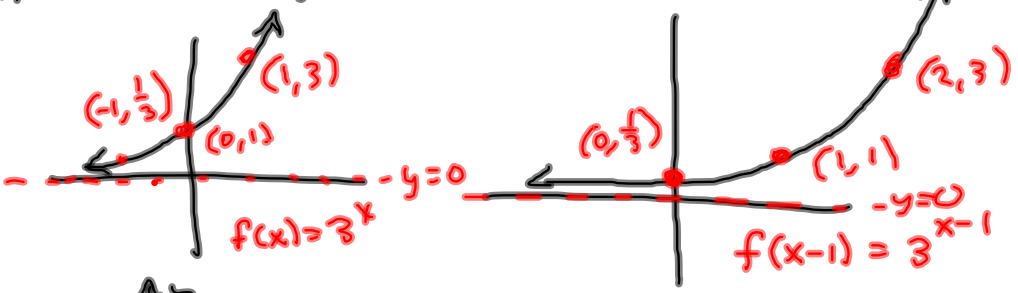
§4.1 I

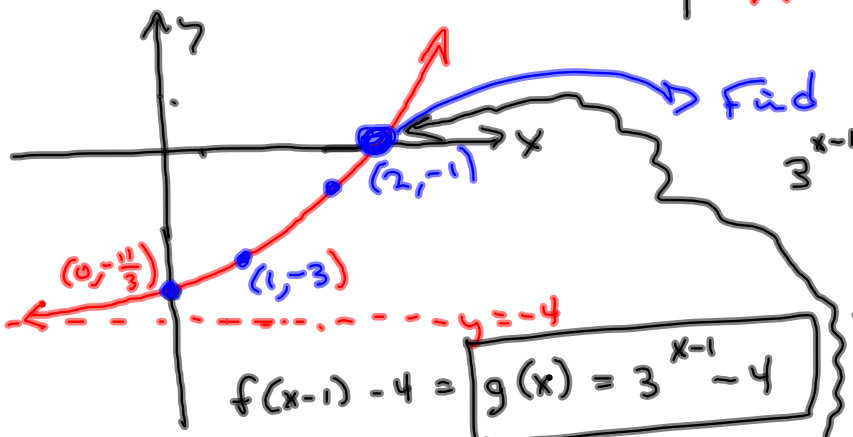
$g(x) = 3^{x-1} - 4$

$f(x) = 3^x$

$f(x-1) = 3^{x-1}$ Right 1

$f(x-1) - 4 = 3^{x-1} - 4 = g(x)$ Down 4





Find x-intercept

$3^{x-1} - 4 = 0$

$3^{x-1} = 4$

$\log_3(3^{x-1}) = \log_3(4)$

$x - 1 = \log_3(4)$

$x = \log_3(4) + 1$

$(\log_3(4) + 1, 0)$

$\log_3(4) + 1 = \frac{\ln 4}{\ln 3} + 1 \approx 2.26185951$

Ask for the intercept exactly & then ask for decimal approx.

$g(x) = 3^{1-x} - 4$

$x \longrightarrow x+1 \longrightarrow -x+1 = 1-x$

$(1-x) = -(x-1)$

$x \longrightarrow -x \longrightarrow -(x-1)$

$f(x) = 3^x$

$f(-x) = 3^{-x}$ Flip $\longleftrightarrow$

$f(-(x-1)) = 3^{-(x-1)}$ Right 1

$f(-(x-1)) = 3^{-(x-1)} - 4$ Down 4

$f(x) = 3^x$

$f(x+1) = 3^{x+1}$ Left 1

$f(-x+1) = 3^{-x+1}$ Flip $\longleftrightarrow$

$f(1-x) = 3^{1-x}$ Down 4

Same Picture.

