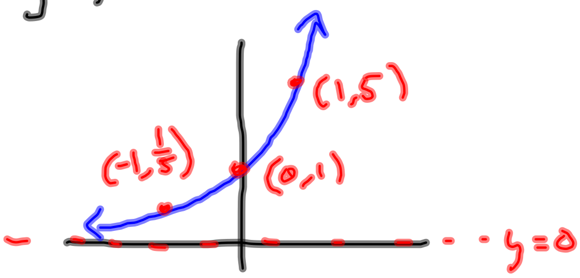
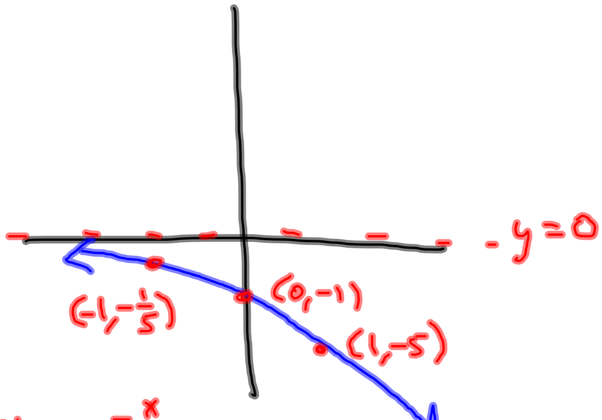
like 4.1 #55

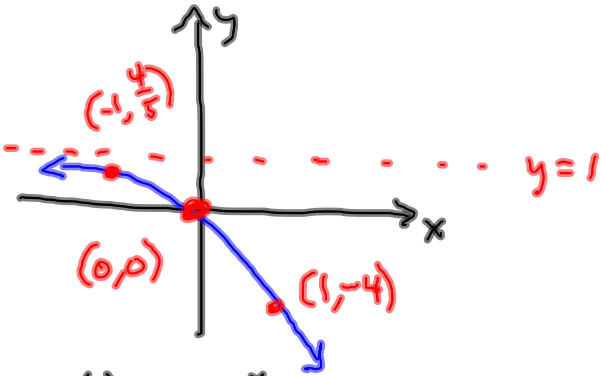
$g(x) = -5^x + 1$




$f(x) = 5^x$




$-f(x) = -5^x$




$g(x) = 1 - 5^x$

$= -5^x + 1$

$= -f(x) + 1$

Recall principles

A = Future Value

P = Principal = Present Value

r = Interest Rate

m = the # of periods per year (compounding)

t = time, in years.

$$A = P\left(1 + \frac{r}{m}\right)^{mt}$$

Recall – For enrichment

$$e = \lim_{n\to\infty}\left(1 + \frac{1}{n}\right)^{n}$$

n = 50 :

n = 1000 :

```
e^(1)
          2.718281828
```

| X | Y1 |
|---|---|
| 50 | 2.6916 |
| 1000 | 2.7169 |
| 1E8 | 2.7183 |
| 6 | 2.5216 |
| 9 | 2.5812 |
| 12 | 2.613 |
| 10 | 2.5937 |

X=6

$$\underbrace{\text{As } m \to \infty,\ P\left(1+\frac{r}{m}\right)^{mt} \longrightarrow Pe^{rt}}_{\text{Continuous Compounding}}$$

Very close to compounding daily.

→ m = 360, sometimes.

$$\left(1+\frac{r}{m}\right)^{mt}$$

$$= \left(1+\frac{1}{\frac{m}{r}}\right)^{\frac{m}{r}\cdot rt}$$

$m \to \infty$, then $\frac{m}{r} \to \infty$

$$= \left(\left(1+\frac{1}{\frac{m}{r}}\right)^{\frac{m}{r}}\right)^{rt}$$

$$= \left(\left(1+\frac{1}{\square}\right)^{\square}\right)^{rt}$$

$$\xrightarrow{\square \to \infty} e^{rt}$$

$$\left(1+\frac{1}{n}\right)^{n}$$

$$a^{bc} = \left(a^{b}\right)^{c}$$

So, $P\left(1+\frac{r}{m}\right)^{mt}$ becomes $Pe^{rt}$ when compounding takes place continuously.

what's the value, after 5 years of \$1000, if APR = .08, and compounded...

