§6.2 #'s $\underbrace{1, 5, 9, \ldots, 45}_{\text{Every } 4^{th}}$, 53, 57

§6.1

$$x - y = 2$$
$$3x - y = 12$$

$$\begin{array}{c|ccc} -3R1 & -3 & 3 & -6 \\ R2 & 3 & -1 & 12 \\ \hline -3R1+R2 & 0 & 2 & 6 \end{array}$$

$$\left[\begin{array}{cc|c} 1 & -1 & 2 \\ 3 & -1 & 12 \end{array}\right]\begin{array}{l} R1 \\ R2 \end{array}$$

$$-3R1+R2 \to R2 \quad \left[\begin{array}{cc|c} 1 & -1 & 2 \\ 0 & 2 & 6 \end{array}\right]$$

$$\tfrac{1}{2}R2 \to R2 \quad \left[\begin{array}{cc|c} 1 & -1 & 2 \\ 0 & 1 & 3 \end{array}\right]$$

$$R2+R1 \to R1 \quad \left[\begin{array}{cc|c} 1 & 0 & 5 \\ 0 & 1 & 3 \end{array}\right]$$

$x=5$
$y=3$

$\{(5,3)\}$

$$\left[\begin{array}{cc|c} * & * & * \\ * & * & * \end{array}\right] \sim \left[\begin{array}{cc|c} 1 & * & * \\ * & * & * \end{array}\right] \sim \left[\begin{array}{cc|c} 1 & * & * \\ 0 & * & * \end{array}\right]$$

$$\sim \left[\begin{array}{cc|c} 1 & * & * \\ 0 & 1 & * \end{array}\right] \sim \left[\begin{array}{cc|c} 1 & 0 & * \\ 0 & 1 & * \end{array}\right]$$

3-variables

$$\left[\begin{array}{ccc|c} * & * & * & * \\ * & * & * & * \\ * & * & * & * \end{array}\right] \sim \left[\begin{array}{ccc|c} 1 & * & * & * \\ * & * & * & * \\ * & * & * & * \end{array}\right]$$

The basic scheme

$$\left[\begin{array}{ccc|c} 1 & * & * & * \\ 0 & * & * & * \\ 0 & * & * & * \end{array}\right] \sim \left[\begin{array}{ccc|c} 1 & * & * & * \\ 0 & 1 & * & * \\ 0 & * & * & * \end{array}\right]$$

$$\left[\begin{array}{ccc|c} 1 & 0 & * & * \\ 0 & 1 & * & * \\ 0 & 0 & * & * \end{array}\right] \sim \left[\begin{array}{ccc|c} 1 & 0 & 0 & \star \\ 0 & 1 & 0 & \odot \\ 0 & 0 & 1 & \clubsuit \end{array}\right]$$

$x = \star$
$y = \odot$
$z = \clubsuit$

$$\begin{aligned} x \quad\quad - z &= -2 \\ 2x - y \quad\quad &= 1 \\ y + 3z &= 15 \end{aligned}$$

$$\left[\begin{array}{ccc|c} 1 & 0 & -1 & -2 \\ 2 & -1 & 0 & 1 \\ 0 & 1 & 3 & 15 \end{array}\right](-2)$$

$-2R1$	-2	0	2	4
$R2$	2	-1	0	1
	0	-1	2	5

$$-2R1 + R2 \rightarrow R2 \quad \left[\begin{array}{ccc|c} 1 & 0 & -1 & -2 \\ 0 & -1 & 2 & 5 \\ 0 & 1 & 3 & 15 \end{array}\right]$$

$$-R2 \rightarrow R2 \quad \left[\begin{array}{ccc|c} 1 & 0 & -1 & -2 \\ 0 & 1 & -2 & -5 \\ 0 & 1 & 3 & 15 \end{array}\right]$$

$$-R2 + R3 \rightarrow R3 \quad \left[\begin{array}{ccc|c} 1 & 0 & -1 & -2 \\ 0 & 1 & -2 & -5 \\ 0 & 0 & 5 & 20 \end{array}\right]$$

$$\frac{1}{5}R3 \quad \left[\begin{array}{ccc|c} 1 & 0 & -1 & -2 \\ 0 & 1 & -2 & -5 \\ 0 & 0 & 1 & 4 \end{array}\right]$$

$R3$	0	0	1	4
$R1$	1	0	-1	-2
	1	0	0	2

$$\begin{array}{l} R3 + R1 \rightarrow R1 \\ 2R3 + R2 \rightarrow R2 \end{array} \quad \left[\begin{array}{ccc|c} 1 & 0 & 0 & 2 \\ 0 & 1 & 0 & 3 \\ 0 & 0 & 1 & 4 \end{array}\right]$$