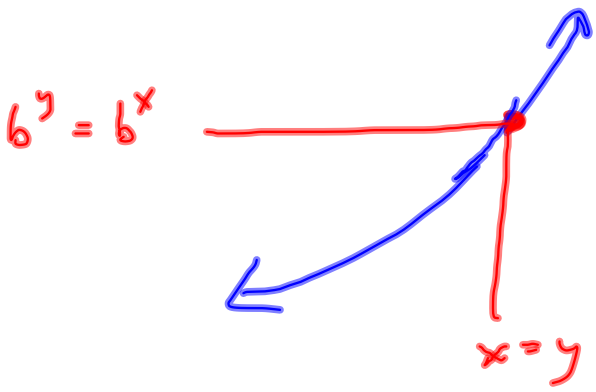
See #50 in Home, §4.1

§ 4.1 II

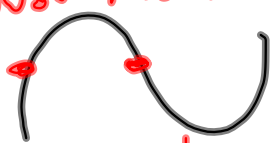
$A=P(1+\frac{r}{m})^{mt}$ Evaluating Future Value of investment

$A=Pe^{rt}$ Population Growth

$b^x = b^y \Rightarrow x=y$ 1-to-1

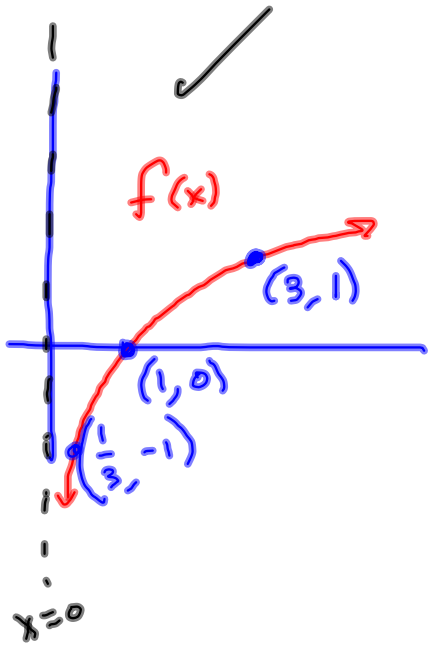


Not 1-to-1



Two spots where

$f(x) = L$

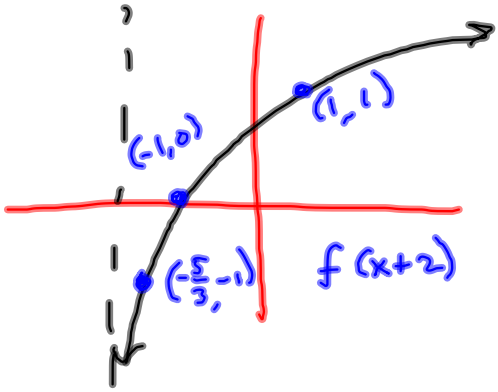


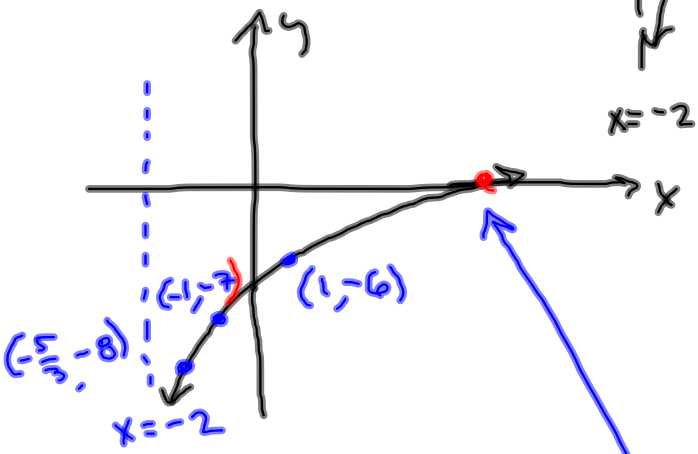
$g(x) = \log_3(x+2) - 7$

$f(x) = \log_3(x)$

$f(x+2) = \log_3(x+2)$ left 2

$f(x+2) - 7 = \log_3(x+2) - 7$ down 7





Find the x-intercept:

$\log_3(x+2) - 7 = 0$

$\log_3(x+2) = 7$

$3^{\log_3(x+2)} = 3^7$

$x+2 = 3^7$

$x = 3^7 - 2$

$(3^7 - 2, 0)$

$(2185, 0)$

Different ways we can pose these probs.