... Monthly $A = P\left(1+\frac{r}{m}\right)^{mt}$

$= 1000\left(1+\frac{.08}{12}\right)^{12(5)} \approx \$1489.84570 8$

$\approx \boxed{\$1489.85}$

```
     1.489845708
1000(1+.08/12)^(
12*5)
     1489.845708
1000*(1+.08/12)^
(12*5)
     1489.845708
```

... Weekly $A = 1000\left(1+\frac{.08}{52}\right)^{(52)(5)} \approx \$1491.37$

... Daily $A = 1000\left(1+\frac{.08}{365}\right)^{(365)(5)} \approx \$1491.76$

close

... Continuously $A = 1000e^{(.08)(5)} \approx \$1491.82$

```
1000*(1+.08/52)^
(52*5)
     1491.366215
e^(.08*5)
     1.491824698
Ans*1000
     1491.824698
```

§4.1 II #s 63, 64, 65, 69, 79, 85, 89, 103, 105, 113

Finish §4.1 : Radioactive Decay

Exponential Decay.

$A(t) = Pe^{-kt}$

Carbon-14 Dating.

C-14 decays over time.

The amt of C-14 present (remaining) tells us how long something's been dead.

$k$ = decay rate (The coefficient of $t$ in the exponent is the rate of growth/decay).

Suppose there were 200 grams of a substance with a decay rate of .1%, at the start. How much is left after 50 years?

$$A(t) = 200e^{(-.001)(50)} \approx 190.2458849$$

$$\approx \boxed{190\ g}$$

A scientist knows how much C-14 was in the atmosphere in the past. She measures how much is in the sample. From this, she finds the age.

Solve $A = Pe^{-kt}$ for $t$.

We'll learn this, soon.

§4.2 Logarithmic Functions & Applications.

$b > 0,\ b \neq 1$

Logarithmic Function, base $b$ is

$$f(x) = \log_b(x)$$

$y = \log_b x$ means $b^y = x$

$4 = \boxed{\log_3 81}$ because $3^4 = 81$

→ Says "Write 81 as a power of 3. Report the power."

$$\log_3(81) = \underbrace{\log_3(3^4)} = 4$$

See? It's like $3^x$ & $\log_3 x$ are inverse functions.

$$\log_{\frac{1}{2}} 8 = \log_{\frac{1}{2}}(2^3) = \log_{\frac{1}{2}}\left(\left(2^{(-1)(-1)}\right)^3\right)$$

$$= \log_{\frac{1}{2}}\left(\left(2^{-1}\right)^{(-1)(3)}\right) = \log_{\frac{1}{2}}\left(\left(\tfrac{1}{2}\right)^{-3}\right) = -3$$

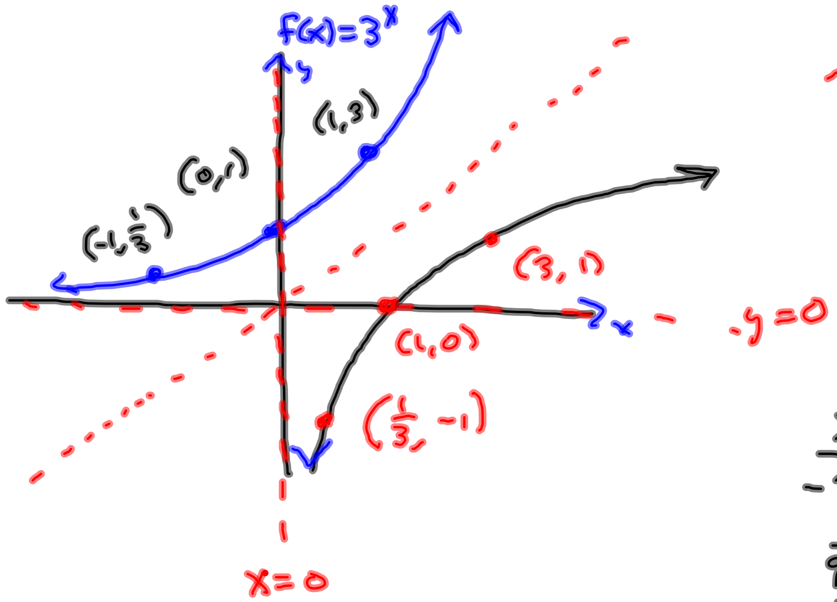
$$= \log_{\frac{1}{2}}\left(\left(\left(\tfrac{1}{2}\right)^{-1}\right)^3\right) = \log_{\frac{1}{2}}\left(\left(\tfrac{1}{2}\right)^{-3}\right) =$$

Common Log $\log_{10} x = \log x$

Natural Log $\log_e x = \ln x$

Calculator Keys.

Calculators don't have $\log_3 x$ or $\log_5 x$ . . .




