$$R(x) = \frac{(x+2)(x^2-1)}{(x+1)(x-3)} = \frac{(x+2)(x-1)\cancel{(x+1)}}{\cancel{(x+1)}(x-3)} = \frac{(x+2)(x-1)}{x-3} = R^*(x)$$

$x \neq -1$

$\mathcal{D} = \{x \mid x \neq -1 \text{ \& } x \neq 3\}$

$\boxed{\text{V.A.: } x = 3}$

HOLE: $x = -1$

$R^*(-1) = \frac{(-1+2)(-1-1)}{-1-3} = \frac{-2}{-4} = \frac{1}{2}$

$\boxed{\text{HOLE } (-1, \tfrac{1}{2})}$

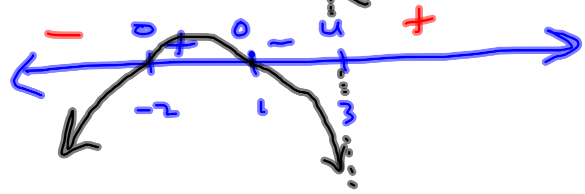
$R^*(0) = \frac{(0+2)(0-1)}{0-3} = \frac{-2}{-3} = \frac{2}{3}$

$\boxed{(0, \tfrac{2}{3}) = \text{y-int}}$

$\boxed{\text{x-int: } (-2,0), (1,0)}$

critical: $x = 3, -2, 1$

$m = 1$ for all



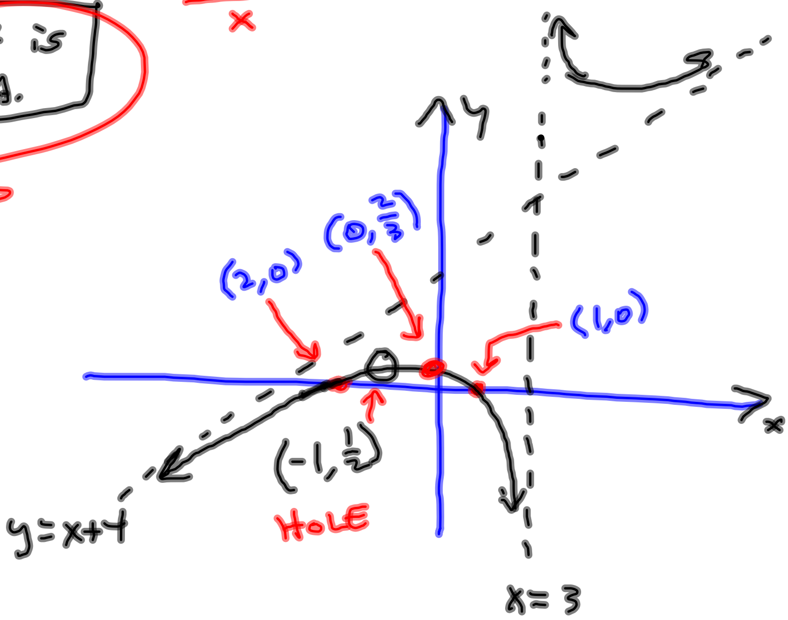
$\boxed{\text{H.A.: None}}$ $\frac{(x+2)(x-1)}{x-3} = \frac{x^2+x-2}{x-3}$

O.A.:

$$\begin{array}{r|rrr} 3 & 1 & 1 & -3 \\ & & 3 & 12 \\ \hline & 1 & 4 & 9 \end{array}$$

$\boxed{y = x+4 \text{ is O.A.}}$

$\frac{(x)(x)}{x} = x$ as $x \to \pm\infty$



$x^5 \leftarrow$ Exponent

$\uparrow$ Base

New! Exponential Functions.

