§3.6 Rational Functions & Inequalities.

Find the domain of f

$f(x) = \frac{3x^2 2x}{x+5}$?

Not proper

$\mathcal{D} = \{x \mid x+5 \neq 0\}$
$= \{x \mid x \neq -5\}$
$= \boxed{(-\infty, -5) \cup (-5, \infty)}$

Similarly,

$f(x) = \frac{x+3}{x^2-7x+12}$

$(x-3)(x-4) = 0 \Rightarrow$
$x \in \{3, 4\}$
Ditch

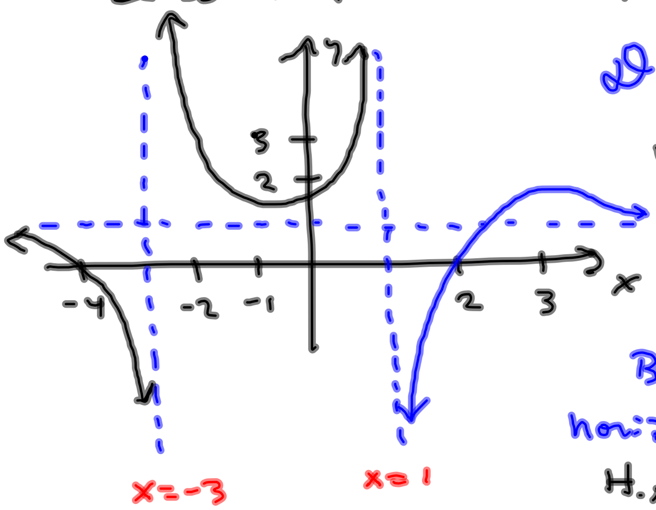
$\mathcal{D} = \{x \mid x^2 - 7x + 12 \neq 0\}$
$= \{x \mid x \neq 3 \text{ and } x \neq 4\}$
$= \boxed{(-\infty, 3) \cup (3, 4) \cup (4, \infty)}$

Is proper
This means $y = 0$ is H.A.

The denominator outstrips the numerator, eventually.

$\frac{\deg = 1}{\deg = 2}$ $2 > 1$

Determine Domain & Eq'ns of asymptotes



$D = \{x \mid x \neq -3 \text{ and } x \neq 1\}$

Vertical Asymptotes

V.A.: $x = -3$, $x = 1$

No touching!

But you CAN cross a horizontal asymptote:

H.A.: $y = 1$

For this one we would say that

V.A. $\lim_{x \to -3} f(x)$ DNE It blows up @ $x = -3$

H.A. $\lim_{x \to \infty} f(x) = 1 \longrightarrow y = 1$ is H.A.

$$\frac{x^5 - 27x^4 + 9}{x^5 + 3x^2 - 11} \begin{array}{l} \leftarrow \text{deg} = 5 \\ \leftarrow \text{deg} = 5 \end{array} \Big\} \text{Same.}$$

Just look at the big stuff $\frac{x^5}{x^5} = 1$ $y = 1$ is H.A.

$$\begin{array}{l} \text{deg}=2 \rightarrow \\ \text{deg}=2 \rightarrow \end{array} \frac{3x^2 - 5}{4x^2 + 2x - 11} \Longrightarrow \lim_{x \to \infty} f(x) \approx \frac{3x^2}{4x^2} = \boxed{\frac{3}{4} = y}$$

H.A.

Find all asymptotes

$f(x) = \frac{3}{x+7}$

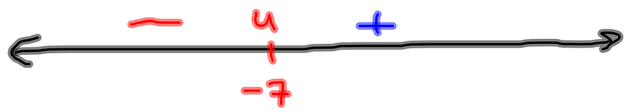
$\boxed{\begin{array}{l} H.A.: y = 0 \\ V.A.: x = -7 \end{array}}$

Extra: Let's graph it.

