$$a^x a^y = a^{x+y} \; *$$

$$(a^x)^y = a^{xy}$$

$$a^{-x} = \frac{1}{a^x} \; *$$

$$\frac{a^x}{a^y} = a^{x-y} \; *$$

$$\log(xy) = \log(x) + \log(y)$$

$$\ln(a^x) = x \cdot \ln(a)$$

$$\ln(a^{-x}) = -x \cdot \ln(a)$$

$$\ln\left(\frac{x}{y}\right) = \ln(x) - \ln(y)$$

$$b^{\log_b(x)} = x$$

$$\log_b(b^x) = x$$

$$\log_b(x) = \frac{\ln(x)}{\ln(b)}$$

$$\log_3(27) = \frac{\ln(27)}{\ln(3)} = 3$$

$0^x$

$\log_x(16) = 4$

$x^{\log_x(16)} = x^4$

$16 = x^4$

$16^{\frac{1}{4}} = (x^4)^{\frac{1}{4}}$

$2 = |x|$

But we know already that $x > 0$

$\boxed{x = 2}$

$-\frac{5}{2} = (-5)(\frac{1}{2}) =$

$(5)(-\frac{1}{2}) = (\frac{1}{2})(-5)$

$\log_4(x) = -\frac{5}{2}$

$4^{\log_4(x)} = 4^{-\frac{5}{2}}$

$x = (4^{\frac{1}{2}})^{-5}$

$= 2^{-5}$

$= \frac{1}{2^5} = \boxed{\frac{1}{32} = x}$

$\log(x) + \log(y) = \log(xy)$

$\log_6(w-1) + \log_6(w-2) = 1$

$\log_6\left((w-1)(w-2)\right) = 1$

$6^{\log_6\left((w-1)(w-2)\right)} = 6^1$

$(w-1)(w-2) = 6$

$w^2 - 2w - 1w + 2 = 6$

$w^2 - 3w - 4 = 0$

$w^2 - 4w + 1w - 4 = 0$

$w(w-4) + 1(w-4) = 0$

$(w-4)(w+1) = 0$

$\boxed{w = 4}$ OR $w = -1$ Ditch it.

$\notin \mathcal{D}$

$\boxed{w \in \{4\}}$ Final ANS

$\log_6(3) + \log_6(2)$
$= \log_6(2 \cdot 3)$
$= \log_6(6)$
$= \log_6(6^1)$
$= 1$

$\mathcal{D} = \{w \mid w-1>0$ and $w-2>0\}$
$= \{w \mid w>1$ and $w>2\}$
$= \{w \mid w>2\}$

$$6^{x+y} = 6^x 6^y$$

$$6^{\log_6(w-1) + \log_6(w-2)} = 6^1$$

$$6^{\log_6(w-1)}\, 6^{\log_6(w-2)} = 6$$

$$(w-1)(w-2) = 6, \text{etc.}$$

$$\log_3(x-6) - \log_3(2x) = 4$$

$$\log_3\left(\frac{(x-6)}{2x}\right) = 4$$

$$3^{\log_3\left(\frac{x-6}{2x}\right)} = 3^4$$

$$\frac{x-6}{2x} = 81$$

$$x-6 = 81(2x) = 162x$$

$$-161x = 6$$

$$x = -\frac{6}{161} \notin D$$

$$\boxed{\varnothing}$$

$$\log_3(x) + \log_3\left(\frac{1}{x}\right) = 0$$

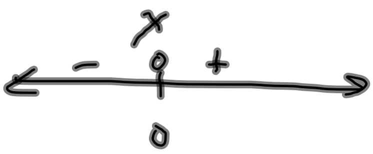
$$\log_3\left(x \cdot \frac{1}{x}\right) = 0$$

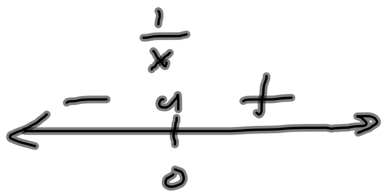
$$\log_3(1) = 0$$

$$\log_3(3^0) = 0$$

$$0 = 0$$

True for all $x \in \mathcal{D} = \{x \mid x > 0 \text{ \& } \frac{1}{x} > 0\} = \boxed{(0, \infty)}$



Pay me or pay me later

Find answer to 4 decimal places.