(1) $\frac{1}{2}$-life

$Pe^{-kt} = \frac{1}{2}P$

$\frac{1}{2}$-life is 5600 yrs. How old is sample that has 20% of C-14 remaining?

$Pe^{-5600k} = \frac{1}{2}P$

Find k. Use it.

(2) 13% remains

$Pe^{-kt} = .13P$

13% left after 12 hrs. How much is left after 30 hrs?

$Pe^{-12k} = .13P$

Find k. Use it.

(3) How much is left (23 grams?

$30e^{-kt} = 23$

Decayed from 30 grams to 23 grams in 100 yrs. What's the $\frac{1}{2}$-life.

$30e^{-100k} = 23$

Find k. Use it.

(1)

$Pe^{-5600K} = \frac{1}{2}P$

Find K. Use it.

$e^{-5600K} = \frac{1}{2}$

$-5600K = \ln(\frac{1}{2})$

$K = \frac{\ln(\frac{1}{2})}{-5600}$

20% remains. How old?

$Pe^{-Kt} = .2P$

Solve for t. Plug in K.

$e^{-Kt} = .2$

$-Kt = \ln(.2)$

$t = \frac{\ln(.2)}{-K}$

$= \frac{\ln(.2)}{-\frac{\ln(\frac{1}{2})}{-5600}} \approx 13,002.79732$

Common Sense Test:

0	100%
5600	50%
11200	25%
16800	12.5%

we're in here.

13% left after 12 hrs. How much is left after 30 hrs?

what %

$Pe^{-12K} = .13P$

Find K. Use it.

$e^{-12K} = .13$

$-12K = \ln(.13)$

$K = \frac{\ln(.13)}{-12}$

x = fraction remaining

$Pe^{-30K} = xP$

$e^{-30K} = x$

$x = e^{-30\left(\frac{\ln(.13)}{-12}\right)}$

$x \approx .006093382$

$\approx$ 0.6% left.

(3)

Decayed from 30 grams to 23 grams in 100 yrs.

What's the ½-life.

$30e^{-100K} = 23$

Find K. Use it.

Pe^{-Kt}

$30e^{-100K} = 23$

$e^{-100K} = \frac{23}{30}$

$-100K = \ln\left(\frac{23}{30}\right)$

$K = \frac{\ln(23/30)}{-100}$

½-life

$Pe^{-Kt} = \frac{1}{2}P$

$e^{-Kt} = \frac{1}{2}$

$-Kt = \ln\left(\frac{1}{2}\right)$

$t = \frac{\ln(\frac{1}{2})}{-K} = \frac{\ln(\frac{1}{2})}{\ln(23/30)}(-100)$

Extract time from interest Rate & Future Value.

$2000 appreciates to $5000 @ APR 7% compounded monthly. How long did it take?

$A = 5000$
$P = 2000$
$r = .07$
$m = 12$

$A = P(1+\frac{r}{m})^{mt}$ Solve for t.

$\frac{A}{P} = \left(1+\frac{r}{m}\right)^{mt}$

$\ln\left(\frac{A}{P}\right) = \ln\left(\left(1+\frac{r}{m}\right)^{mt}\right)$

$\frac{\ln(\frac{A}{P})}{m\ln(1+\frac{r}{m})} = \frac{mt\ln(1+\frac{r}{m})}{m\ln(1+\frac{r}{m})}$

Plug in.

$\frac{\ln(\frac{A}{P})}{m\ln(1+\frac{r}{m})} = t$