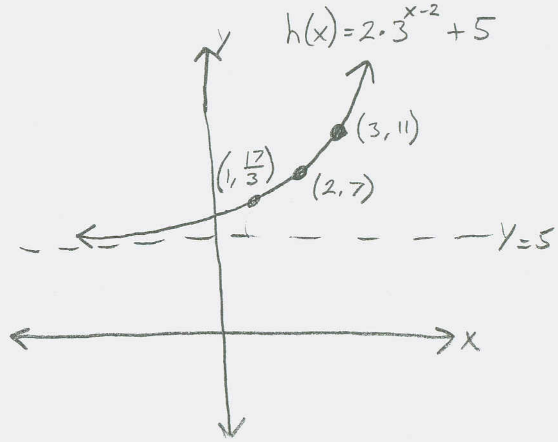
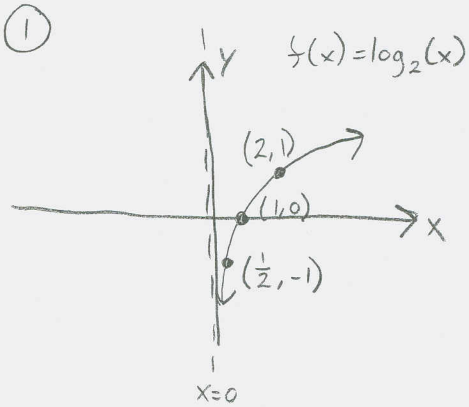


ii. $J(x) = \log_2(x+4) - 1$

1. $f(x) = \log_2(x)$
2. $f(x+4) = \log_2(x+4)$
3. $f(x+4) - 1 = \log_2(x+4) - 1$

1



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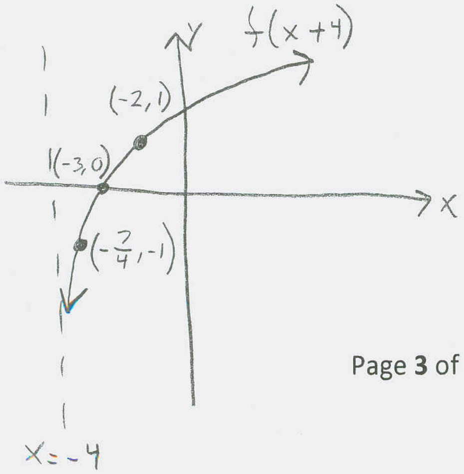

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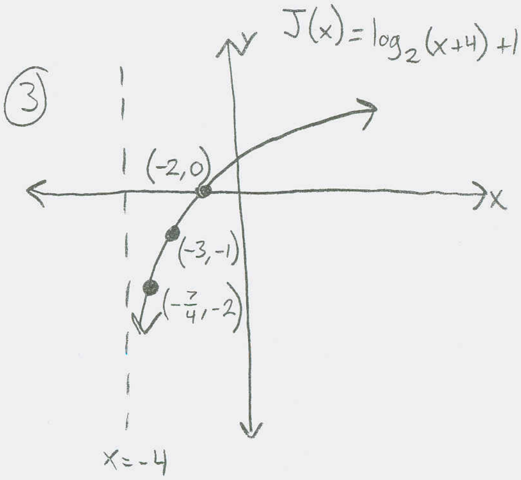


6. Solve correct to four decimal places: $2^{3x} = 3^{2x-4}$. You will need to have a calculator to find the approximate final answer.

$\ln 2^{3x} = \ln 3^{2x-4}$

$\Rightarrow 3x\ln 2 = (2x-4)\ln 3$

$\Rightarrow 3x\ln 2 = 2x\ln 3 - 4\ln 3$

$\Rightarrow 3x\ln 2 - 2x\ln 3 = -4\ln 3$

$\Rightarrow x(3\ln 2 - 2\ln 3) = -4\ln 3$

$\Rightarrow x = -\dfrac{4\ln(3)}{3\ln(2) - 2\ln(3)}$

$x = \dfrac{-\ln(3^4)}{\ln(2^3) - \ln(3^2)}$

$x = \dfrac{-\ln(81)}{\ln(8) - \ln(9)}$

$\to x \approx 37.3096952$

$\boxed{x \approx 37.3097}$

7. In the space for the graph #1, sketch the graph of $g(x) = \log_3 x$ on the same set of axes as your graph of $f(x) = 3^x$.

