Practice Test 1
MAT 121-G11
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Work these problems in full. Show your work. I suggest writing down EVERY equation you use before you plug your numbers in. This should help you remember the formulas. If you get stuck on a problem, move on to another problem, or start writing down everything you know about the problem. Sometimes you will write something down that will give you insight to what the next step is. Have a good test!

Solve the equations. State whether the equations are identities, conditional or inconsistent equations.

1) $5 + 7|x + 6| = 19$

$\Rightarrow 7|x+6| = 14$

$\Rightarrow |x+6| = 2$

$\Rightarrow x+6 = 2$ or $x+6 = -2$

$\Rightarrow x = -4$ or $x = -8$

$x \in \{-8, -4\}$ conditional

2) $9 + |3x - 9| = 4$

$|3x - 9| = -5$

Nope, an absolute value will NOT equal a negative number

$\emptyset$ or $\{\ \}$ Inconsistent

3) $\frac{3}{2}x + \frac{1}{3} = \frac{1}{4}x - \frac{1}{6}$

$\Rightarrow 12\left(\frac{3}{2}x\right) - 12\left(\frac{1}{4}x\right) = 12\left(-\frac{1}{3}\right) - 12\left(\frac{1}{6}\right)$

$\Rightarrow 18x - 3x = -4 - 2$

$\Rightarrow 15x = -6$

$\Rightarrow x = -\frac{6}{15} = -\frac{2}{5}$

$x \in \left\{-\frac{2}{5}\right\}$ conditional

4) $\cancel{\frac{x(x+2)}{x+2} = x}$

$\frac{x^2+2x}{x+2} = x$

$\Rightarrow \frac{x(x+2)}{x+2} = x$

$\Rightarrow x = x$ Identity

$\{x \mid x \neq 0\}$

5) Determine the midpoint of the line between the points P(1, 3), P(4, 7).

$x_1\ y_1 \quad x_2\ y_2$

Midpoint Formula: $\left(\frac{x_2 - x_1}{2}, \frac{y_2 - y_1}{2}\right)$

$\Rightarrow \left(\frac{4-1}{2}, \frac{7-3}{2}\right)$

$\Rightarrow \left(\frac{3}{2}, 2\right)$ is midpoint

6) Find the distance between the two points in problem 5.

Distance Formula: $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

$\Rightarrow d = \sqrt{(4-1)^2 + (7-3)^2} = \sqrt{3^2 + 4^2} = \sqrt{9+16} = \sqrt{25} = \boxed{5}$

7) Find the center and radius of the circle and sketch the graph.

$(\frac{1}{2} \cdot b)^2$

$x: (\frac{1}{2} \cdot 10)^2 = (5)^2 = 25$

$y: (\frac{1}{2} \cdot -4)^2 = (-2)^2 = 4$

$$x^2 + y^2 - 9 = -10x + 4y$$

$(x-h)^2 + (y-k)^2 = r^2$

$x^2 + 10x + y^2 - 4y = 9$

$x^2 + 10x + (5)^2 + y^2 - 4y = 9 + 25$

$(x+5)^2 + y^2 - 4y + (-2)^2 = 34 + 4$

$(x+5)^2 + (y-2)^2 = 36$

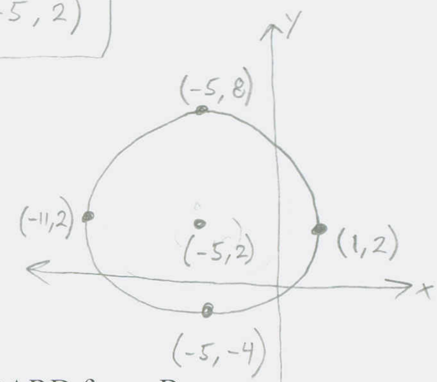
$(x-h)^2 + (y-k)^2 = r^2$

$\boxed{\text{Center} = (h,k) = (-5, 2)}$

radius: $r^2 = 36$

$r = \sqrt{36}$

$\boxed{r = 6}$



8) a) Sketch a graph of the linear equation then put the equation in STANDARD form. Be sure to clearly show and label both intercepts.