$2R3$	0	0	2	8
$R2$	0	1	-2	-5
	0	1	0	3

$x = 2$

$y = 3$

$z = 4$

$$\boxed{\{(2, 3, 4)\}}$$

$$x - 2y + z = 1$$
$$2x + y - z = 5$$
$$4x - 3y + z = 7$$

Dependent system

$$\left[\begin{array}{ccc|c} 1 & -2 & 1 & 1 \\ 2 & 1 & -1 & 5 \\ 4 & -3 & 1 & 7 \end{array}\right]$$

$-2R1 + R2 \rightarrow R2$
$-4R1 + R3$

$$\left[\begin{array}{ccc|c} 1 & -2 & 1 & 1 \\ 0 & 5 & -3 & 3 \\ 0 & 5 & -3 & 3 \end{array}\right] \sim \left[\begin{array}{ccc|c} 1 & -2 & 1 & 1 \\ 0 & 5 & -3 & 3 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

$\frac{1}{5}R2$
$$\left[\begin{array}{ccc|c} 1 & -2 & 1 & 1 \\ 0 & 1 & -\frac{3}{5} & \frac{3}{5} \\ 0 & 0 & 0 & 0 \end{array}\right]$$

$$\frac{6}{5} + 1 = \frac{6}{5} + \frac{5}{5} = \frac{11}{5}$$

$2R2$	0	2	$-\frac{6}{5}$	$\frac{6}{5}$
$R1$	1	-2	1	1
	1	0	$-\frac{1}{5}$	$\frac{11}{5}$

$2R2 + R1$
$$\begin{array}{cccc} x & y & z & \\ \end{array}$$
$$\left[\begin{array}{ccc|c} 1 & 0 & -\frac{1}{5} & \frac{11}{5} \\ 0 & 1 & -\frac{3}{5} & \frac{3}{5} \\ 0 & 0 & 0 & 0 \end{array}\right]$$

$$x - \frac{1}{5}z = \frac{11}{5}$$
$$x = \frac{1}{5}z + \frac{11}{5}$$
$$y - \frac{3}{5}z = \frac{3}{5}$$
$$y = \frac{3}{5}z + \frac{3}{5}$$

Sol'n Set

$$\left\{ (x, y, z) \,\middle|\, x = \frac{1}{5}z + \frac{11}{5},\ y = \frac{3}{5}z + \frac{3}{5},\ z \in \mathbb{R} \right\}$$
$$= \left\{ \left(\frac{1}{5}z + \frac{11}{5},\ \frac{3}{5}z + \frac{3}{5},\ z\right) \,\middle|\, z \text{ is real} \right\}$$

Either answer suffices.
Convention is to let the last variable be free & define other variable in terms of the free variables.

$$x - 2y + z = 1$$
$$2x + y - z = 5$$
$$4x - 3y + z = 11$$

$$\left[\begin{array}{ccc|c} 1 & -2 & 1 & 1 \\ 2 & 1 & -1 & 5 \\ 4 & -3 & 1 & 11 \end{array}\right] \sim \left[\begin{array}{ccc|c} 1 & -2 & 1 & 1 \\ 0 & 5 & -3 & 3 \\ 0 & 5 & -3 & 7 \end{array}\right]$$

No sol'n

$$\left[\begin{array}{ccc|c} 1 & -2 & 1 & 1 \\ 0 & 1 & -\frac{3}{5} & \frac{3}{5} \\ 0 & 0 & 0 & 4 \end{array}\right] \quad 0 = 4!?$$

No sol'n

$$\boxed{\varnothing}$$

$$\begin{array}{c|c} & \\ \begin{array}{cc} & \\ A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} & \begin{matrix} 1 \\ 2 \end{matrix} \Big\} \text{2 ROWS} \end{array} \end{array}$$

$\underbrace{1 \quad 2}_{\text{2 columns}}$

2 x 2 matrix

$a_{12} = 2$ = 1st row, 2nd column

$a_{22} = 4$

$B = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix}$ 2 ROWS

3 cols

B is a 2 x 3
2 Rows, 3 Columns.

$b_{22} = 5$

b_{32} = they ain't one.

But $b_{23} = 6$

$3B = \begin{bmatrix} 3 & 6 & 9 \\ 12 & 15 & 18 \end{bmatrix}$

A+B can't do different sizes.