See graph on page 1

8. Solve the following equations. (check to make sure your final answers work in the original equation.)

i. $\ln x = \ln(6-2x)$

$e^{\ln x} = e^{\ln(6-2x)}$

$x = 6-2x$

$3x = 6$

$x = 2$

check:

$\ln(2) = \ln(6-2\cdot 2)$

$\ln(2) = \ln(6-4)$

$\ln(2) = \ln(2)$ ✓

$\boxed{x \in \{2\}}$

ii. $\log_5(x-4) + \log_5(x+2) = \log_5 7$.

$\log_5[(x-4)(x+2)] = \log_5 7$

$5^{\log_5(x^2-2x-8)} = 5^{\log_5 7}$

$x^2-2x-8 = 7$

$x^2-2x-15 = 0$

$(x-5)(x+3) = 0$

$x = 5$ OR $x = -3$

Check: $x = 5$

$\log_5(5-4) + \log_5(5+2) = \log_5(7)$

$\Rightarrow \log_5(1) + \log_5(7) = \log_5(7)$

$0 + \log_5(7) = \log_5(7)$ ✓

$x = -3$: $\log_5(-3-4) + \log_5(-3+2) = \log_5 7$

Not in $\mathcal{D}$...

$x = -3$ does not check

$\boxed{x \in \{5\}}$

iii. $4^{2x-1} = \frac{1}{2}$

$\Rightarrow (2^2)^{2x-1} = (2)^{-1}$

$(2)^{4x-2} = (2)^{-1}$

$4x - 2 = -1$

$4x = 1$

$x = \frac{1}{4}$

check: $4^{(2)(\frac{1}{4})-1} \overset{?}{=} \frac{1}{2}$

$4^{\frac{1}{2}-1} = \frac{1}{2}$

$4^{-\frac{1}{2}} = \frac{1}{2}$

$\Rightarrow \frac{1}{\sqrt{4}} = \frac{1}{2}$

$\frac{1}{2} = \frac{1}{2}$ ✓

$\boxed{x \in \{\frac{1}{4}\}}$

iv. $\log_x 16 = 2$

$\Rightarrow x^{\log_x 16} = x^2$

$\Rightarrow 16 = x^2$

$|x| = \sqrt{16}$

$x = \pm 4$

$-4 \notin \mathcal{D}$

$\boxed{x \in \{4\}}$

v. $3^{25x^2} = 81$

$3 \,|\, 81$
$3 \,|\, 27$
$3 \,|\, 9$
$3 = 3^4$

$\Rightarrow 3^{25x^2} = 3^4$

$\Rightarrow \log_3 3^{25x^2} = \log_3 3^4$

$\Rightarrow 25x^2 = 4$

$x^2 = \frac{4}{25}$

$|x| = \sqrt{\frac{4}{25}}$

$x = \pm \frac{2}{5}$

both answers check ✓

$\boxed{x \in \{-\frac{2}{5}, \frac{2}{5}\}}$

9. Write the following as the logarithm of a single expression. Assume that variables represent positive numbers. $3\log_5(x+2) - 4\log_5(x-7) + \log_5 9$.

$\Rightarrow \log_5[(x+2)^3] - \log_5[(x-7)^4] + \log_5 9 = \boxed{\log_5\left[\frac{(x+2)^3(9)}{(x-7)^4}\right]}$