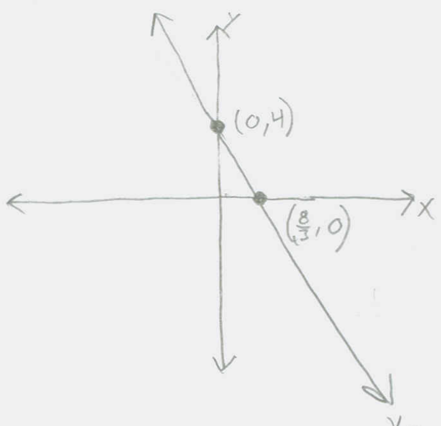
$$2y = -3x + 8$$

$y = -\frac{3}{2}x + 4$

$2y = -3x + 4$

$\boxed{3x + 2y = 4}$

$Ax + By = C$



y-intercept: let $x = 0$,

$y = -\frac{3}{2}(0) + 4 = 4$

y-int: $(0, 4)$

x-intercept: let $y = 0$,

$0 = -\frac{3}{2}x + 4$

$\frac{3}{2}x = 4$

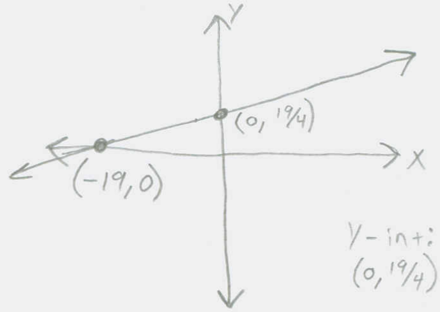
$x = \frac{8}{3}$

x-int: $(\frac{8}{3}, 0)$

Remember to label your axes on your graphs!

b) Find the equation of the line that passes through the points (-3, 4), (5, 6). Sketch the graph. Show and label both intercepts. Write your equation in both point-slope form and in slope-intercept form.

$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{6-4}{5-(-3)} = \frac{2}{8} = \frac{1}{4}$



$y = m(x - x_1) + y_1$

$y = \frac{1}{4}(x - (-3)) + 4$

$\boxed{y = \frac{1}{4}(x+3) + 4}$ point slope form

$y = \frac{1}{4}(x+3) + 4$

$y = \frac{1}{4}x + \frac{3}{4} + \frac{16}{4}$

$\boxed{y = \frac{1}{4}x + \frac{19}{4}}$ slope int. form $y = mx + b$

y-int: $y = \frac{1}{4}(0) + \frac{19}{4}$

$(0, 19/4)$

x-int: $0 = \frac{1}{4}x + \frac{19}{4}$

$-\frac{1}{4}x = \frac{19}{4}$

$x = \frac{19}{4}(-4) = 19$

$(-19, 0)$

c) find the equation of a line that is perpendicular to the line in part b. (hint: use point-slope form) $m_\perp$ ← perpendicular line

{think "negative reciprocal" for slope of perpendicular line}

$m_\perp = -\frac{1}{m} = -\frac{1}{\frac{1}{4}} = -4$

$y_\perp = -4(x - x_1) + y_1 = -4(x - (-3)) + 4 = -4x - 12 + 4 = -4x - 8$, $\boxed{y_\perp = -4x - 8}$

9) Compute the discriminant for the following equations and state how many solutions there will be and whether they are real or not real.

