§ 4.1 #79,

4.2 #97, 63

$2 \cdot 2^{2x} = 4^x + 64$

2^x

$4^x = (2^2)^x = 2^{2x}$

$2 \cdot 2^{2x} = 2^{2x} + 64$

$2 \cdot 2^{2x} - 2^{2x} = 64$

$2^{2x}(2-1) = 64$

$2^{2x} = 64 = 2^6$

$2x = 6$

$x = 3$

2 | 64
2 | 32
2 | 16
2 | 8
2 | 4
2

4^x

$2^{2x} = (2^2)^x = 4^x$

$2 \cdot 4^x = 4^x + 64$

$2 \cdot 4^x - 4^x = 64$

$4^x = 4^3$

$\boxed{x = 3}$

§4.2 #63

$\log_{10}(1000) = z$

$10^{\log_{10}(1000)} = 10^z$

$1000 = 10^z$

$\boxed{10^z = 1000}$

$\log_{10}(1000) = \log(10^3) = 3$

$\boxed{(f^{-1} \circ f)(x) = x}$

1

$3^x = 11$

$\boxed{\log_3(3^x)} = \log_3(11)$

$x = \log_3(11)$

Tell me what power of 3 3^x is.

x

#97 §4.2

$\ln 3 = \ln 3$ ✓

$\ln(x-3) = \ln(2x-9)$

$e^{\ln(x-3)} = e^{\ln(2x-9)}$

$x-3 = 2x-9$

$-2x+3 = -2x+3$

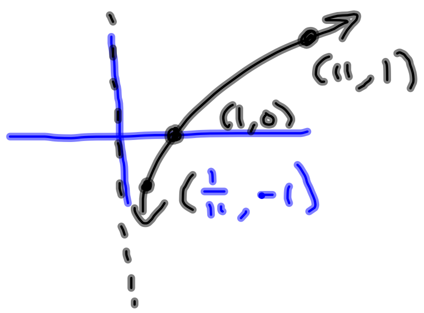
$-x = -6$

$\boxed{x=6}$

$\ln(A) = \ln(B)$
iff
$A = B$

Raise e to the power of both sides to extract x from inside the logarithm.

$$\lim_{x \to 0^+} \log_{11}(x) = -\infty$$



$\log_{11}(11) = 1$

$\frac{1}{11} = 11^{-1}$

$\log_{11}\left(\frac{1}{11}\right) = -1$

Finding Inverses.

$f(x) = 3 \cdot 7^{2x-1} + 5$ Find $f^{-1}(x)$

$x = 3 \cdot 7^{2y-1} + 5$ Solve for y

$\cancel{3 \cdot 7^{2y-1} = 21^{2y-1}}$

$3 \cdot 7^{2y-1} + 5 = x$

$3 \cdot 7^{2y-1} = x - 5$

$7^{2y-1} = \frac{x-5}{3}$

$\log_7\left(7^{2y-1}\right) = \log_7\left(\frac{x-5}{3}\right)$

$2y - 1 = \log_7\left(\frac{x-5}{3}\right)$

$\boxed{\text{See \#87}}$

$2y = \log_7\left(\frac{x-5}{3}\right) + 1$

$y = \boxed{\frac{1}{2}\log_7\left(\frac{x-5}{3}\right) + \frac{1}{2} = f^{-1}(x)}$

x inside the log

#89

$\log_2(x) = 16$

$\boxed{x = 2^{16}}$ → This is OK, but...

$2^{\log_2(x)} = 2^{16}$ ← My thought process

$\boxed{x = 2^{16}}$

x is the base!

$\log_x(16) = 4$

$x^{\log_x(16)} = x^4$

$16 = x^4$

$16^{\frac{1}{4}} = \left(x^4\right)^{\frac{1}{4}}$

$\left(2^4\right)^{\frac{1}{4}} = 2^1 = 2 = x$

$2 \,|\, 16$
$2 \,|\, 8$
$2 \,|\, 4$
2

Assumption, here, is $\boxed{x > 0}$

§4.2 #121 assist.

continuous

cnt$\underline{s}$

want \$10 to grow to \$20 in time t (unknown), if $r = 7\%$ and interest is compounded cnt$\underline{s}$ly.