$$5^{3x} = 29$$

$$\log_5(5^{3x}) = \log_5(29)$$

$$3x = \log_5(29)$$

$$x = \frac{\log_5(29)}{3} = \frac{1}{3}\log_5(29) = \frac{1}{3}\frac{\ln(29)}{\ln(5)}$$

$$\approx .6974061781$$

$$\boxed{x \approx .6974}$$

```
200e^(-.001*50)
      190.2458849
ln(27)/ln(3)
                3
1/3*ln(29)/ln(5)
      .6974061781
```

$$5^{3x} = 29$$

$$\ln(5^{3x}) = \ln(29)$$

$$3x \ln(5) = \ln(29)$$

$$x = \frac{1}{3} \cdot \frac{\ln(29)}{\ln(5)}$$

Same as before, withOUT change-of-base in the last step.

42 $2^x = 3^{x-1}$

$$\ln(2^x) = \ln(3^{x-1})$$

$$x\ln(2) = (x-1)\ln(3) = x\ln(3) - 1\ln(3)$$

$$x\ln(2) - x\ln(3) = -\ln(3)$$

$$x\left(\ln(2) - \ln(3)\right) = -\ln(3)$$

$$x = \frac{-\ln(3)}{\ln(2) - \ln(3)} = \frac{-\ln(3)}{\ln\left(\frac{2}{3}\right)}$$

42 $2^x = 3^{x-1}$

$\ln(2^x) = \ln(3^{x-1})$

$x \ln(2) = (x-1)\ln(3)$

Let $a = \ln(2)$, $b = \ln(3)$

$xa = (x-1)b$

$ax = b(x-1) = bx - b$

$ax = bx - b$

$ax - bx = -b$

$(a-b)x = -b$

$x = \dfrac{-b}{a-b} = \dfrac{b}{b-a} = \dfrac{\ln(3)}{\ln(3)-\ln(2)}$

$\boxed{x \approx 2.7095}$

2.7095112 91

Sometimes, substitution makes the manipulation more sleek.

$\frac{1}{2}$-life of Millsium is 10,000 years.

What's its decay rate?

Book $A = A_0 e^{-kt}$ $\boxed{A = Pe^{-kt}}$ Mills

$$\boxed{A = Pe^{-k \cdot 10000} = \frac{1}{2}P}$$

$$e^{-10000k} = \frac{1}{2}$$

$$\ln\left(e^{-10000k}\right) = \ln\left(\tfrac{1}{2}\right)$$

$$-10000k = \ln\left(\tfrac{1}{2}\right)$$

$$\boxed{k = \frac{\ln\left(\frac{1}{2}\right)}{-10000}}$$

$$= \frac{\ln(2^{-1})}{-10000}$$

$$= \frac{-\ln(2)}{-10000}$$

$$= \frac{\ln(2)}{10000}$$

$10000k = \ln(2)$

$\approx \boxed{6.931471806 \times 10^{-5}}$

$= \boxed{.00006931471806}$ is decay rate

$= .006931471806\ \%$

Ashes from fire pit contain 15% of naturally occurring C-14. If $\frac{1}{2}$-life of C-14 is 5730 yrs, how old is the fire pit?

① Build Model $A = Pe^{-kt}$

② Use Model $15\% \Longrightarrow A = .15P$

① $A = Pe^{-kt}$

$$Pe^{-5730k} = \frac{1}{2}P$$

$$e^{-5730k} = \frac{1}{2}$$

$$-5730k = \ln\left(\frac{1}{2}\right)$$

$$k = \frac{\ln\left(\frac{1}{2}\right)}{-5730}$$

② $Pe^{-kt} = .15P$

$$e^{-kt} = .15$$

$$-kt = \ln(.15)$$

$$t = \frac{\ln(.15)}{-k}$$

$$t = \frac{\ln(.15)}{-\left(\frac{\ln\left(\frac{1}{2}\right)}{-5730}\right)}$$

$$= \frac{\ln(.15)}{\ln\left(\frac{1}{2}\right)} \cdot 5730$$

$$\approx 15{,}682.81285$$

$$\approx \boxed{15{,}683 \text{ yrs ago}}$$

§4.4 #s 1, 7, 15, 17, 19, 21, 26, 30, 35, 39,
41, 51*, 53, 65, 67, 69, 75

#51 The square is NOT inside the log on the LHS.

$\log(x^y) = y\log(x)$ doesn't apply to $(\log(x))^y$