10. Find f^{-1}: $f(x) = 3^{x+2}$

$f(x) = y = 3^{x+2}$

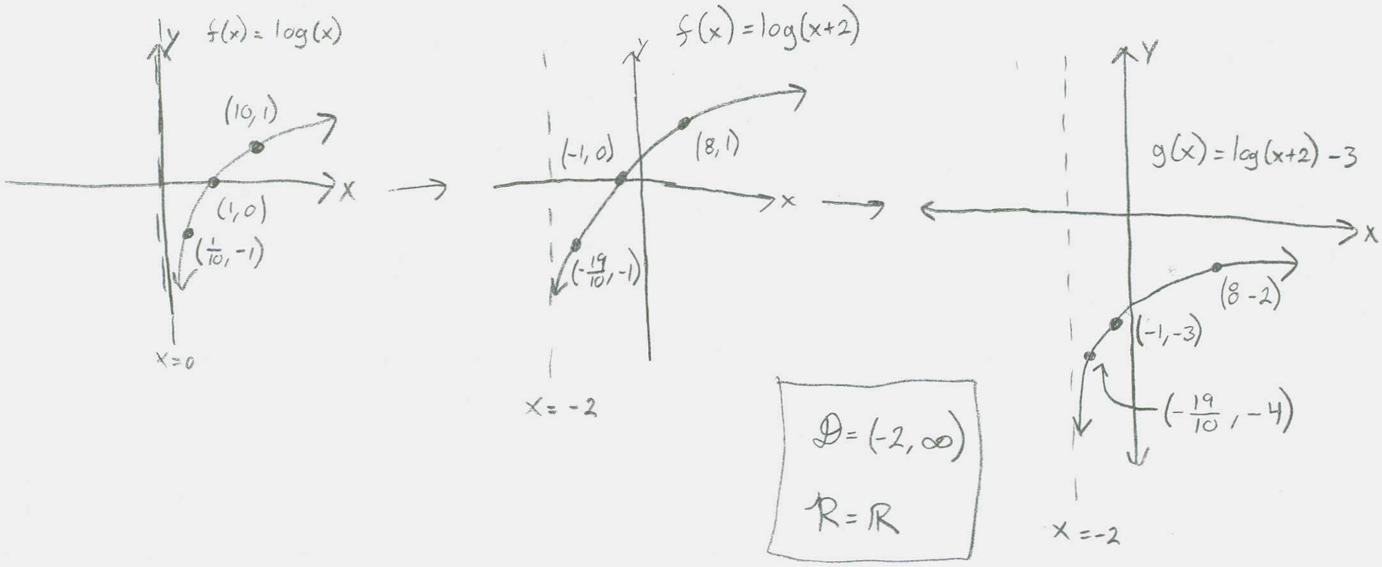
switch: $x = 3^{y+2}$

$\log_3 x = \log_3 3^{y+2}$

$\log_3 x = y + 2$

$\boxed{y = \log_3(x) - 2 = f^{-1}(x)}$

11. Sketch the graph of $g(x) = -3 + \log(x+2)$ and state its domain and range and list its three key points



$$\boxed{\begin{aligned} \mathcal{D} &= (-2, \infty) \\ R &= \mathbb{R} \end{aligned}}$$

12. Find the amount that results from each investment.

i. $100 invested at 11% compounded daily after a period of 3 years.

$$\boxed{A = P\left(1 + \frac{r}{m}\right)^{mt}}$$

r = interest rate = 11% = 0.11

t = time in years = 3

m = # of compoundings per year = 365

P = principal = initial amount = \$100

A = final amount = ?

$$A = 100\left(1 + \frac{.11}{365}\right)^{(365)(3)}$$

$$= 100\left(1 + \frac{.11}{365}\right)^{(1095)}$$

$$\approx \$139.0898977$$

$$\boxed{A \approx \$139.09}$$

ii. $120 invested at 12% for 12 years, compounded continuously.

r = interest rate = 12% = 0.12

t = time in years = 12

m = # of compounding periods per year = e

continuous compounding

P = principle = \$120

A = final amount = ?

$$A = Pe^{rt}$$

$$A = 120e^{(.12)(12)}$$

$$A \approx \$506.483498$$

$$\boxed{A \approx \$506.48}$$

13. If Joe Monty with SI publications wants to make an investment of \$20 at 4% APR compounded continuously, how long will he have to wait until his investment triples?? (i.e. find the tripling time).

$A = Pe^{rt}$

r = interest rate = 4% = 0.04

t = time in years = ?

