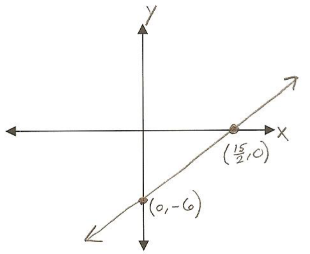
11. Let $f(x) = \frac{4}{5}x - 6$

a. Determine the slope and y-intercept.

$y = mx + b$

$m = \text{slope} = \frac{4}{5}$

$b = y\text{-intercept} = (0, -6)$



b. Use the slope and y-intercept to graph f here.

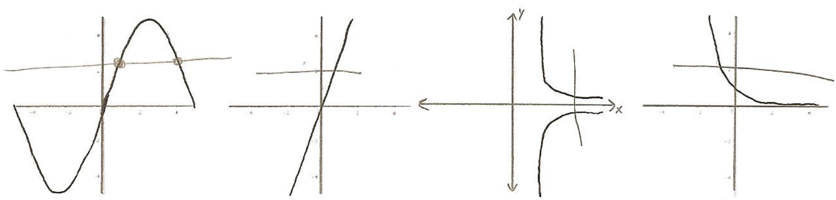
c. Determine the average rate of change of f.

$$\frac{f(x_2) - f(x_1)}{x_2 - x_1} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{\Delta y}{\Delta x} = m = \frac{4}{5}$$

d. Is f increasing, decreasing or constant?

Increasing

12. Determine which of the following are one-to-one. Indicate by writing "Yes" or "No" on the graphs. State which one isn't a function.



No | Yes | Not a function | Yes

13. For $f(x) = 3x$, and $g(x) = 2x^2 - 1$, find :

a. $(f \circ f)(2)$

$f(f(2)) = f(3(2)) = f(6) = 3 \cdot 6 = \boxed{18}$

b. $(f \circ g)(2)$

$f(g(2)) = f(2(2)^2 - 1) = f(8 - 1) = f(7) = 3 \cdot 7 = \boxed{21}$

c. $(g \circ g)(2)$

$g(g(2)) = g(2(2)^2-1) = g(7) = 2(7)^2 - 1 = 2(49) - 1 = 98 - 1 = \boxed{97}$

14. For $f(x) = \frac{1}{x+3}$ *and* $g(x) = \frac{2}{x} + 3$, find $(f \circ g)(x)$ and its domain.

$f(g(x)) = f\left(\frac{2}{x}+3\right) = \frac{1}{\left(\frac{2}{x}+3\right)+3} = \frac{1}{\frac{2}{x}+6} = \frac{1}{\frac{2}{x}+\frac{6x}{x}} = \frac{1}{\frac{2+6x}{x}} \cdot \frac{x}{x} = \frac{x}{2+6x} = (f \circ g)(x)$

Domain:

$\mathcal{D}_g = \{x \mid x \neq 0\}$; for the composition, we need $g(x)+3 \neq 0$

$\frac{2}{x}+3+3 \neq 0 \Rightarrow \frac{2}{x}+6 \neq 0 \Rightarrow \frac{2}{x} \neq -6 \Rightarrow 2 \neq -6x \Rightarrow x \neq -\frac{1}{3}$

so $\boxed{\mathcal{D} = \{x \mid x \neq 0,\ x = -\frac{1}{3}\}}$

also $\mathcal{D} = \mathbb{R} \setminus \{0, -\frac{1}{3}\}$

15. The velocity v of a falling object is directly proportional to the time t of the fall. If after 4 seconds, the velocity is 88 feet per second, what will the velocity be after 6 seconds?

$V = kt$

$88 = k4$

$k = \frac{88}{4}$

$k = 22$

$V = 22t$

$V = 22(6)$

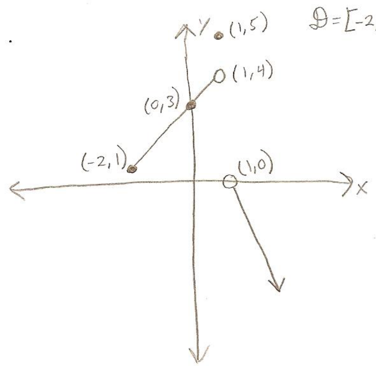
$\boxed{V = 132 \text{ ft/s after 6 seconds}}$

