10/8 - Solutions thru 3.3 up, today.

My sense is that the class is not reading ahead, as I have insisted they do. Now the question is whether to reward the few who do, with an open-note quiz on new material, or to punish those who don't, with an open-note quiz on new material. Hmmm.

3.3 Questions?

3.2

18 $(x-(3-\sqrt{5}))(x-(3+\sqrt{5}))$

$(x-\square)(x-\Delta)$

$(a-b)(a+b)=a^2-b^2$

$= x^2-\Delta x-\square x+\square\Delta$

$= x^2-(3+\sqrt{5})x-(3-\sqrt{5})x+(3-\sqrt{5})(3+\sqrt{5})$

$= x^2-3x-\sqrt{5}x-3x+\sqrt{5}x+3^2-(\sqrt{5})^2$

$= x^2-6x+4$

$(a-bi)(a+bi)$
$=a^2-(bi)^2$
$=a^2+b^2$

$\left(x-(3-i\sqrt{5})\right)\left(x-(3+i\sqrt{5})\right)$

$= x^2-(3+i\sqrt{5})x-(3-i\sqrt{5})x+(3-i\sqrt{5})(3+i\sqrt{5})$

$= x^2-3x-3x+3^2+\sqrt{5}^2$

$= x^2-6x+14$

(Ignored the $-i\sqrt{5}x$ & $+i\sqrt{5}x$ because of their suicide pact.) CONJUGATE PAIRS Theorem.

§ 3.4 Misc. Eq'ns.

Square Root, Rational Exponents, Quadratic Forms, Absolute Value

#s 1–10 Find all $\mathbb{R}$ & imaginary sol'ns.

(1) $x^3+3x^2-4x-12=0$

$\Rightarrow x^2(x+3)-4(x+3)=0$

$\Rightarrow (x+3)\left(\frac{x^2\cancel{(x+3)}}{\cancel{(x+3)}}-\frac{4\cancel{(x+3)}}{\cancel{(x+3)}}\right)=0$

$\Rightarrow (x+3)(x^2-4)=0$

$\Rightarrow (x+3)(x-2)(x+2)=0$

$\Rightarrow x\in\{-3,-2,2\}$

(6) $b^3+20b=9b^2$

$\Rightarrow b^3-9b^2+20b=0$

$\Rightarrow b(b^2-9b+20)=0$

$\Rightarrow b(b-4)(b-5)=0$

$\Rightarrow b\in\{0,4,5\}$

(9) $a^4-16=0$

Let $u=a^2$. Then

$u^2-16=0 \Rightarrow$

$(u-4)(u+4)=0 \Rightarrow$

$(a^2-4)(a^2+4)=0 \Rightarrow$

$(a-2)(a+2)(a-2i)(a+2i)=0$

$\Rightarrow a\in\{-2,2,-2i,2i\}$

11 $\sqrt{x+1} = x-5$

$\implies \left(\sqrt{x+1}\right)^2 = (x-5)^2$

$\implies x+1 = x^2 - 10x + 25$

$\implies x^2 - 10x + 25 = x + 1$

$\implies x^2 - 11x + 24 = 0$

$\implies (x-8)(x-3) = 0$

$\implies x \in \{3, 8\}$

Doesn't check

Final Solution Set is $\{8\}$

$\sqrt{3+1} \overset{?}{=} 3-5$

$\sqrt{4} = -2$

$2 = -2$

$x=3$ is an extraneous root. The squaring step is legit, but it's not reversible.

22 $\sqrt{x} + \sqrt{x-36} = 2$

$(a+b)^2 = a^2 + 2ab + b^2$

$\Rightarrow \left(\sqrt{x} + \sqrt{x-36}\right)^2 = 2^2$

$\Rightarrow (\sqrt{x})^2 + 2\sqrt{x}\sqrt{x-36} + \left(\sqrt{x-36}\right)^2 = 4$

Mark

$x + \underline{2}\sqrt{(x)(x-36)} + x - 36 = 4$

$2\sqrt{x^2-36x} + 2x - 36 = 4$

$\sqrt{x^2-36x} + x - 18 = 2$

$\sqrt{x^2-36x} = 20 - x$

S.I. Leader

$\left(\sqrt{x^2-36x}\right)^2 = (20-x)^2$

$\underline{x^2} - 36x = 400 - 40x + \underline{x^2}$

$40x - 400 - 36 = 0$

$40x - 436 = 0$

$40x = 436$

$x = \frac{436}{40} = \frac{109}{10}$

This problem has no sol'n anyway.