$\log_3 x$

$\log_3 x$

| x | y |
|---|---|
| -2 | Not in Domain silly!.. |
| $\frac{1}{9}$ | -2 |
| $\frac{1}{3}$ | -1 |
| 1 | 0 |
| 3 | 1 |

$\log_3(-2) = ?$

$-2 = 3^{?}$

$\log_3(0) =$

$0 = 3^{?}$

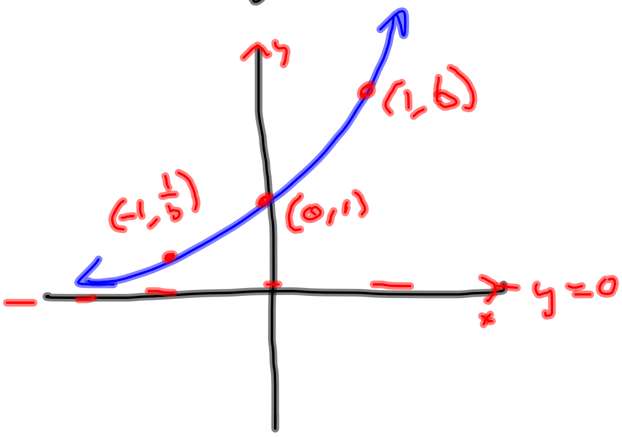
$\log_3 1 = ?$

$1 = 3^{?}$

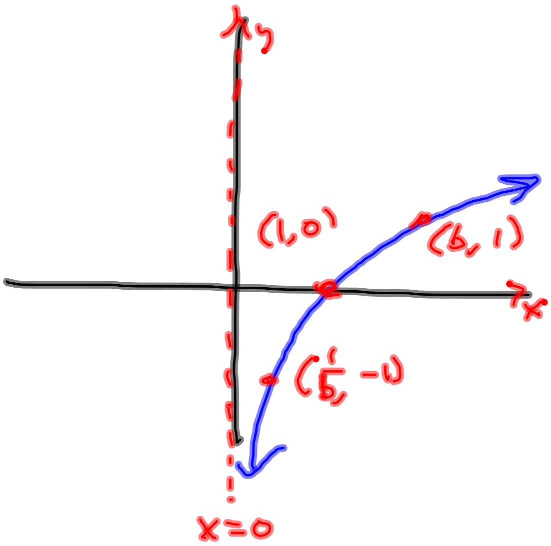
$1 = 3^0$

$f(x) = b^x$

$b > 1$




$g(x) = \log_b x$, $b > 1$




Increasing: $(-\infty, \infty)$

$\mathcal{D} = (-\infty, \infty)$

$\mathcal{R} = (0, \infty)$

H.A.: $y = 0$

Increasing: $(0, \infty)$

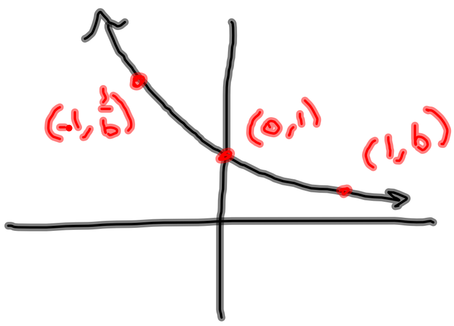
$\mathcal{D} = (0, \infty)$

$\mathcal{R} = (-\infty, \infty)$

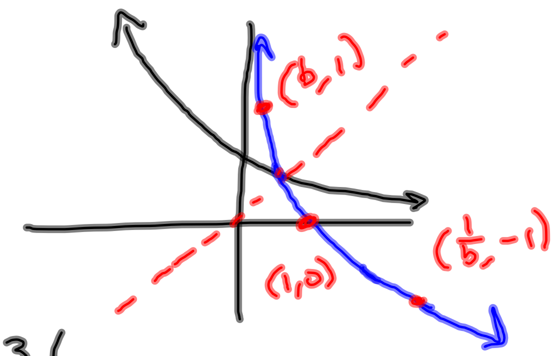
V.A.: $x = 0$

Seed: finding $\mathcal{R}$ does amount to finding $\mathcal{D}$ for the inverse!

$b^x \quad 0<b<1$




$\log_b x \quad 0<b<1$




§4.2 #s 9, 13, 17, 21, 25, 29, 31,
34, 38, 47, 49, 51, 59, 63, 67,
69, 73, 79, 83, 87, 89, 93,
97, 99, 101, 107, 113, 121, 127, 129

$\log_{\frac{1}{3}} x$

$\log_{.5} x$