$-2 \le x < 1$

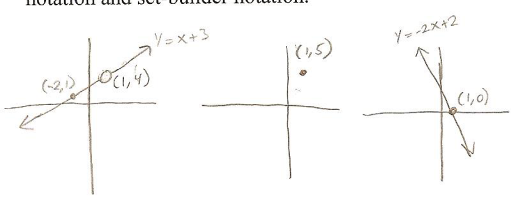
$R = (-\infty, 0) \cup [1, 4) \cup \{5\}$

$\mathcal{D} = [-2, \infty)$

16. Sketch the graph of $f(x) = \begin{cases} x+3 & if\ -2 \le x \cancel{\le} 1 \\ 5 & if\ x = 1 \\ -2x+2 & if\ x > 1 \end{cases}$.



Include all intercepts. State the domain and range in both interval notation and set-builder notation.



17. At the corner Shell station, the revenue R varies directly with the number g of gallons of gasoline sold. If the revenue is \$23.40 when the number of gallons sold is 12, find the linear function that relates revenue R to the number of gallons g of gasoline. Then find the revenue R when the number of gallons of gasoline is 10.5.

$R(g) = kg$

$R(12) = k12 = 23.4$

$k = \frac{23.4}{12}$

$k = 1.95$

$\boxed{R(g) = 1.95g}$

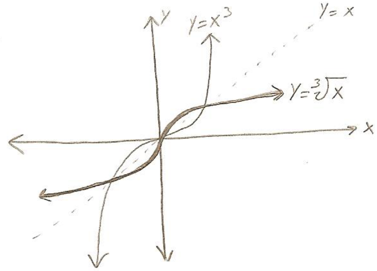
$R(10.5) = 1.95(10.5) = \boxed{\$20.48}$

18. Let $f(x) = x^3$. Find $f^{-1}(x)$, and graph both f and f^{-1} on the same coordinate axes

$f(x) = y = x^3$

$x = y^3 \leftarrow$ Switch variables

$\sqrt[3]{x} = y = f^{-1}$



19. Let $f(x) = \frac{2x-3}{x+4}$. Find $f^{-1}(x)$ by using the switch-and-solve method. Check your answer.

$y = \frac{2x-3}{x+4}$

Switch: $x = \frac{2y-3}{y+4}$

$(y+4)x = 2y-3$

$yx + 4x = 2y-3$

$yx - 2y = -4x-3$

$y(x-2) = -4x-3$

$y = -\frac{4x+3}{x-2}$

check: $f(f^{-1}(x)) = x$

$f(f^{-1}(x)) = \frac{2\left(-\frac{4x+3}{x-2}\right)-3}{\left(-\frac{4x+3}{x-2}\right)+4}$

$= \frac{-\frac{8x+6}{x-2} - \frac{3(x-2)}{x-2}}{-\frac{4x+3}{x-2} + \frac{4(x-2)}{x-2}}$

$= \frac{\frac{-8x-6-3x+6}{x-2}}{\frac{-4x-3+4x-8}{x-2}} \cdot \frac{(x-2)}{(x-2)}$

$= \frac{-11x}{-11}$

$f(f^{-1}(x)) = x$ ✓

MAT 121-G13 Practice Test 1

1. State whether the relations below represent a function (yes/no). If not, why? State the domain and range of each function. State whether the function is one to one, if it is, state the domain and range of the inverse.
$f = \{(2,-1),(-3,-1),(6,4),(-3,-1),(1,2)\}$

Is a function, Not 1-to-1 since -1 is an output of 2 & -3

$\mathcal{D} = \{-3, 1, 2, 6\}$

$g = \{(3,1),(-2,4),(1,3),(2,5)\}$ Is a function. $\mathcal{D}_g = \{-2, 1, 2, 3\}$; $\mathcal{R}_g = \{1, 3, 4, 5\}$

Is 1-to-1

$\mathcal{D}_{g^{-1}} = \mathcal{R}_g = \{1, 3, 4, 5\}$; $\mathcal{R}_{g^{-1}} = \mathcal{D}_g = \{-2, 1, 2, 3\}$

2. Determine whether the equation $y - 9 = (x-2)^2$ defines y as a function of x. If it does not, show/explain why not, either by a general argument, or by finding an x-value in the domain that corresponds to more than one y-value in the range.

