Rough Sketches

$f(x) = (x-3)^2(x+2)(x-1)$

$\lim_{x\to\infty} f(x) = \lim_{x\to\infty} (x)^2(x)(x) = \lim_{x\to\infty} x^4 = +\infty$

zeros: $x = -2, 1, 3$

$m = 1, 1, 2$

c, c, t

'c' means "cross."

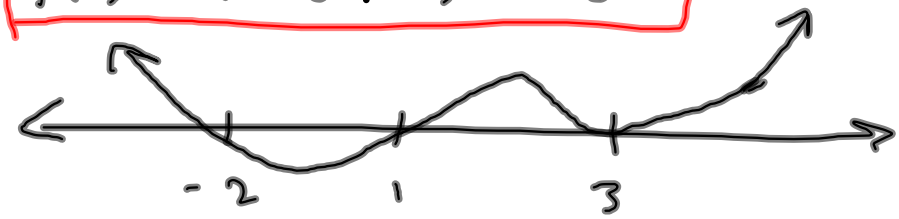
't' .. "touch."

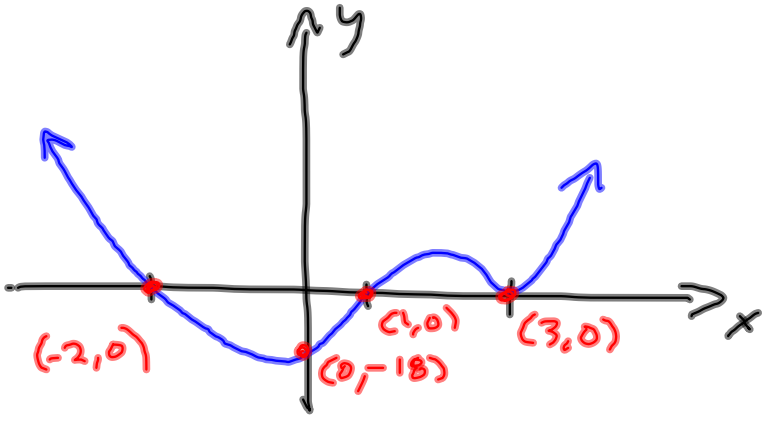
Calista

$f(0) = (-3)^2(2)(-1) = 6 \rightsquigarrow (0,6)$

Nope. Missed the "2" in the power.

$f(0) = (-3)^2(2)(-1) = -18$





Rough Sketches

$f(x) = (x-3)^2(x+2)(x-1)$

I'd do the same work (almost) to solve $(x-3)^2(x+2)(x-1) > 0$

$$\lim_{x\to\infty} f(x) = \lim_{x\to\infty} (x)^2(x)(x) = \lim_{x\to\infty} x^4 = +\infty$$

zeros: $x = -2, 1, 3$

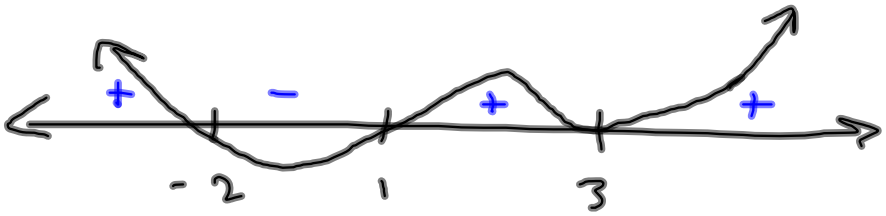
$m = 1, 1, 2$

c, c, t

'c' means "cross."

't' .. "touch."

$f(0) = (-3)^2(2)(-1) = -18$ ← Throwing in a test point.

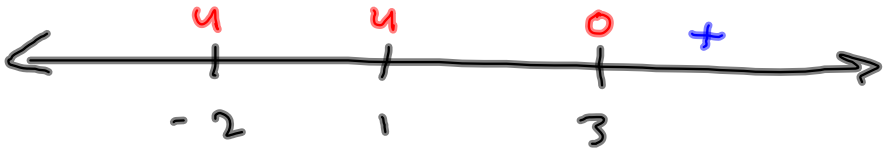


$(x-3)^2(x+2)(x-1) > 0$ is easy to solve with this info.