I'm tired & stupid.

#21 DOES have a solution.

#s 27-34 Find all real solutions.

28 $x^{\frac{2}{3}} = \frac{1}{2}$

$\left(x^{\frac{2}{3}}\right)^{\frac{3}{2}} = \left(\frac{1}{2}\right)^{\frac{3}{2}}$

$x = \left(\frac{1}{2}\right)^{\frac{3}{2}} = \frac{1^{\frac{3}{2}}}{2^{3/2}} = \frac{1}{2^{\frac{3}{2}}} = \frac{1}{(2^3)^{\frac{1}{2}}}$

$= \frac{1}{8^{\frac{1}{2}}} = \frac{1}{\sqrt{8}} = \frac{1}{\sqrt{4 \cdot 2}} = \frac{1}{\sqrt{4}\sqrt{2}} = \frac{1}{2\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}}$

$= \frac{\sqrt{2}}{2 \cdot 2} = \frac{\sqrt{2}}{4} = x$

$x \in \left\{ \frac{\sqrt{2}}{4} \right\}$

$\left(\frac{\sqrt{2}}{4}\right)^{\frac{2}{3}} = \left(\frac{(\sqrt{2})^2}{4^2}\right)^{\frac{1}{3}} = \left(\frac{2}{16}\right)^{\frac{1}{3}} = \left(\frac{1}{8}\right)^{\frac{1}{3}} =$

$= \frac{1^{\frac{1}{3}}}{8^{\frac{1}{3}}} = \frac{1}{\sqrt[3]{8}} = \frac{1}{2}$

#s 35-52 All real & imag....

34 $x^4+10=7x^2$

Quadratic in Form

$\Longrightarrow x^4-7x^2+10=0$

Let $u=x^2 \Longrightarrow$

$u^2-7u+10=0$

$x^2-5=0$

$\Longrightarrow (u-5)(u-2)=0$

$x^2=5$

$\Longrightarrow (x^2-5)(x^2-2)=0$

$x=\pm\sqrt{5}$

$\Longrightarrow (x-\sqrt{5})(x+\sqrt{5})(x-\sqrt{2})(x+\sqrt{2})=0$

$\Longrightarrow x\in\{\pm\sqrt{5},\pm\sqrt{2}\}$

42 $\left(\frac{b-5}{6}\right)^2 - \left(\frac{b-5}{6}\right) - 6 = 0$

Let $u = \frac{b-5}{6}$ $\Longrightarrow$

$u^2 - u - 6 = 0$

$\Longrightarrow$ $(u-3)(u+2) = 0$

$\Longrightarrow$ $u = -2$ or $u = 3$

$\Longrightarrow$ $\frac{b-5}{6} = -2$ or $\frac{b-5}{6} = 3$

$\Longrightarrow$ $b-5 = -12$ or $b-5 = 18$

$\Longrightarrow$ $b = -7$ or $b = 23$

$\Longrightarrow$ $b \in \{-7, 23\}$

49 $q-7q^{\frac{1}{2}}+12=0$

Let $u=q^{\frac{1}{2}}$

$u^2-7u+12=0$

$(u-3)(u-4)=0$

$u=3$ or $u=4$

$\Rightarrow q^{\frac{1}{2}}=3$ or $q^{\frac{1}{2}}=4$

$(q^{\frac{1}{2}})^2=3^2$ or $(q^{\frac{1}{2}})^2=4^2$

$q=3^2$ or $q=16$

$q=9$

$q\in\{9,16\}$ Be sure to check.