y-int: $(0, \frac{3}{7})$ $f(0) = \frac{3}{0+7} = \frac{3}{7}$

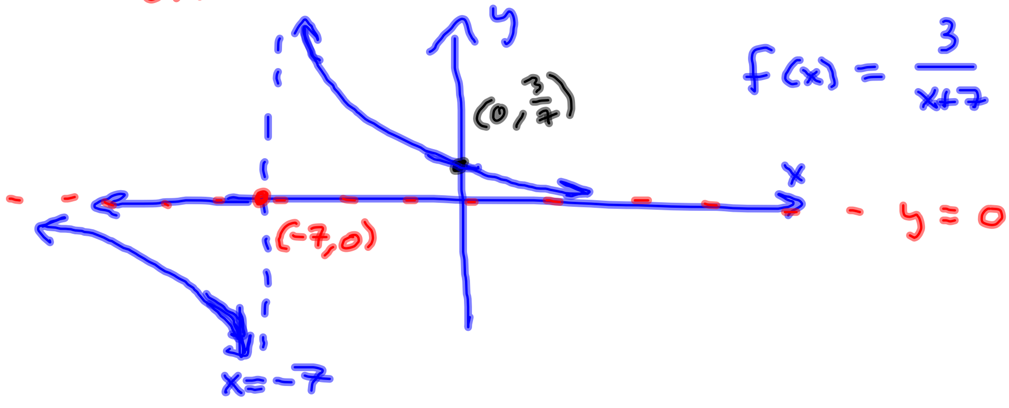
x-int: $f(x) = 0$

$\frac{3}{x+7} = 0 \Longrightarrow 3 = 0$ Nevah



Test: $x = -8$: $\frac{3}{-8+7} = -3$

$x = -6$: $\frac{3}{-6+7} = 3$



$f(x) = \frac{3}{x+7}$

Oblique Asymptotes (SPECIAL) when the degree of the numerator is greater than the degree of the denominator.

$f(x) = \dfrac{3x^2+4}{x-1}$ ← deg = 2 (numerator), ← deg = 1 (denominator)

2 = 1 + 1
They differ by 1.

Divide & the quotient will be the oblique asymptote.

$$\begin{array}{r|rrr} 1 & 3 & 0 & 4 \\ & & 3 & 3 \\ \hline & 3 & 3 & 7 \\ & x^1 & c & r \end{array}$$

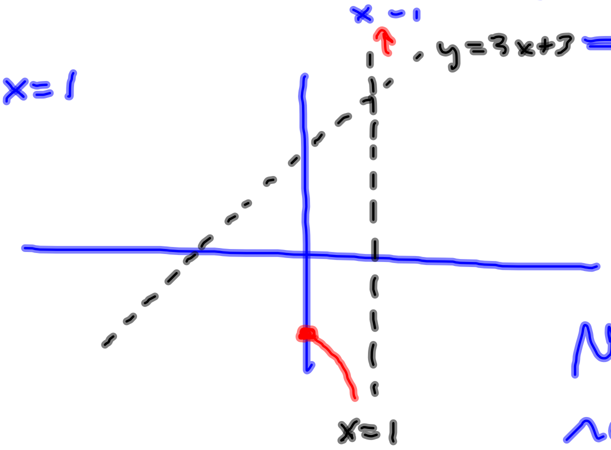
$$\frac{3x^2+4}{x-1} = 3x+3 + \frac{7}{x-1}$$

Oblique Asymptote.

Quick Sketch: $f(0) = \frac{4}{-1} = -4 \rightsquigarrow (0,-4)$

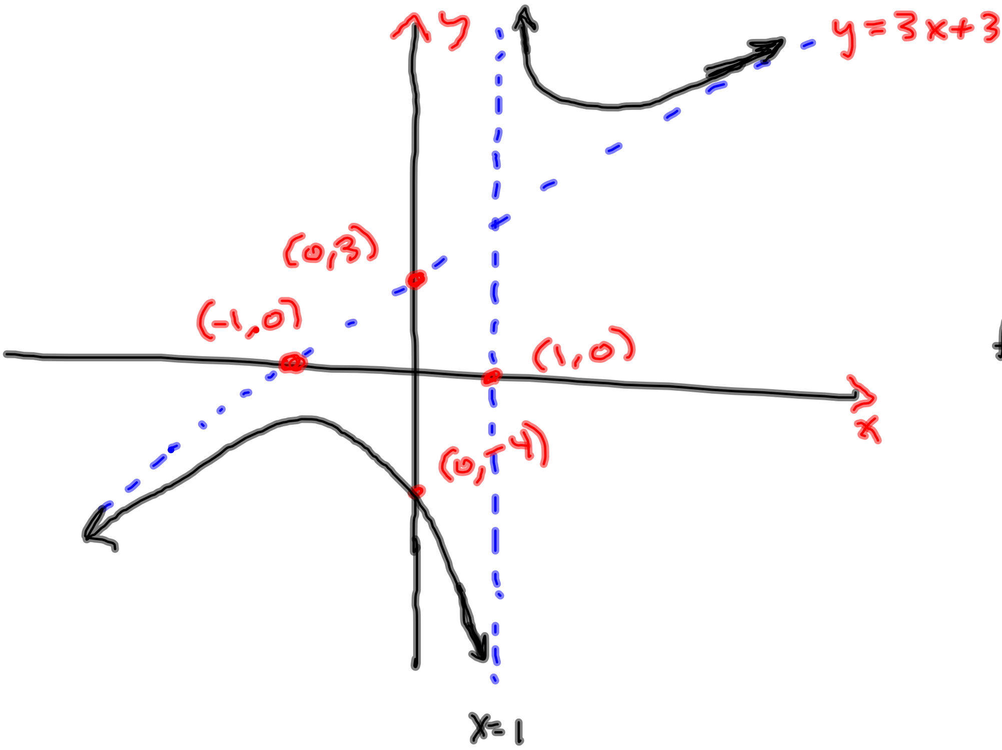
x-int:
$$\frac{3x^2+4}{x-1} = 0 \Longrightarrow 3x^2+4 = 0 \Longrightarrow \text{No real sol'n.}$$