$f(x) = b^x$ $\quad b \neq 1$, $b > 0$

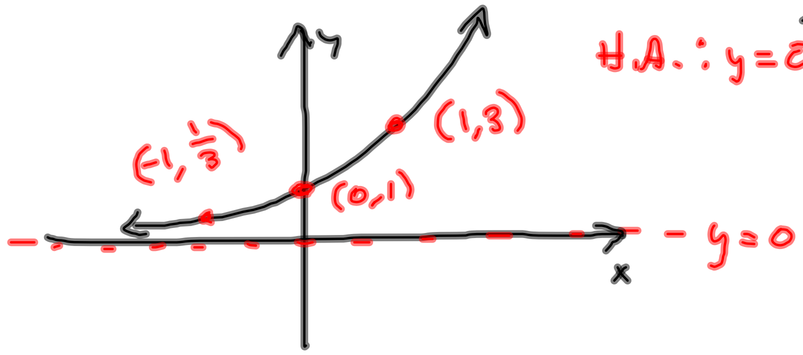
$(-3)^x$ No way

$1^x = 1$ Boring!

$f(x) = 3^x$

$D = (-\infty, \infty)$, $R = (0, \infty)$

H.A.: $y = 0$

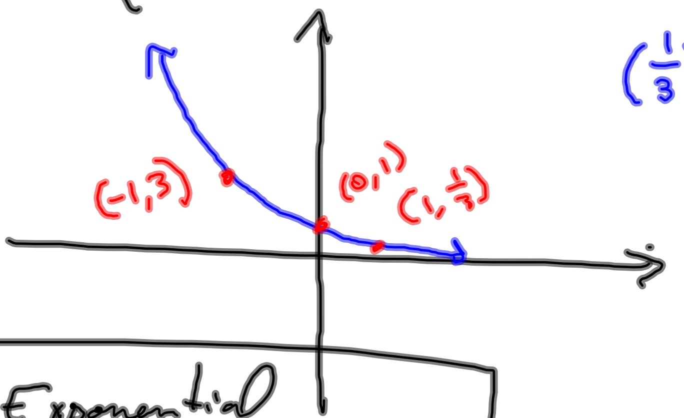


Exponential Growth.

$b > 1$

$b = 3$ in this one.

$\left(\frac{1}{3}\right)^x$



$\frac{1}{3} = 3^{-1}$

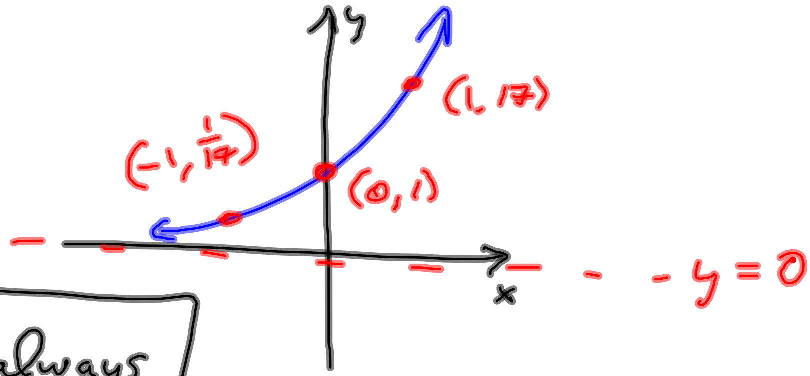
$\left(\frac{1}{3}\right)^x = \left(3^{-1}\right)^x = 3^{-x}$

mirror image of $f(x) = 3^x$!