$y - 9 = (x-2)^2$

$y = (x-2)^2 + 9$ Is a function

Now try $(y-9)^2 = x - 2$

$\sqrt{(y-9)^2} = \sqrt{x-2}$

$|y-9| = \sqrt{x-2}$

$y - 9 = \pm\sqrt{x-2}$

$y = 9 \pm \sqrt{x-2}$ Not a function since one x-value will give two y-values (outputs).

3. Find the inverse function of $f(x) = 2x^3 - 7$ by reversing the composition.

$x \xrightarrow{\text{cube}} x^3 \xrightarrow{\text{multiply by 2}} 2x^3 \xrightarrow{\text{subtract 7}} 2x^3 - 7 = f(x)$

$x \xrightarrow{+7} x+7 \xrightarrow{\div 2} \frac{x+7}{2} \xrightarrow{\sqrt[3]{\ }} \sqrt[3]{\frac{x+7}{2}} = f^{-1}(x)$

4.) Let $f(x) = x^2 + 5$.

a. Simplify the difference quotient $\frac{f(x+h)-f(x)}{h}$. You may use the alternative version of this given by $\frac{f(x)-f(c)}{x-c}$.

$$\frac{f(x+h)-f(x)}{h} = \frac{((x+h)^2+5)-(x^2+5)}{h} = \frac{x^2+2xh+h^2+5-x^2+5}{h}$$

$$= \frac{2xh+h^2}{h} = \frac{h(2x+h)}{h} = \boxed{2x+h}$$

b. find the average rate of change of *f* from x = -1 to x = 1.

$$\frac{f(x)-f(c)}{x-c} = \frac{f(-1)-f(1)}{-1-1} = \frac{((-1)^2+5)-((1)^2+5)}{-2} = \frac{6-6}{-2} = \frac{0}{-2} = \boxed{0}$$

5. Let $f(x) = \sqrt{2x+4}$ *and* $g(x) = 4x - 2$.

a. Determine each of the following functions and state domain of each.

i. $(f+g)(x) = \sqrt{2x+4} + 4x - 2$; $\mathcal{D}g = \mathbb{R}$; $\mathcal{D}f$: Need $2x+4 \geq 0$

$2x \geq -4$

$x \geq -2$

$\boxed{\mathcal{D} = \{x \mid x \geq -2\}}$

ii. $(f-g)(x) = \sqrt{2x+4} - (4x-2) = \sqrt{2x+4} - 4x + 2$

$\mathcal{D} = \{x \mid x \geq -2\}$

iii. $(f \cdot g)(x) = (\sqrt{2x+4})(4x-2) = 4x\sqrt{2x+4} - 2\sqrt{2x+4}$

$\mathcal{D} = \{x \mid x \geq -2\}$

iv. $\left(\frac{f}{g}\right)(x) = \frac{\sqrt{2x+4}}{4x-2}$; $\mathcal{D}g$: Need $4x-2 \neq 0$ $\quad \mathcal{D}g = \{x \mid x \neq \frac{1}{2}\}$

$4x \neq 2$

$x \neq \frac{1}{2}$ $\quad \mathcal{D}f = \{x \mid x \geq -2\}$

$\boxed{\mathcal{D} = [-2, \tfrac{1}{2}) \cup (\tfrac{1}{2}, \infty)}$

v. $\left(\frac{g}{f}\right)(x) = \frac{4x-2}{\sqrt{2x+4}}$; $\mathcal{D}_g = \mathbb{R}$; $\mathcal{D}_f$: Need, $2x+4 \geq 0$ and, Need $\sqrt{2x+4} \neq 0$

$x \geq -2$

$2x+4 \neq 0$

$2x \neq -4$

$x \neq -2$

$\boxed{\mathcal{D} = (-2, \infty)}$

6. Determine algebraically whether the following functions are even, odd, or neither.

a. $h(x) = \frac{x^4 - x^2 - 3}{x^2} \Rightarrow h(-x) = \frac{(-x)^4 - (-x)^2 - 3}{(-x)^2} = \frac{x^4 - x^2 - 3}{x^2} = h(x)$ even function

b. $g(x) = \frac{\cancel{\sqrt[3]{x^2}}\, x^2 + x^3 - 5}{x^5 - x} \Rightarrow g(-x) = \frac{(-x)^2 + (-x)^3 - 5}{(-x)^5 - (-x)} = \frac{x^2 - x^3 - 5}{-x^5 + x} \neq g(x)$ & $\neq -g(x)$