Want ">0". Want "+"

$x \in (-\infty, -2) \cup (1, 3) \cup (3, \infty)$

Graph $R(x) = \dfrac{(x-3)^2}{(x+2)(x-1)}$



Same exact sign pattern, except it's undefined @ $x = -2, 1$

$$\lim_{x\to\infty} R(x) = \lim_{x\to\infty} \frac{(x)^2}{(x)(x)} = \lim_{x\to\infty} \frac{x^2}{x^2} = 1$$

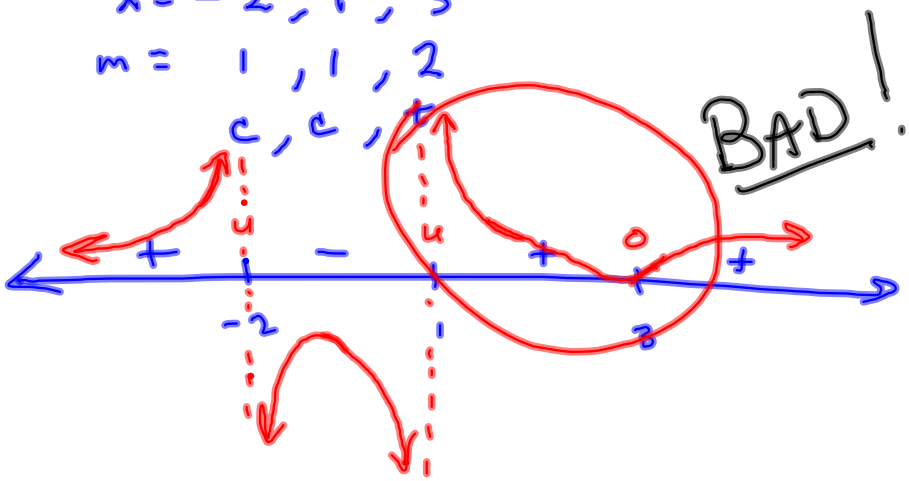
$y = 1$ is H.A.

Critical values:

$x = -2, 1, 3$

$m = 1, 1, 2$

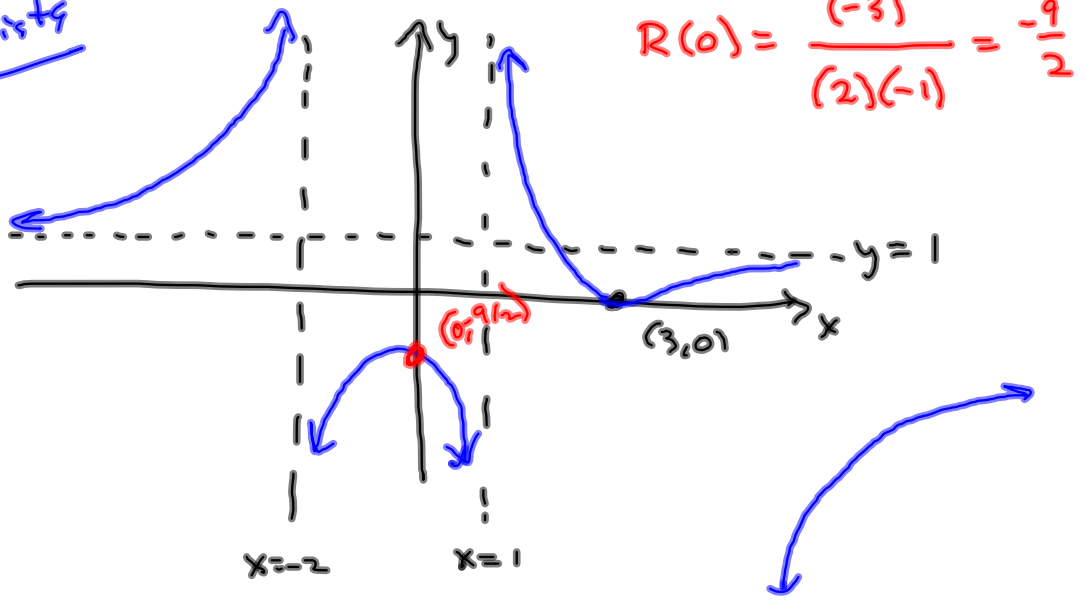
c, c, +



BAD!

