

### Mastery Learning

Two questions from Test 3 - Perfect score &  
I'll split the difference with you.

Two questions from Test 4 - Perfect score, etc.  
Do this one week from today.

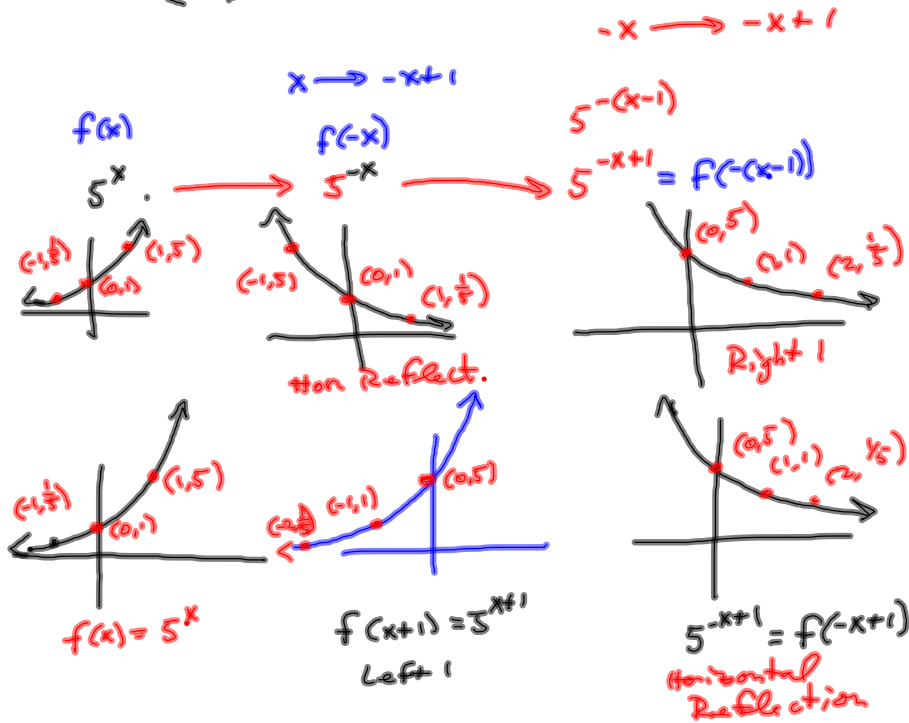
$5^{1-x}$  how to deal with it?

- vertical shift  $f(x) \pm k$
- horizontal shift  $f(x \pm k)$
- $\longleftrightarrow$  horizontal reflect  $f(-x)$
- $\updownarrow$  vertical reflect  $-f(x)$

$x \rightarrow 1-x$ ?

①  $x \xrightarrow{\text{left 1}} x+1 \xrightarrow{\text{reflect}} -x+1 = 1-x$

②  $x \xrightarrow{\text{reflect}} -x \xrightarrow{\text{right 1}} -(x-1) = 1-x$



most common screwup:

$f(x) \xrightarrow{\text{Horiz. Flip}} f(-x) \xrightarrow{\text{Left 1}} f(-x+1)$

**Bad!**

I'm not replacing  $x$  by  $x+1$   
 I'm replacing  $-x$  by  $-x+1$ . within the transformations that are legit, this is replacing  $x$  by  $x-1$   
 $-x+1 = -(x-1)$

Last Time, we may've been shaky on systems with infinitely many solutions in the 3-variable case

$$2x - y + z = 7 \quad E1$$

$$y + z = 5 \quad E2$$

I eliminated  $y$  in  $E1$  to get  $x$  &  $y$  in terms of  $z$ :

Replace  $E1$  by  $E1 + E2$

$$2x - y + z = 7$$

$$y + z = 5$$

$$\hline \begin{array}{r} 2x \quad + 2z = 12 \\ x \quad + z = 6 \end{array}$$

More variables than restrictions.

By convention, let  $z$  be "free" and  $x$  &  $y$  depend on  $z$ .

New System:

$$\begin{array}{r} x \quad + z = 6 \\ y \quad + z = 5 \end{array}$$

$$\Rightarrow x = -z + 6$$

$$y = -z + 5$$

Sol'n set

$$\left\{ (-z+6, -z+5, z) \mid z \text{ is free} \right\}$$

This is a line.

$$2x - y + z = 7 \quad E_1$$

$$y + z = 5 \quad E_2$$

Solve  $E_2$  for  $y$ !

$$y = -z + 5$$

Send it to  $E_1$

$$2x - (-z + 5) + z = 7$$

$$2x + z - 5 + z = 7$$

$$2x + 2z - 5 = 7$$

$$2x + 2z = 12$$

$$x + z = 6$$

Two eq'ns

3 var's

Let  $z$  be free.