Function is neither odd nor even

7. Determine whether each function is one to one.

i) $h(x) = 3x - 7$

Suppose $h(x_1) = h(x_2)$

$3x_1 - 7 = 3x_2 - 7$

$3x_1 = 3x_2$

$x_1 = x_2$

Since $h(x_1) = h(x_2)$ when $x_1 = x_2$, the function is 1-to-1.

ii) $g(x) = \frac{x-4}{x+1}$

Suppose $g(x_1) = g(x_2)$

$\frac{x_1 - 4}{x_1 + 1} = \frac{x_2 - 4}{x_2 + 1}$

$(x_2+1)(x_1-4) = (x_2-4)(x_1+1)$

$x_2x_1 - 4x_2 + x_1 - 4 = x_2x_1 + x_2 - 4x_1 - 4$

$-4x_2 + x_1 \cancel{-4} = x_2 - 4x_1 \cancel{-4}$

$-5x_2 = -5x_1$

$x_2 = x_1$

The function is 1-to-1

iii) $v(x) = 3x^2 - 2$

Suppose $V(x_1) = V(x_2)$

$3(x_1)^2 - 2 = 3(x_2)^2 - 2$

$3(x_1)^2 = 3(x_2)^2$

$\to (x_1)^2 = (x_2)^2$

$\sqrt{x_1^2} = \sqrt{x_2^2}$

$|x_1| = \sqrt{(x_2)^2}$

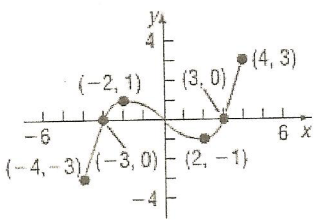
$x_1 = \pm\sqrt{(x_2)^2}$ Not 1-to-1

8. Find functions f and g so that $(f \circ g)(x) = H$, given that $H = (2x - 3)^4$.

$H = (\boxed{2x-3})^4$

let $f(x) = x^4$ and $g(x) = 2x-3$, then $(f \circ g)(x) = f(2x-3) = (2x-3)^4$

9. Use the graph of the function f below to answer the following questions:



a. The intercepts
(Express answers as ordered pairs.)

i. (3 pts) x-intercept(s): $(-3, 0)$, $(3, 0)$, $(0, 0)$

ii. (3 pts) y-intercept(s): $(0, 0)$

b. (3 pts) The domain and range:

$\mathcal{D} = [-4, 4]$ $R = [-3, 3]$

c. Intervals of increase/decrease:

i. (3 pts) f is increasing on $[-4, -2]$ and $[2, 4]$.

ii. (3 pts) f is decreasing on $[-2, 2]$.

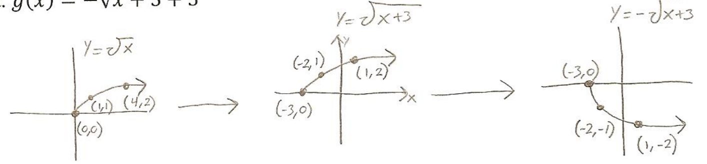
d. Extrema:

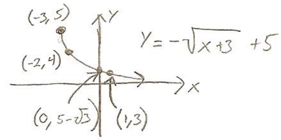
i. (3 pts) f has local minimum of -1 at 2.

ii. (2 pts) f has a local maximum of 1 at -2.

10. Graph each of the following functions using techniques of shifting. Compressing, and stretching, and/ or reflecting. Start with the graph of the basic function and show all stages in separate sketches. Track 3 key points through the transformations and show the *y*-intercept in the final sketch.

