§5.1 #s 1-7, 10, 12, 25, 30, 34, 39, 41, 45, 59, 61, 63, 74

§5.2 #s 7, 9, 11, 17, 31, 33, 41, 47, 60, 63, 64, 65, 71

Setup only:
List variables, verbal description, units
Write system of equations.

A solution for a system of linear equations is an ordered pair (x, y) that satisfies all equations.

This is 2-variable version.

3-variable version ... an ordered triple ...

5- .. .. 5-tuple ...

n- .. .. n-tuple

$(x,y) = (-1,2)$ is a solution of

$x + y = 1$ $\quad -1 + 2 = 1$ ✓

$2x - 3y = -8$ $\quad 2(-1) - 3(2) = -8$ ✓

Is $(3,2)$ a soln for

$2x + 4y = 16$ $\quad 2(3) + 4(2) = 14$ No.

$3x - y = 7$

Solving a system by substitution.

$-2x+y=1 \Rightarrow y=2x+1$

$3x-4y=1$

$3x-4(2x+1)=1$

$3x-8x-4=1$

$-5x=5$

$x=-1$

$\Rightarrow y=2(-1)+1$

$=-2+1$

$=-1=y$

$\{(-1,-1)\}$ is solution set.

One solution:

System is

INDEPENDENT

$y - 3x = 5 \Rightarrow y = 3x + 5$

$3(x+1) = y - 2$

$3x + 3 = 3x + 5 - 2$

$3x + 3 = 3x + 3$

$-3 = \quad -3$

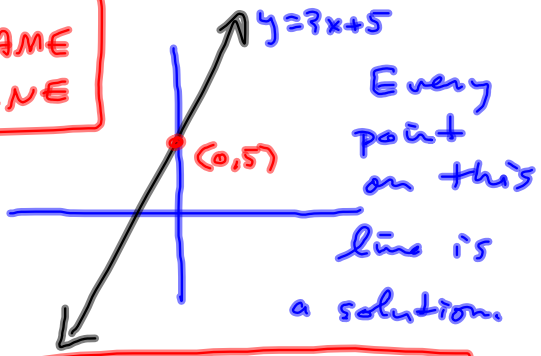
$3x = 3x$

$-3x = -3x$

$0 = 0$

SAME LINE

$\{(x,y) \mid y = 3x + 5\}$



This system is

DEPENDENT

$\{(x,y) \mid y - 3x = 5\}$

$\{(x,y) \mid 3(x+1) = y - 2\}$

More in keeping with 5.2:

$\{(x, 3x+5) \mid x \text{ is real}\}$

$2y = 1 - 4x$

$2x + y = 0 \Rightarrow y = -2x$

Parallel Lines

$2(-2x) = 1 - 4x$

$-4x = 1 - 4x$

$+4x = +4x$

$0 = 1$!?

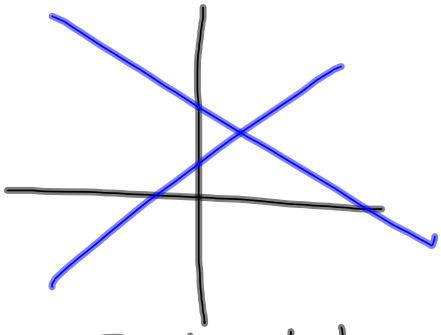
Nevah!

No Solution

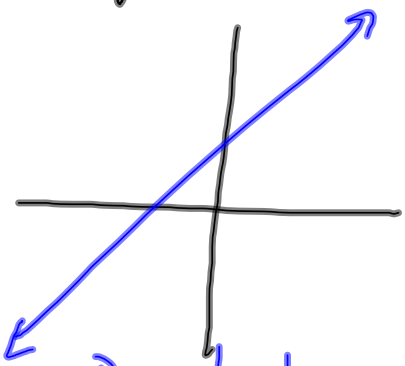
$\emptyset$

Inconsistent

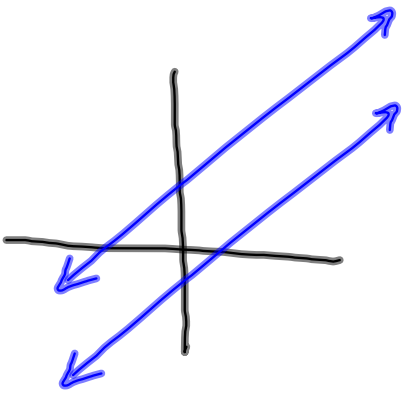
3 possibilities for a system of 2 equations in 2 variables



Independent
Unique Solution



Dependent
Infinite number of solutions (Same line)



Inconsistent
Parallel Lines
No Solution!