New System

$$x + z = 6$$

$$y + z = 5$$

$$\left\{ \begin{array}{l} (-z + 6, -z + 5, z) \\ z \text{ is real} \end{array} \right\}$$

↳ See previous page.

Perimeter of triangle is 60 ft.

$$x + y + z = 60$$

Sum of two shorter sides is 2 ft more than the long side.

$$y + z = x + 2$$

Longest side is 11 ft more than shortest side.

$$x = z + 11$$

Find the lengths of each side

Let  $x$  = length of longest side (in feet)  
 $y$  = " " " " medium " " "  
 $z$  = " " " " shortest " " "

System in standard form

$$\begin{aligned} x + y + z &= 60 \\ -x + y + z &= 2 \\ x - z &= 11 \end{aligned}$$

Augmented matrix for this system.

$$\left[ \begin{array}{ccc|c} 1 & 1 & 1 & 60 \\ -1 & 1 & 1 & 2 \\ 1 & 0 & -1 & 11 \end{array} \right]$$

65 Burger, fries & coke cost \$3.80 last yr.

$$x + y + z = 3.8 \quad E1$$

Price of burger rose 10%

.. .. fries .. 20%

.. .. coke .. 25%

Same meal costs \$4.49 this year.

Solutions online missed this.

Price of coke is **Now** 7¢ less than burger

What was the price of each last year?

$x$  = price of burger LAST year (in \$)

$y$  = .. .. fries .. ..

$z$  = .. .. coke .. ..

Same meal costs \$4.49 this year.

$$1x + .1x = 1.1x = \text{cost of burger this year.}$$

$$1y + .2y = 1.2y = \text{.. .. fries .. ..}$$

$$z + .25z = 1.25z = \text{.. .. coke .. ..}$$

$$1.1x + 1.2y + 1.25z = 4.49 \quad E2$$

$$1.25z = 1.1x + .07 \quad E3$$

↑ should be "-"

Nathan says "less"

$$x + y + z = 3.80$$

$$1.1x + 1.2y + 1.25z = 4.49$$

$$-1.1x \quad +1.25z = \bar{.07}$$

Should be a minus.

### §5.3 Systems of Nonlinear Eq'ns.

$$\begin{array}{l}
 y = -x^2 \\
 y = 3x
 \end{array}
 \Rightarrow
 \begin{array}{l}
 y = y \\
 -x^2 = 3x \\
 -x^2 - 3x = 0 \\
 x^2 + 3x = 0 \\
 x(x+3) = 0 \\
 x = 0 \text{ OR } x = -3
 \end{array}$$

$\{(0,0), (-3,-9)\}$

$\downarrow$   
 $y = 0$   
 from  $y = 3x$

$\rightarrow$   
 $y = -9$   
 from  
 $y = 3x$

$$2x^2 - y = 8 \implies 2x^2 - 8 = y$$

$$7x + y = -4$$

ONE WAY

$$7x + (2x^2 - 8) = -4$$

$$2x^2 + 7x - 8 = -4$$

$$2x^2 + 7x - 4 = 0$$

Another way

$$y = -7x - 4$$

$$2x^2 - (-7x - 4) = 8$$

$$2x^2 + 7x + 4 = 8$$

$$2x^2 + 7x - 4 = 0$$

etc.

$$(2x - 1)(x + 4) = 2x^2 + 8x - 1x - 4 \checkmark$$

$$2x - 1 = 0$$

$$2x = 1$$

$$x = \frac{1}{2}$$

$$y = 2x^2 - 8$$

$$= 2\left(\frac{1}{2}\right)^2 - 8$$

$$= 2\left(\frac{1}{4}\right) - 8$$

$$= \frac{1}{2} - 8$$

$$= \frac{1 - 16}{2} = -\frac{15}{2}$$

$$\left(\frac{1}{2}, -\frac{15}{2}\right)$$

$$x + 4 = 0$$

$$x = -4$$

$$y = 2x^2 - 8$$

$$= 2(-4)^2 - 8$$

$$= +32 - 8$$

$$= 24$$

$$(-4, 24)$$



$$y = x^2$$

$$x = y^2$$

Σ 5.3 #s 5, 7, 11, 23, 33, 37, 41, 45

① Send  $y = x^2$  to  $E2$  :

$$x = (x^2)^2$$

$$x = x^4$$

$$x^4 - x = 0$$

$$x(x^3 - 1) = 0$$

$$x = 0$$

$$\text{OR } x^3 - 1 = 0$$

$$y = 0$$

$$x^3 = 1$$

$$x = \sqrt[3]{1} = 1$$

$$y = 1$$

$$\{(0,0), (1,1)\}$$

② Send  $x = y^2$  to  $E1$

$$y = (y^2)^2$$

$$y = y^4$$

⋮

$$y = 0 \quad \text{OR} \quad y = 1$$

$$x = 0 \quad \text{OR} \quad x = 1$$