$\boxed{\text{Exponential Decay} \quad 0 < b < 1 \quad f(x) = b^x}$

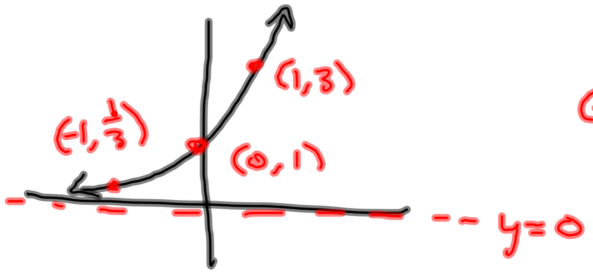
$\left(\frac{1}{3}\right)^{-1} = \frac{1^{-1}}{3^{-1}} = \frac{1}{\frac{1}{3}} = 1 \cdot \frac{3}{1} = 3$

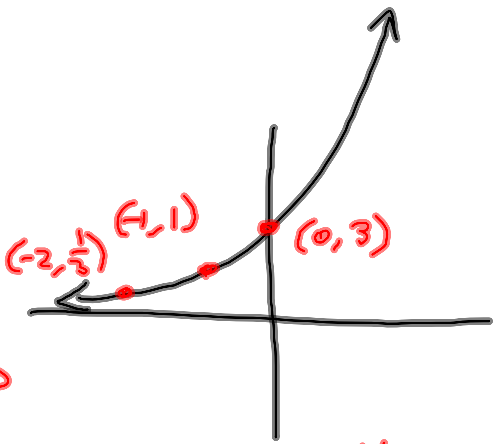
17^x



$\boxed{b^x > 0 \text{ always}}$

$3^{x+1} - 2$

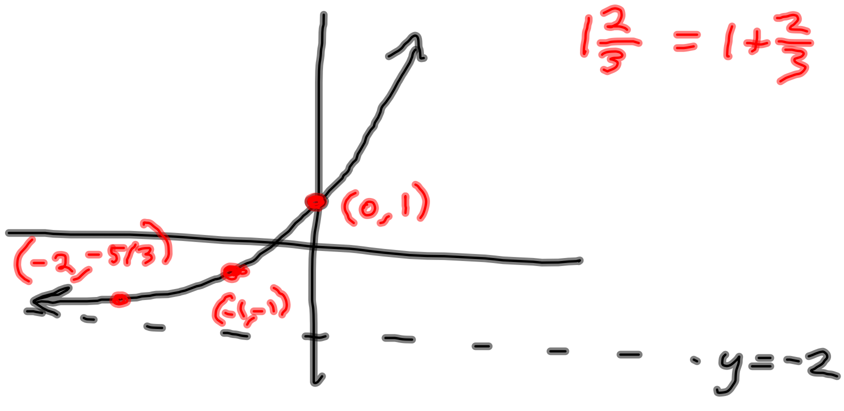




$f(x) = 3^x \xrightarrow{\text{Left 1}} f(x+1) = 3^{x+1}$

$f(x+1) - 2$
Down 2

$1\frac{2}{3} = 1 + \frac{2}{3}$



$\frac{1}{3} - 2$

$= \frac{1}{3} - \frac{6}{3} = -\frac{5}{3}$

See last pg of text for properties of exponents

Exponential Equations.

$A^C = A^B \Longrightarrow C = B$

$f(C) = f(B) \Longrightarrow C = B$

Exponential Function is 1-to-1.

$2^{3x+2} = 2^{5x} \Longrightarrow 3x+2 = 5x$

$-2x = -2$

$\boxed{x=1}$

$3^{-x} = 9$

$3^{-x} = 3^2$

$-x = 2$

$\boxed{x=-2}$

$\left(\frac{4}{9}\right)^x \cdot \left(\frac{8}{27}\right)^{1-x} = \frac{2}{3}$

$\frac{4}{9} = \frac{2^2}{3^2} = \left(\frac{2}{3}\right)^2$

$(a^b)^c = a^{bc}$

$\frac{8}{27} = \frac{2^3}{3^3} = \left(\frac{2}{3}\right)^3$

$3(1-x) = 3-3x$

$\left(\left(\frac{2}{3}\right)^2\right)^x \left(\left(\frac{2}{3}\right)^3\right)^{1-x} = \frac{2}{3}$