P = principle = \$20

$A = 3P$ = triple the original investment

$A = Pe^{rt} = 3P$

$\frac{Pe^{rt}}{P} = \frac{3P}{P}$

$e^{rt} = 3$

$rt = \ln 3$

$t = \frac{\ln 3}{r} = \frac{\ln 3}{.04}$

$t \approx 27.46530722$

$\boxed{t \approx 27.5 \text{ years}}$

It would take about 27.5 years for Joe's investment to triple.

14. Find the time (to the nearest month) that it takes to pay off a loan of \$100,000 at 9% APR compounded monthly with payments of \$1250 per month

$R = P\frac{\frac{r}{m}}{1-(1+\frac{r}{m})^{-mt}} = P\frac{i}{1-(1+i)^{-mt}}$

R = payment = \$1,250/month

t = time in years = ?

P = principal loan = \$100,000

r = interest rate = 9% APR = 0.09

m = # of compounding periods per year = 12

$i = \frac{r}{m} = \frac{.09}{12} = 0.0075$

$R = P\frac{i}{1-(1+i)^{-mt}}$

$\left[1-(1+i)^{-mt}\right]R = P\frac{i}{1-(1+i)^{-mt}}\left[1-(1+i)^{-mt}\right]$

$\left(1-(1+i)^{-mt}\right)R = Pi$

$1-(1+i)^{-mt} = \frac{Pi}{R}$

$1-\frac{Pi}{R} = (1+i)^{-mt}$

$\ln\left(1-\frac{Pi}{R}\right) = \ln\left[(1+i)^{-mt}\right]$

$\ln\left(1-\frac{Pi}{R}\right) = -mt\ln(1+i)$

$-\frac{\ln\left(1-\frac{Pi}{R}\right)}{m\ln(1+i)} = t$

$t = -\frac{\ln\left(1-\frac{(100,000)(.0075)}{1250}\right)}{(12)(\ln(1+.0075))}$

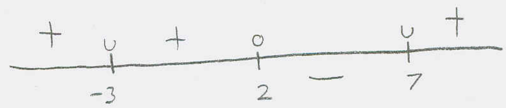
$t \approx 10.2191393 7$ years

$\boxed{t \approx 10 \text{ years and } 3 \text{ months}}$

15. What is the domain of $\sqrt{\dfrac{x-2}{(x+3)^2(x-7)^3}}$? Need $\dfrac{x-2}{(x+3)^2(x-7)^3} \geq 0$ & $(x+3)^2(x-7)^3 \neq 0$

Zeros: $x = 2$
$m = 1$
cross

V.A.: $x = -3$, $m = 2$, NO sign change
$x = 7$, $m = 3$, change sign



$$\boxed{\mathcal{D} = (-\infty, -3) \cup (-3, 2] \cup (7, \infty)}$$

double Check

test $x = 8$:

$$\frac{8-2}{(8+3)^2(8-7)^3} = \frac{6}{(11)^2(1)^3} = \frac{Pos}{Pos}$$

16. What is the domain of $\ln \dfrac{x-2}{(x+3)^2(x-7)^3}$? Same as above, except $\dfrac{x-2}{(x+3)^2(x-7)^3} > 0$

$\mathcal{D}\ \ln(x) = \{x \mid x > 0\}$

$$\boxed{\mathcal{D} = (-\infty, -3) \cup (-3, 2) \cup (7, \infty)}$$

17. Solve: $\log_6(K-1) + \log_6(K-2) = 1$

$\Rightarrow \quad \log_6[(k-1)(k-2)] = 1$

$6^{\log_6[(k-1)(k-2)]} = 6^1$

$(k-1)(k-2) = 6$

$k^2 - 3k + 2 = 6$

$k^2 - 3k - 4 = 0$

$(k-4)(k+1) = 0$

$k = 4 \quad \text{OR} \quad k = -1$

Check:

$k = 4$:

$\log_6(4-1) + \log_6(4-2) \stackrel{?}{=} 1$

$\log_6(3) + \log_6(2) \stackrel{?}{=} 1$

$\log_6[(3)(2)] \stackrel{?}{=} 1$

$\log_6(6) = 1$

$1 = 1 \checkmark$

$k = -1$:

$\log_6(-1-1) + \log_6(-1-2) \stackrel{?}{=} 1$

$\log_6(-2) + \log_6(-3) \stackrel{?}{=} 1$ No!

must be positive!

throw out $x = -1$

$$\boxed{x \in \{4\}}$$

18. The half-life of Carbon-14 is ~5,730 years.

i. Find an exponential model that gives the amount of radioactive Carbon-14 present in a charcoal sample after *t* years.