kalista

$$R(0) = \frac{(-3)^2}{(2)(-1)} = -\frac{9}{2}$$



Holes:

Not (this OR that)

(81) $f(x) = \dfrac{x+1}{x^2-1}$ means (Not this) and (Not that)

Domain: $\mathcal{D} = \{x \mid x^2-1 \neq 0\} = \{x \mid x \neq -1 \text{ and } x \neq 1\}$

$x^2 - 1 = 0$

$x = \pm 1$

Ditch!

$= \{x \mid x \neq \pm 1\}$

Lowest Terms: Why? Because $x = \pm 1$ are candidates for vertical asymptotes, but ...

$$f(x) = \frac{x+1}{(x+1)(x-1)} = \frac{\cancel{x+1}}{\cancel{(x+1)}(x-1)} = \frac{1}{x-1}$$

FACT: $f(x)$ looks JUST LIKE $\frac{1}{x-1}$, except it has a HOLE @ $x = -1$

Graph $\frac{1}{x-1}$ & put a hole @ $x = -1$

$\frac{1}{-1-1} = \frac{1}{-2} = -\frac{1}{2} \rightsquigarrow$ Hole @ $(-1, -\frac{1}{2})$

$g(x) = \frac{1}{x-1}$

$x=1$ is V.A.

$y=0$ $g(x)$ is Proper

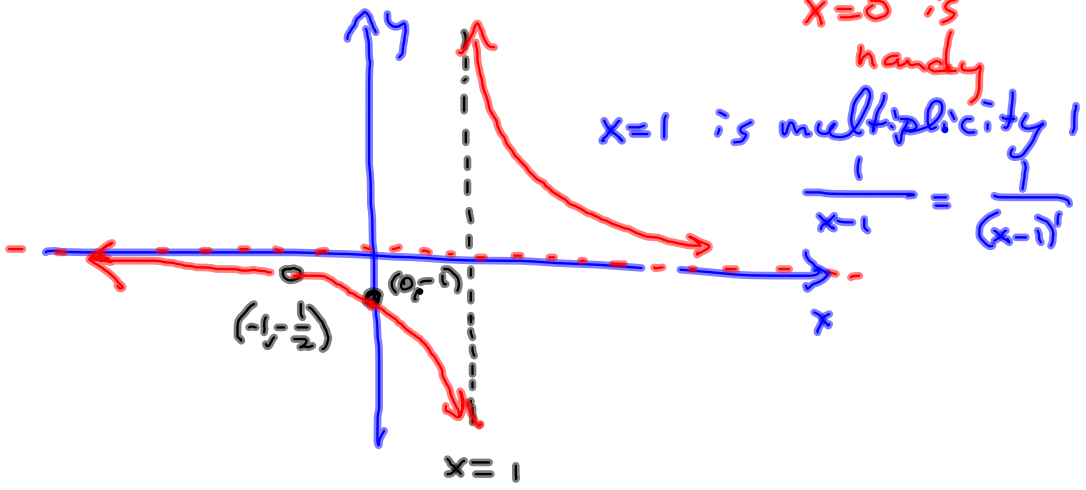
$g(0) = \frac{1}{0-1} = -1 \rightarrow (0,-1)$

Test:

$x=0$ is handy

$x=1$ is multiplicity 1

$\frac{1}{x-1} = \frac{1}{(x-1)^1}$



$R(x) = \dfrac{x^2 - 5x + 6}{x^2 - x - 6}$ Graph it.

Learnne

1. Domain
2. Lowest Terms
3. V.A., Holes
4. H.A.
5. zeros
6. Sign Pattern / Rough sketch
7. Final Sketch (y-int)

$\dfrac{(x-3)(x-2)}{(x-3)(x+2)} = \dfrac{x-2}{x+2} \quad (x \neq 3)$