$a^r \cdot a^s = a^{r+s}$

$\left(\frac{2}{3}\right)^{2x} \left(\frac{2}{3}\right)^{3-3x} = \frac{2}{3}$

$\left(\frac{2}{3}\right)^{2x+3-3x} = \frac{2}{3}$

$\left(\frac{2}{3}\right)^{3-x} = \left(\frac{2}{3}\right)^1$

$3-x = 1$

$-x = -2$

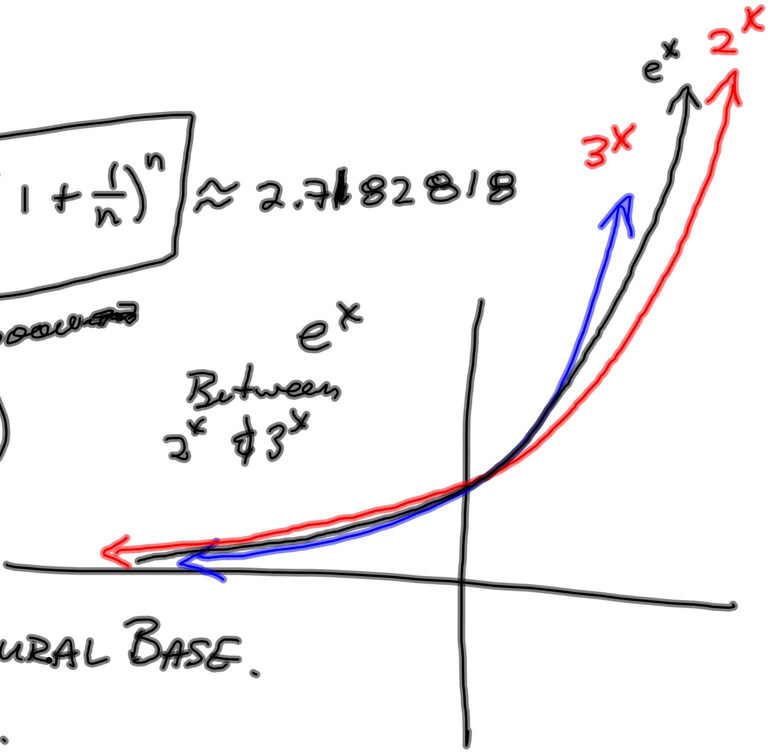
$\boxed{x=2}$

e Euler

$$\boxed{e = \lim_{n\to\infty}\left(1+\frac{1}{n}\right)^n} \approx 2.7182818$$

$$\left(1+\frac{1}{1000000}\right)^{1000000}$$

e^x
Between
2^x & 3^x



e is the NATURAL BASE.

Slope = Height.

It's its own derivative.

Compound Interest

\$100 invested @ 6% APR compounded monthly.

Month	Balance
0	100
1	$100 + \frac{.06}{12}(100)$
2	$100 + \frac{.06}{12}(100) + \left(100 + \frac{.06}{12}(100)\right)\frac{.06}{12}$

$$= 100\left(1 + \frac{.06}{12}\right) + 100\left(1 + \frac{.06}{12}\right)\left(\frac{.06}{12}\right)$$

Factor out $100(1+\frac{.06}{12})$:

$$= 100\left(1 + \frac{.06}{12}\right)\left[1 + \frac{.06}{12}\right]$$

$$= 100\left(1 + \frac{.06}{12}\right)^2$$

⋮

36 months: $100\left(1 + \frac{.06}{12}\right)^{36}$

36 months is 3yrs times 12 periods per year.

$$100\left(1 + \frac{.06}{12}\right)^{12 \cdot 3}$$

$$\boxed{A = P\left(1 + \frac{r}{m}\right)^{mt}}$$

P = Principal
r = interest rate
m = periods per year
t = time, in years

§4.1 I #s 9-20, 21, 24, 27, 30,
39, 40, 41, 42, 47, 50, 51, 55

Next time:

Continuous Compounding

Radioactive Decay

$$P\left(1+\frac{r}{m}\right)^{mt} \xrightarrow{m \to \infty} Pe^{rt}$$