Done with Substitution Method for systems.

Equivalent systems have the same solution set.

Adding a multiple of one equation to the other and replacing either one with the result gives an equivalent system.

Addition Method

$y = 2x + 1 \longrightarrow -2x + y = 1$ E1

$3x - 4y = 1 \qquad\qquad 3x - 4y = 1$ E2

Left-to-Right. Top-to-bottom

$-3E1 \quad 6x - 3y = -3$

$2E2 \quad 6x - 8y = 2$

$-5y = 5$

$y = -1$

NEW SYSTEM:

$-2x + y = 1$

$\boxed{y = -1}$

$-2x + (-1) = 1$

$-2x - 1 = 1$

$-2x = 2$

$\boxed{x = -1}$

$\{(-1, -1)\}$

Addition Method for 3 variables.

(1) Eliminate x in E2 & E3

(2) .. y in E3

write the new system at each stage.

§5.2 #12

$$x + y - z = 0 \quad E1$$
$$3x - y + 2z = 14 \quad E2$$
$$2x - y + 3z = 18 \quad E3$$

Replace E2 by $-3E1 + E2$

$$-3E1 \quad -3x - 3y + 3z = 0$$
$$E2 \quad 3x - y + 2z = 14$$
$$-3E1 + E2 \quad -4y + 5z = 14$$

$$x + y - z = 0 \quad E1$$
$$-4y + 5z = 14 \quad E2$$
$$-3y + 5z = 18 \quad E3$$

Replace E3 by $-2E1 + E3$

$$-2E1 \quad -2x - 2y + 2z = 0$$
$$E3 \quad 2x - y + 3z = 18$$
$$-2E1 + E3 \quad -3y + 5z = 18$$

Leanne says "kill the z's!" She's right. I'm just trying to be systematic.

$x + y - z = 0$

$-4y + 5z = 14$

$\boxed{z = 6}$

$-4y + 5(6) = 14$

$-4y + 30 = 14$

$-4y = -16$

$\boxed{y = 4}$

$-3 \quad 2z \quad 12y - 15z = -42$

$4 \quad 3 \quad -12y + 20z = 72$

$5z = 30$

$z = 6$

$x + 4 - 6 = 0$

$x - 2 = 0$

$\boxed{x = 2}$

$\boxed{\{(2, 4, 6)\}}$

Check:

$x + y - z = 0$

$3x - y + 2z = 14$

$2x - y + 3z = 18$

$2 + 4 - 6 = 0$ ✓

$3(2) - 4 + 2(6) = 14$ ✓

$2(2) - 4 + 3(6) = 18$ ✓

(27) $x + 2y - 3z = -17$ E1
$3x - 2y - z = -3$ E2

More variables than requirements!
More than 1 answer.

$x + 2y - 3z = -17$
$-8y + 8z = 48$
Can't go any further, really.

$-3E1 + E2$:
$-3E1 \quad -3x - 6y + 9z = 51$
$E2 \quad 3x - 2y - z = -3$
$-8y + 8z = 48$

$x + 2y - 3z = -17$
$y - z = -6$
Lincoln sends his love.

Brian
Here's the trick:
Back-eliminate the y.
Eliminate y in E1:
$E1: \quad x + 2y - 3z = -17$
$-2E2: \quad -2y + 2z = +12$
$x - z = -5$

New system:
$x - z = -5$
$y - z = -6$

This gives us this:
$x = z - 5$
$y = z - 6$
Sol'n Set: $\{(z-5, z-6, z) \mid z \text{ is real}\}$