$A = Pe^{rt}$

$20 = 10e^{.07t}$

solve for t.

$\frac{20}{10} = e^{.07t}$

This is your standard doubling time question.

$e^{.07t} = 2$

$Pe^{rt} = 2P$

$Pe^{.07t} = 2P$

$e^{.07t} = 2$

$\vdots$

$t = \frac{\ln(2)}{.07}$

$\ln(e^{.07t}) = \ln(2)$

$.07t = \ln(2)$

$t = \frac{\ln(2)}{.07}$

≈ 9.9 yrs

$\frac{\ln(2)}{.05} \approx \frac{.69}{.05} \approx \frac{69}{5}$

$\frac{1}{2}$ - life is similar.

What's the $\frac{1}{2}$ life of a radioactive substance that decays at a rate of 2.3% per year?

$$A = Pe^{-kt} = Pe^{-.023t} = \frac{1}{2}P$$

Highly Radioactive.

$$e^{-.023t} = \frac{1}{2}$$

$$\ln\left(e^{-.023t}\right) = \ln\left(\tfrac{1}{2}\right)$$

$$-.023t = \ln\left(\tfrac{1}{2}\right)$$

$$\boxed{t = \frac{\ln\left(\frac{1}{2}\right)}{-.023}} \approx 30\text{yrs.}$$

§4.3 Rules of logarithms

$\log_b x$ & b^x are inverse functions.

For every property of exponents there's an evil twin property for logarithms.

PRODUCT RULE

1

Recall

$x^3 x^6 = x^{3+6} = x^9$

$x^M x^N = x^{M+N}$

NEW

$\log_b(MN) = \log_b(M) + \log_b(N)$

$\log_4(8) + \log_4(2)$

$= \log_4(8 \cdot 2) = \log_4(16) =$

$\log_4(4^2) = 2$

Quotient Rule

$$\frac{x^6}{x^2} = x^{6-2} = x^4$$

New

$$\log_b\left(\frac{M}{N}\right) = \log_b(M) - \log_b(N)$$

$$\log_2\left(\frac{32}{2}\right) = \log_2(32) - \log_2(2)$$

$$5 - 1 = 4$$

$$\log_2(16) = \log_2(2^4) = 4$$

Power Rule

$(x^M)^N = x^{MN}$

$(x^2)^5 = x^{10}$

$\log_b(M^N)$

$= N\log_b(M)$

$\log_2(2^3) = 3$

$3\log_2(2^1) = 3\cdot 1 = 3$

$\log_2(2^3) = \log_2(2\cdot 2\cdot 2)$

$= \log_2(2) + \log_2(2) + \log_2(2)$

$= 1 + 1 + 1 = 3$

Calculator only does $\log(x) = \log_{10}(x)$ and $\ln(x) = \log_e(x)$. Can't do $\log_3(x)$ or $\log_7(x)$, directly.

Let $x = \log_b(M)$

Then $b^x = b^{\log_b(M)}$

$b^x = M$

$\log_c(b^x) = \log_c(M)$

$x \log_c(b) = \log_c(M)$

$x = \dfrac{\log_c(M)}{\log_c(b)}$

$$\boxed{\log_b(M) = \frac{\log_c(M)}{\log_c(b)}}$$

Change of Base FORMULA.

$\log_7(13) = \dfrac{\ln(13)}{\ln(7)} = \dfrac{\log_{10}(13)}{\log_{10}(7)}$

≈ 1.318123223

SI's from 7:30-8:30 BH 106 Thursday Only

§ 4.3 #s 5, 9, 13, 17, 21, 25, 29, 31, 33, 35, 39-63 odds, 67, 71, 75, 77, 81, 85, 96.