a. $g(x) = -\sqrt{x+3} + 5$



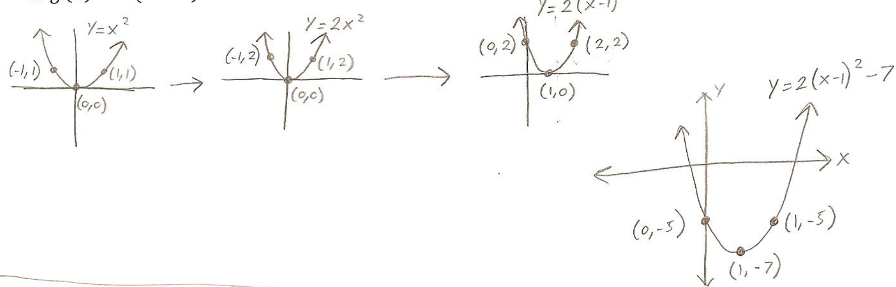


Y-int: $Y = -\sqrt{0+3} + 5$

$Y = 5 - \sqrt{3}$

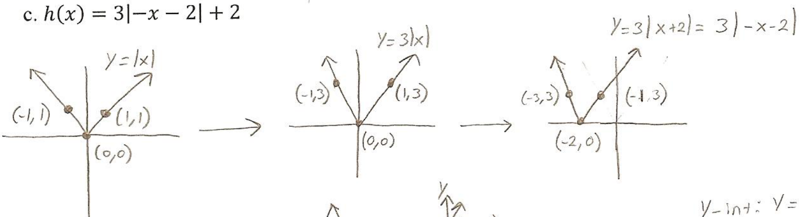
Y-int: $(0, 5-\sqrt{3})$

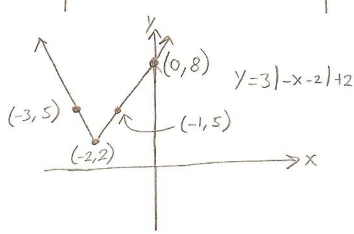
b. $g(x) = 2(x-1)^2 - 7$



$= 3|-1||x+2| + 2$

c. $h(x) = 3|-x-2| + 2$





Y-int: $Y = 3|0-2| + 2$

$Y = 3(2) + 2 = 8$

$(0,8) =$ y-int

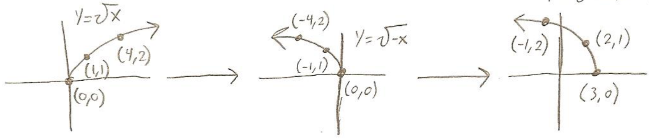
$= \sqrt{3-x} = \sqrt{-x+3} = \sqrt{-(x-3)}$

d. $f(x) = \sqrt{3 - x} - 1$

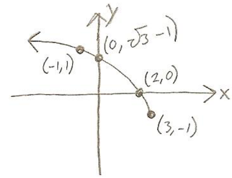
Y-int:

$y = \sqrt{3} - 1$

$y = \sqrt{3-x} = \sqrt{-(x-3)}$



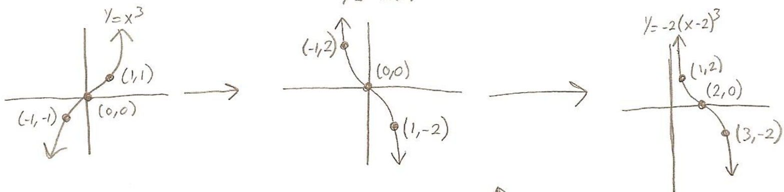
$y = \sqrt{3-x} - 1$

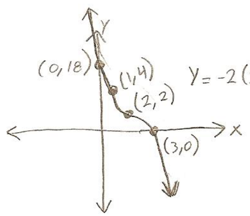


e. $j(x) = -2(x-2)^3 + 2$

$y = -2(x)^3$

$y = -2(x-2)^3$





$y = -2(x-2)^3 + 2$

Y-int:

$y = -2(-2)^3 + 2$

$y = -2(-8) + 2$

$y = 16 + 2 = 18$

Y-int: (0,18)

f. $f(x) = -4(2 - x)^3 - 5$

$y = -4(x)^3$

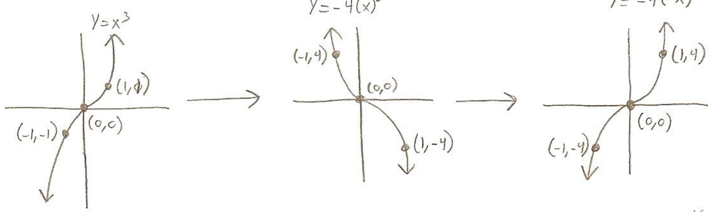
$y = -4(-x)^3$

Y-int:

$y = -4(2-0)^3 - 5$

$y = -4(8) - 5$

$y = -32 - 5 = -37$



$y = -4(-x+2)^3 = -4[-(x-2)]^3$