V.A.: $x = 1$



No horizontal Asymptote.

Not enuff room.
re-sketch.



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$$f(x) = \frac{3x^2+4}{x-1}$$

Graph $f(x) = \dfrac{-x^2+7x-9}{x^2-6x+9}$

$x^2-6x+9 = (x-3)^2 = 0 \Rightarrow x=3$

$\mathcal{D} = \{x \mid x \neq 3\} = \boxed{(-\infty,3)\cup(3,\infty)}$

$-x^2+7x-9 \overset{\text{set}}{=} 0$

$x^2-7x+9 = 0$

$x^2-7x = -9$

$x^2-7x+\left(\frac{7}{2}\right)^2 = -9+\frac{49}{4} = \frac{-36+49}{4} = \frac{13}{4}$

$\left(x-\frac{7}{2}\right)^2 = \frac{13}{4}$

$x-\frac{7}{2} = \pm\frac{\sqrt{13}}{2}$

$x = \frac{7\pm\sqrt{13}}{2}$ are the zeros of $f(x)$.

≈ 1.70 or 5.30

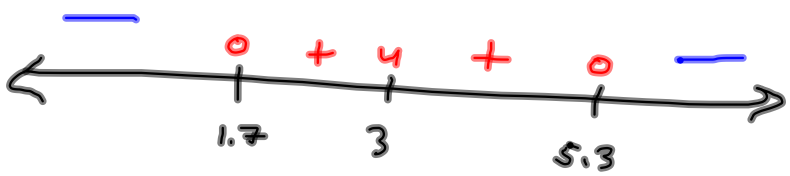
$\boxed{\text{H.A.: } y=-1}$ $\quad f(x)=\dfrac{-x^2+7x-9}{x^2-6x+9} \longrightarrow \dfrac{-x^2}{x^2}=-1=y$

$\boxed{\text{V.A.: } x=3}$ (by domain work) Lincoln & numerous others.

y-int: $(0,-1)$

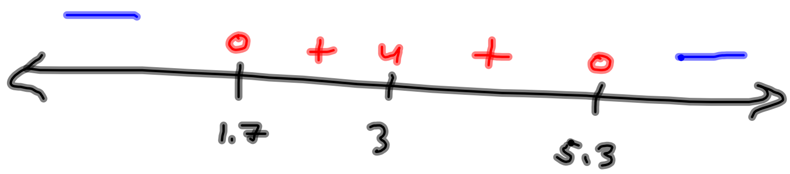
x-int: $\left(\dfrac{7-\sqrt{13}}{2},0\right),\left(\dfrac{7+\sqrt{13}}{2},0\right)$

$\approx (1.70,0),\ (5.30,0)$



Our work showed

$$f(x)=\frac{-\left(x-\frac{7+\sqrt{13}}{2}\right)^1\left(x-\frac{7-\sqrt{13}}{2}\right)^1}{(x-3)^2}$$



Our work showed

$$f(x) = \frac{-\left(x - \frac{7+\sqrt{13}}{2}\right)^1 \left(x - \frac{7-\sqrt{13}}{2}\right)^1}{(x-3)^2}$$

$$f(x) = \frac{-x^2+7x-9}{x^2-6x+9}$$