$b^2 - 4ac \equiv$ discriminant

a) $f(x) = 2x^2 + 3x - 9$

a b c=-9

$b^2 - 4ac$

$3^2 - 4(2)(-9) = 9 + 72 = 81$

$\boxed{81 > 0 \text{ Two real solutions}}$

If $b^2 - 4ac > 0$ Two real solns

If $b^2 - 4ac < 0$ Two non-real solns

b) $g(x) = 4x - 3x^2 + 7$

a b=-3 c=7

$b^2 - 4ac$

$(-3)^2 - 4(4)(7) = 9 - 112 = -103$

$\boxed{-103 < 0 \text{ Two non-real solutions}}$

If $b^2 - 4ac = 0$ one real "repeated" solution

c) $f(x) = 2x^2 - 8x + 8$

a b=-8 c=8

$b^2 - 4ac$

$(-8)^2 - 4(2)(8) = 64 - 64 = 0$ one real "repeated" solution

10) Solve the equation $2x^2 = 12 - 8x$ by

a) completing the square

$$2x^2 + 8x = 12$$

$$\left(\frac{1}{2}\cdot 4\right)^2 = (2)^2 = 4$$

$$x^2 + 4x = 6$$

$$x^2 + 4x + (2)^2 = 6 + 4$$

$$(x+2)^2 = 10$$

$$(x+2)^2 = 10$$

$$\sqrt{(x+2)^2} = \sqrt{10}$$

$$|x+2| = \sqrt{10}$$

$$x + 2 = \pm\sqrt{10}$$

$$x = -2+\sqrt{10} \text{ or } x = -2-\sqrt{10}$$

$$\boxed{x \in \{-2-\sqrt{10}, -2+\sqrt{10}\}}$$

b) using the quadratic formula.

$$\underset{a}{2x^2} + \underset{b}{8x} \underset{c}{-12} = 0$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-8 \pm \sqrt{8^2 - 4(2)(-12)}}{2(2)} = \frac{-8 \pm \sqrt{64+96}}{4} = \frac{-8 \pm \sqrt{160}}{4} = \frac{-8 \pm \sqrt{16 \cdot 10}}{4}$$

$$= \frac{-8 \pm \sqrt{16}\sqrt{10}}{4} = \frac{-8 \pm 4\sqrt{10}}{4} = \frac{4(-2 \pm \sqrt{10})}{4} = -2 \pm \sqrt{10}$$

$$\boxed{x \in \{-2-\sqrt{10}, -2+\sqrt{10}\}}$$

11) Solve by factoring.

$$f(x) = x^2 + x - 12$$

$$x^2 + x - 12 = 0$$

$$(x+4)(x-3) = 0$$

$$x + 4 = 0 \quad \text{OR} \quad x - 3 = 0$$

$$x = -4 \qquad x = 3$$

$$x \in \{-4, 3\}$$

12) Solve the following inequalities. Put your answer in both set-builder and interval notation.

a) $\frac{7-3x}{2} \geq -3$

$$7 - 3x \geq -6$$

$$-3x \geq -13$$

$$x \leq \frac{13}{3}$$

$x \in \left(-\infty, \frac{13}{3}\right]$ Interval Notation

$\left\{x \mid -\infty < x \leq \frac{13}{3}\right\}$ Set-builder

b) $3\left|\frac{x-2}{2}\right| + 6 > 12$

$$3\left|\frac{x-2}{2}\right| > 6$$

$$\left|\frac{x-2}{2}\right| > 2$$

$$\frac{x-2}{2} > 2 \quad \text{OR} \quad \frac{x-2}{2} < -2$$

$$x - 2 > 4 \qquad x - 2 < -4$$

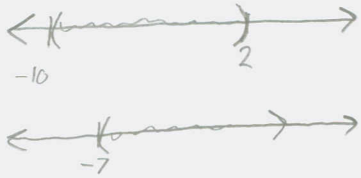
$$x > 6 \qquad x < -2$$

$$x \in (-\infty, -2) \cup (6, \infty)$$

$$\{x \mid x < -2 \text{ OR } x > 6\}$$

13) Write as a single interval.

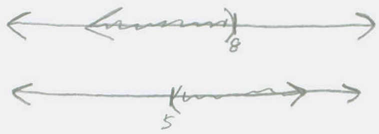
a) $(-10, 2) \overset{\text{and}}{\cap} [-7, \infty)$



We only want the values that are common to both lines... we want the "overlap" of the two lines.

$x \in (-7, 2)$; $\{x \mid -7 < x < 2\}$

b) $(-\infty, 8) \overset{\text{OR}}{\cup} (5, \infty)$



we want all the values that are contained in either of the two lines

$x \in (-\infty, \infty)$

$x \in \mathbb{R}$

14) Solve the compound inequality. Give your answer in both set-builder and interval notation.

