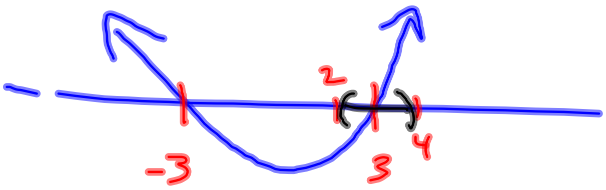
Show that $f(x) = x^2 - 9$ has a zero in the interval $(2, 4)$ without solving.

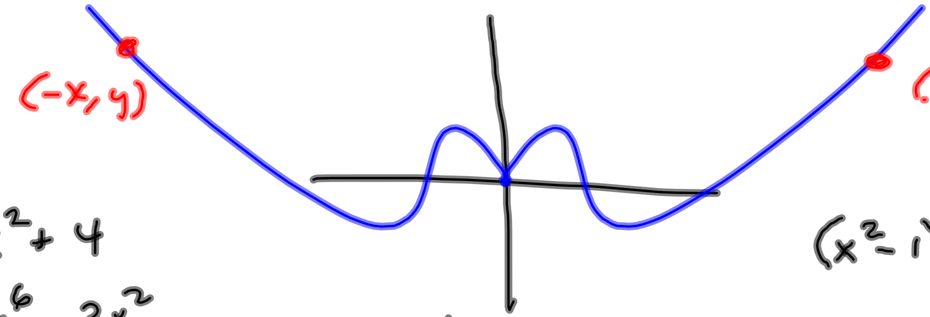
$f(2) = 4 - 9 = -5 < 0$
$f(4) = 16 - 9 = 7 > 0$ $\Longrightarrow$ There $\exists$ a $c \in (2,4)$ such that $f(c) = 0$ by IVT.



Polynomials are continuous. IVT works for ANY continuous function.

Symmetry.

Even functions are symmetric about the y-axis $\rightarrow f(-x) = f(x)$



$x^2 + 4$

$(x^2 - 1)^2$

$f(x) = x^6 - 3x^2$

$-2x^{16} + 4x^2$

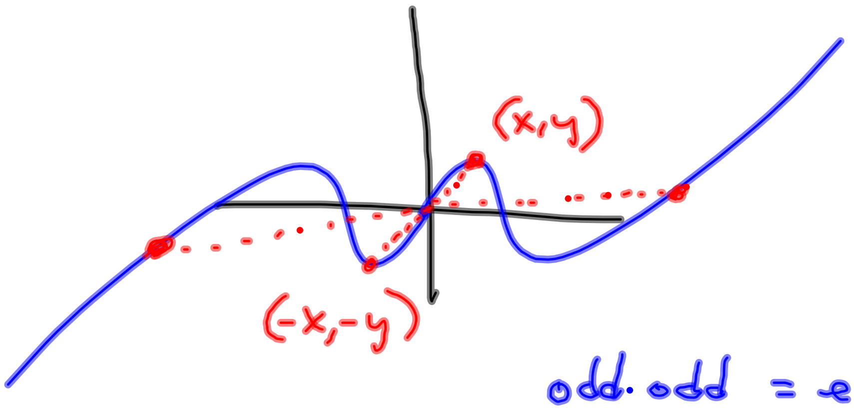
$f(-x) = (-x)^6 - 3(-x)^2$

$= x^6 - 3x^2 = f(x)$

odd functions

$f(-x) = -f(x)$ Symmetric thru the origin.

$x^3 - x$



odd · odd = even

even · odd = odd

$$f(x) = \frac{15x^2 + 72x^4}{x^3 - x} \Longrightarrow$$

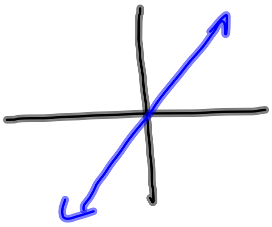
$$f(-x) = \frac{15x^2 + 72x^4}{-x^3 + x} = \frac{15x^2 + 72x^4}{-(x^3 - x)}$$

$$= -\frac{15x^2 + 72x^4}{x^3 - x} = -f(x)$$

Leading coefficient test.