#s 53-64

58 $|2x^2 - x - 2| = 1$

$2x^2 - x - 2 = 1$ OR $2x^2 - x - 2 = -1$

$2x^2 - x - 3 = 0$ $2x^2 - x - 1 = 0$

$(2x - 3)(x + 1) = 0$ $(2x + 1)(x - 1) = 0$

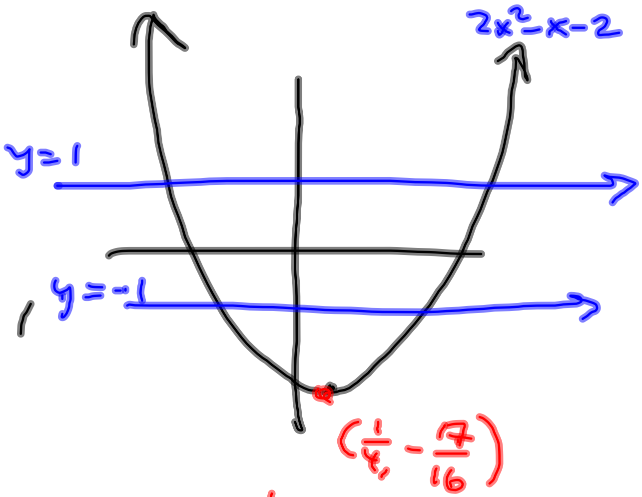
$x \in \{\frac{3}{2}, -1, -\frac{1}{2}, 1\}$ Be sure to check.

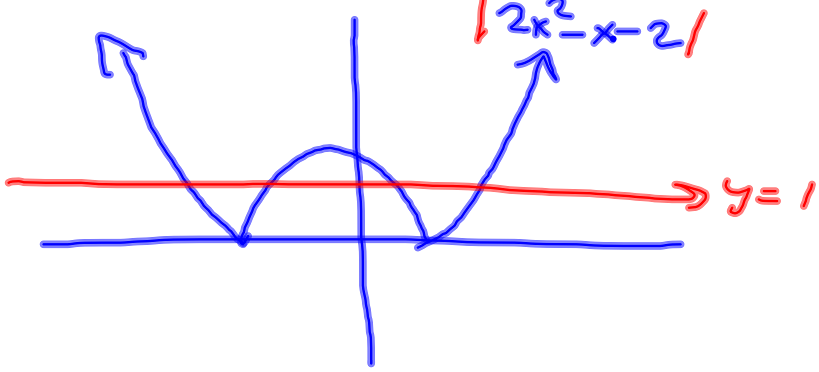
$y = 2x^2 - x - 2$

$\frac{y}{2} = x^2 - \frac{1}{2}x - 1$

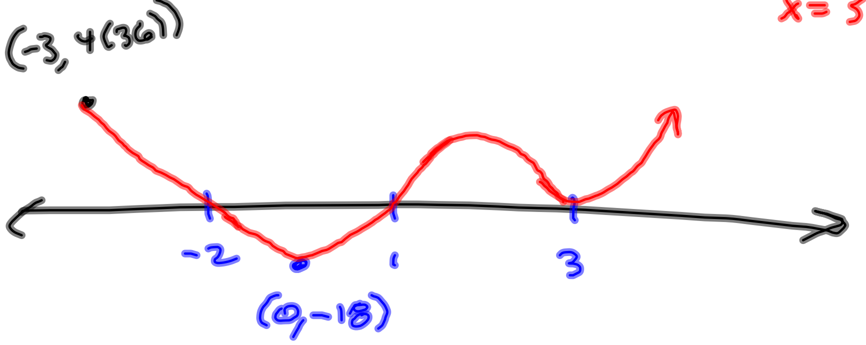
$= x^2 - \frac{1}{2}x + \left(\frac{1}{4}\right)^2 - \frac{1}{16} - 1$

$= \left(x - \frac{1}{4}\right)^2 - \frac{17}{16}$





$P(x) = (x-1)^1(x+2)^1(x-3)^{2}$ → Touches @ $x=3$. Doesn't cross.



Test: $x = 0$ $\quad P(0) = (-1)(2)(-3)^2 = -18$

$x = -3$ $\quad P(3) = (-4)(-1)(-6)^2$

$= (4)(36)$

$(x+5)^2(x-3)(x+6)(x-2)^2$