$\frac{1}{2}(x + 1) > 3$ *or* $\cancel{0 < 7 - x}$

$(x+1) > 6$ $\quad$ $0 < 3 - x$

$x > 5$ $\quad$ $x < 3$

$x \in (-\infty, 3) \cup (5, \infty)$

$\{x \mid x < 3 \text{ OR } x > 5\}$

15) How many liters of a 15% alcohol solution and how may liters of a 10% alcohol solution should be mixed together to obtain 15 liters of a 14% alcohol solution?

Liters total $= 15$

Liters 15% $= x$

Liters 10% $= 15 - x$

$15\%(x) + 10\%(15 - x) = 14\%(15)$

$.15(x) + .1(15 - x) = .14(15)$

$.15x + 1.5 - .1x = 2.1$

$.05x = 0.6$

$x = \frac{0.6}{.05} = 12$

$15 - x = 15 - 12 = 3$

Need 12 L of 15% alcohol soln and 3 L of 10% alcohol solution to make 15 L of 14%

16) George can paint a house in 8 hours working alone. Bill can do the same job alone in 12 hours. If Bill starts painting at 8 A.M. and George joins him at 10 A.M., then at what ~~tine~~ time will they have the entire house painted?

	Rate	Time	How much of job done
Bill	$\frac{1}{12}\frac{\text{house}}{\text{hr}}$	x	$\frac{1}{12}x$
George	$\frac{1}{8}\frac{\text{house}}{\text{hr}}$	$x+2$	$\frac{1}{8}(x+2)$

x = the time it takes Bill to get the house painted when working with George (in hours).

$$\frac{1}{12}x + \frac{1}{8}(x+2) = 1$$

$$\frac{1}{12}x + \frac{1}{8}x + \frac{2}{8} = 1$$

$$\frac{2}{24}x + \frac{3}{24}x = \frac{8}{8} - \frac{2}{8}$$

$$\frac{5}{24}x = \frac{6}{8}$$

$$\frac{5}{24}x = \frac{3}{4}$$

$$x = \frac{3}{4}\left(\frac{24}{5}\right)$$

$$x = \frac{3 \cdot 6}{1 \cdot 5}$$

$$x = \frac{18}{5} = 3\frac{3}{5}$$

Bill painted for 3.6 hrs, so.
George painted for 5.6 hrs
George started at 8 A.M.
5.6 hrs after 8 A.M. is

$\boxed{\text{1:36 P.M.}}$

Note: $\frac{3}{5}\text{hr} \cdot \frac{60\text{min}}{1\text{hr}} = 36\text{min}$

EXTRA) Derive the quadratic formula from the standard form of a quadratic equation given as

$ax^2 + bx + c = 0$ Note: you will probably definitely not see this on the test, but if you can do this, then you should have a very good understanding of what is going on with quadratic equations!

$$ax^2 + bx = -c$$

$$x^2 + \frac{b}{a}x = -\frac{c}{a}$$

$$\left(\frac{1}{2}\cdot\frac{b}{a}\right)^2 = \left(\frac{b}{2a}\right)^2 = \frac{b^2}{4a^2}$$

$$x^2 + \frac{b}{a}x + \left(\frac{b}{2a}\right)^2 = -\frac{c}{a} + \frac{b^2}{4a^2}$$

$$\left(x + \frac{b}{2a}\right)^2 = -\frac{c}{a}\left(\frac{4a}{4a}\right) + \frac{b^2}{4a^2}$$

$$\left(x + \frac{b}{2a}\right)^2 = -\frac{4ac}{4a^2} + \frac{b^2}{4a^2}$$

$$\left|x + \frac{b}{2a}\right| = \sqrt{\frac{b^2 - 4ac}{4a^2}}$$

$$x + \frac{b}{2a} = \pm\frac{\sqrt{b^2 - 4ac}}{\sqrt{4a^2}}$$

$$x = -\frac{b}{2a} \pm \frac{\sqrt{b^2 - 4ac}}{2a}$$

$$\boxed{x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}}$$