$A = Pe^{-kt}$

A = amount of C-14 present after t years = ?

P = Initial amount of C-14

k = decay rate

t = time in years

$A = Pe^{-kt} = \frac{1}{2}P$

$\frac{Pe^{-kt}}{P} = \frac{\frac{1}{2}P}{P}$

$\ln e^{-kt} = \ln(\frac{1}{2})$

$-kt = \ln(\frac{1}{2})$

$k = -\frac{\ln(\frac{1}{2})}{t}$

$k = -\frac{\ln(\frac{1}{2})}{5{,}730}$

$A = Pe^{-\left(-\frac{\ln(\frac{1}{2})}{5730}\right)t}$

$\boxed{A = Pe^{\left[\left(\frac{\ln(\frac{1}{2})}{5730}\right)t\right]}}$

ii. According to mainstream science, how old should a sample from a Neolithic fire pit be if it is found that 32% of naturally occurring Carbon-14 is present in the sample? (round answer to nearest year)

$A = .32P$

$A = Pe^{\left(\frac{\ln(\frac{1}{2})}{5730}\right)t} = .32P$

$\frac{Pe^{\left(\frac{\ln(\frac{1}{2})}{5730}\right)t}}{P} = \frac{.32P}{P}$

$e^{\left(\frac{\ln(\frac{1}{2})}{5730}\right)t} = .32$

$\left(\frac{\ln(\frac{1}{2})}{5730}\right)t = \ln(.32)$

$t = \frac{\ln(.32)}{\frac{\ln(\frac{1}{2})}{5730}} = \frac{\ln(.32)}{\ln(\frac{1}{2})}(5730)$

$t \approx 9{,}419.295967$

$\boxed{t \approx 9{,}419.30 \text{ years}}$

19. This one involves some chapter 3 stuff. May not see this on this test, but it is important to keep up on this stuff for the final.

Solve $\dfrac{e^x + e^{-x}}{2} = 1$

$\Rightarrow e^x + e^{-x} = 2$

$e^x(e^x + e^{-x}) = (2)e^x$

$(e^x)^2 + e^0 = 2e^x$

$(e^x)^2 - 2e^x + 1 = 0$

let $u = e^x$

$u^2 - 2u + 1 = 0$

$(u-1)(u-1) = 0$

$(u-1)^2 = 0$

$u - 1 = 0$

$e^x - 1 = 0$

$e^x = 1$

$\ln e^x = \ln 1$

$x = 0$

check:

$\dfrac{e^0 + e^{-0}}{2} \stackrel{?}{=} 1$

$\dfrac{1+1}{2} = 1$

$\dfrac{2}{2} = 1$

$1 = 1$ ✓

$\boxed{x \in \{0\}}$

20. Solve $\ln(x) - \ln(x+1) = \ln(x+3) - \ln(x+5)$

$\Rightarrow \ln\left(\dfrac{x}{x+1}\right) = \ln\left(\dfrac{x+3}{x+5}\right)$

$\Rightarrow e^{\ln\left(\frac{x}{x+1}\right)} = e^{\ln\left(\frac{x+3}{x+5}\right)}$

$\dfrac{x}{x+1} = \dfrac{x+3}{x+5}$

$x(x+5) = (x+1)(x+3)$

$\cancel{x^2} + 5x = \cancel{x^2} + 4x + 3$

$5x - 4x = 3$

$x = 3$

check:

$\ln(3) - \ln(3+1) = \ln(3+3) - \ln(3+5)$

$\ln\left(\dfrac{3}{4}\right) = \ln\left(\dfrac{6}{8}\right)$

$\ln\left(\dfrac{3}{4}\right) = \ln\left(\dfrac{3}{4}\right)$ ✓

$\boxed{x \in \{3\}}$