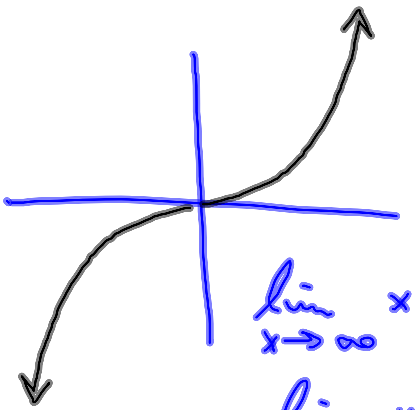
Power Functions Review.

x



$x^3, x^5, x^7, x^9, \ldots$

$27x^3$



up forever!

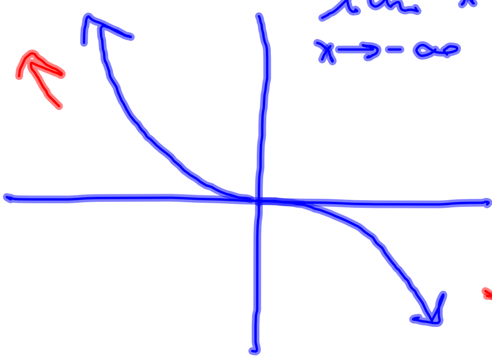
$\lim_{x\to\infty} x^{17} = \infty$

$\lim_{x\to-\infty} x^{9} = -\infty$

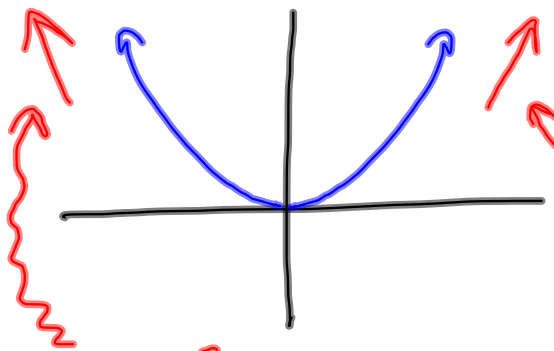
The point → is 3.5

$-x^3, -x^5, \ldots$

$-39x^{17}$



Even-powered power functions



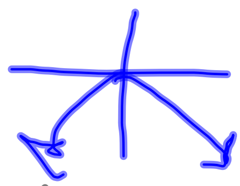
$x^2, x^4, x^{28}, x^{13132}$

$13x^2$

$\lim_{x \to -\infty} x^{28} = \infty$

$\lim_{x \to \infty} x^{14} = \infty$

$-x^4$



$\lim_{x \to \infty} -x^{28} = -\infty$

$\lim_{x \to -\infty} -27x^{16} = -\infty$

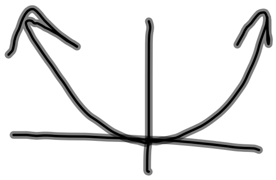
Turns out the leading term (highest degree term) "controls" end behavior.

$$f(x) = -13x^3 + \underline{44x^6} - 11$$

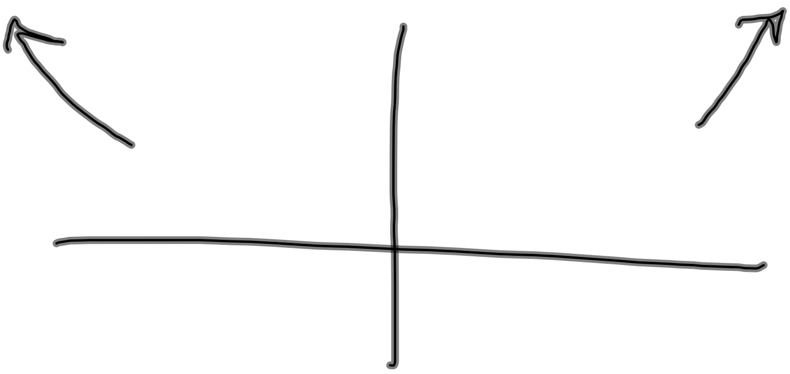
controls end behavior.

$$\lim_{x \to \infty} f(x) = +\infty$$

$44x^6$



whatever f(x) looks like, we know this much:



Behavior near x-intercepts.

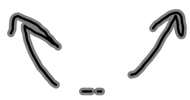
$$f(x) = (x-2)(x+3)^2(x-5)^3$$

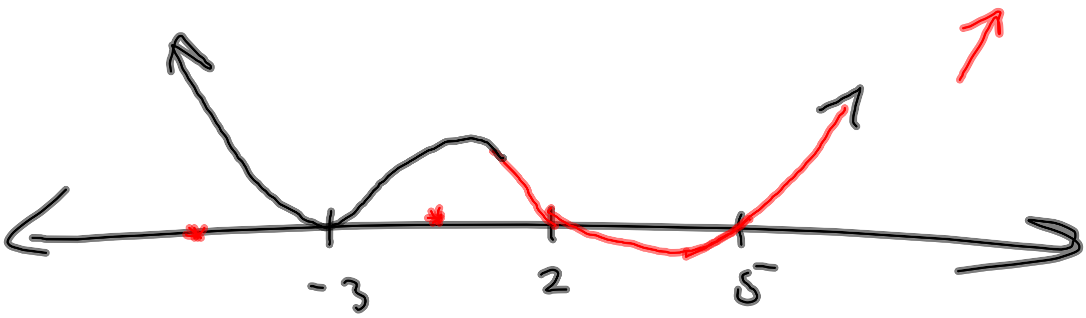
Zeros:

	m	
$x = -3$	2	touches x-axis
$x = 2$	1	crosses
$x = 5$	3	crosses

End Behavior:

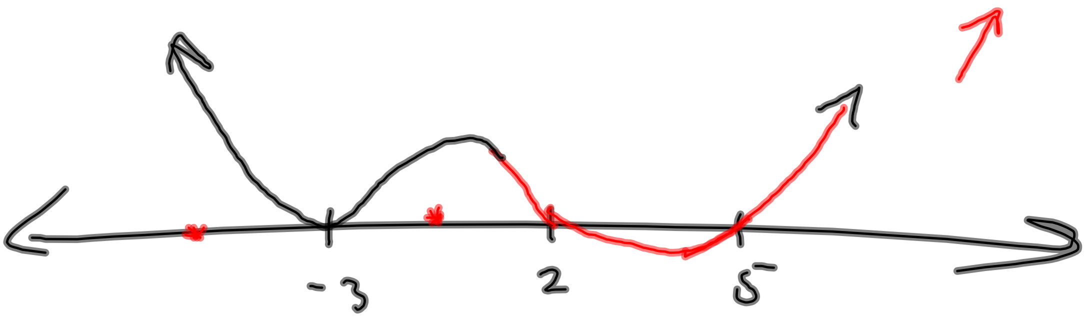
Big Stuff : $(x)(x)^2(x)^3 = x^6$





Solve $f(x) > 0$

$$f(x) = (x-2)(x+3)^2(x-5)^3$$



Solve $f(x) > 0$

$f(x) = (x-2)(x+3)^2(x-5)^3$

Sign Pattern

+ 0 + 0 − 0 +

−3 2 5

Interpret in context of $f(x) > 0$

+ 0 + 0 − 0 +

Y −3 Y 2 N 5 Y

"+" means ">0"

$(-\infty,-3) \cup (-3,2) \cup (5,\infty)$

Thx, Calista.

Test values are slow, but sure.

Try $x = -4$

$$f(-4) = (-4-2)(-4+3)^2(-4-5)^3$$
$$= (-6)(-1)^2(-9)^3$$

(Neg)(Pos)(Neg) = Positive Good

$f(x) = x^4 + x^3 - 15x^2 - 3x + 36$ Graph it!

Descartes: 2 or 0 pos. roots

$f(-x) = x^4 - x^3 - 15x^2 + 3x + 36$: 2 or 0 neg. roots.

$\frac{p}{q}$: $\pm 1, \pm 2, \pm 3, \pm 4, \pm 6, \pm 9, \pm 12, \pm 18, \pm 36$

3	1	1	−15	−3	36
		3	12	−9	−36
	1	4	−3	−12	0

$(x-3)(x^3 + 4x^2 - 3x - 12)$

$= (x-3)(x+4)(x-\sqrt{3})(x+\sqrt{3})$

$x^2(x+4) - 3(x+4)$

$= (x+4)(x^2 - 3)$

$= (x+4)(x-\sqrt{3})(x+\sqrt{3})$

$\lim_{x\to\infty} f(x) = +\infty$

x^4 controls