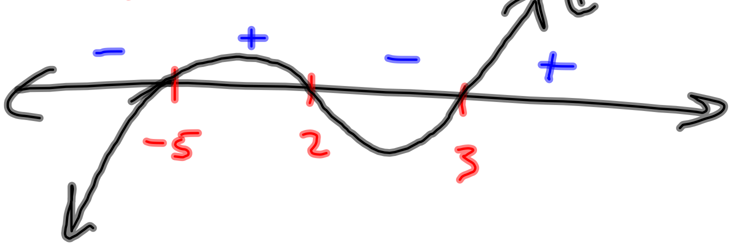
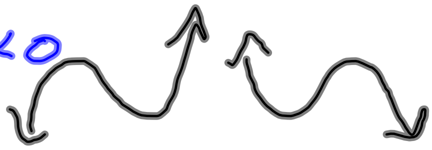
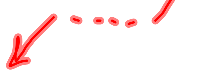
$(x-2)(x-3)(x+5) < 0$

$= x^3 + \text{smaller}$







$x \in (-\infty, -5) \cup (2,3)$

§3.2

$$\frac{277}{3}$$

$$\begin{array}{r} 92 \text{ r } 1 \\ 3\overline{)277} \\ -(27) \\ \hline 7 \\ -6 \\ \hline 1 \end{array}$$

Brian,
Phillip.

Interpretations

$$\frac{277}{3} = 92 + \frac{1}{3}$$

$$277 = (3)(92) + 1$$

Dividend = Divisor · Quotient + Remainder

$$P(x) = (x-c)Q(x) + R(x)$$

Teacher Scratch:

$(x-2)(x+5)+7$

$= x^2+3x-10+7$

$= x^2+3x-3$

Interpretations

$x^2+3x-3=(x-2)(x+5)+7$

Dividend = Divisor · Quotient + Remainder

$P(x)=(x-c)\,Q(x)+R(x)$

$$\frac{x^2+3x-3}{x-2}=x+5+\frac{7}{x-2}$$

Divide x^2+3x-3 by $x-2$

$$
\begin{array}{r l}
 & x+5 \text{ r } 7 \\
x-2 \,\big) & x^2+3x-3 \\
 & -(x^2-2x) \\ \hline
 & 5x-3 \\
 & -(5x-10) \\ \hline
 & +7
\end{array}
$$

$\frac{x^2}{x}=x$

$x(x-2)=x^2-2x$

$\frac{5x}{x}=5$

$5(x-2)=5x-10$

$x^2+3x-3=(x-2)(x+5)+7$

$P(x)=x^2+3x-3 \Longrightarrow$

$P(2)=2^2+3(2)-3$
$=7$ ✓

$P(2)=7$ This is the Remainder Theorem.

$P(c)$ = the remainder when $P(x)$ is divided by $x-c$

Synthetic Division:

Divide x^2+3x-3 by $x-2$

$$\begin{array}{r|rrr} 2 & 1 & 3 & -3 \\ & & 2 & 10 \\ \hline & 1 & 5 & 7 \end{array}$$

$x+5$ r 7

$P(x) = 3x^5-8x^4+3x^3-25x^2+20x-7$

Find $P(3)$

Divide by $x-3$

$$\begin{array}{r|rrrrrr} 3 & 3 & -8 & 3 & -25 & 20 & -7 \\ & & 9 & 3 & 18 & -21 & -3 \\ \hline & 3 & 1 & 6 & -7 & -1 & \boxed{-10 = P(3)} \end{array}$$

$$\begin{array}{r} 243 \\ \times\ 3 \\ \hline 729 \end{array} \qquad \begin{array}{r} 81 \\ \times\ 8 \\ \hline 648 \end{array}$$

$P(3) = 3(3)^5 - 8(3)^4 + 3(3)^3 - 25(3)^2 + 20(3) - 7$

$729 - 648 + 81 - 225 + 60 - 7$

$81 + 141 - 232$

$-232 + 222 = \boxed{-10 = P(3) \text{ the hard way}}$

$P(x) = 3x^5 - 8x^4 + 3x^3 - 25x^2 + 20x - 7$

Find $P(3)$

Divide by $x-3$

$$\begin{array}{r|rrrrrr} 3 & 3 & -8 & 3 & -25 & 20 & -7 \\ & & 9 & 3 & 18 & -21 & -3 \\ \hline & 3 & 1 & 6 & -7 & -1 & \boxed{-10 = P(3)} \end{array}$$

Interpret:

This says

$3x^5 - 8x^4 + 3x^3 - 25x^2 + 20x - 7$

$= (x-3)(3x^4 + x^3 + 6x^2 - 7x - 1) - 10$

What happens when $P(c) = 0$?

" " " the remainder is zero when I divide by $x-c$?

$$P(x) = (x-c)\,Q(x) + 0$$

Then $x-c$ is a factor of $P(x)$

That's the $\boxed{\text{factor Theorem.}}$

Use the fact that $x=5$ is a zero of $P(x) = x^3 - 2x^2 - 13x - 10$ to find all zeros of $P(x)$. Write $P(x)$ in factored form

$x=5$ is a zero

Split off a factor of $x-5$:

$$\begin{array}{r|rrrr} 5 & 1 & -2 & -13 & -10 \\ & & 5 & 15 & 10 \\ \hline & 1 & 3 & 2 & 0 \end{array}$$

So $P(x) = (x-5)\underline{(x^2+3x+2)}$

Reduces the question to solving a quadratic equation:

$$x^2 + 3x + 2 = 0$$

$$(x+2)(x+1) = 0$$

This says

$$\boxed{\begin{array}{l} P(x) = (x-5)(x+2)(x+1) \quad \text{and} \\ x = 5, -2, -1 \text{ are zeros of } P(x). \end{array}}$$

Teacher scratch

$(2x+3)(3x+5) = 6x^2 + 19x + 15$

zeros are $x = -\frac{3}{2}, -\frac{5}{3}$

Notice:

$x = -\frac{3}{2}$: 3 divides 15 & 2 divides 6

3 is a factor of $a_0 = 15$

2 " " " " $a_2 = 6$

$x = -\frac{5}{3}$: 3 is a factor of $a_2 = 6$

5 " " " " $a_0 = 15$

Rational Zeros Theorem.

$P(x) = a_n x^n + a_{n-1} x^{n-1} + \dots + a_2 x^2 + a_1 x + a_0$

If $\frac{p}{q}$ is a zero of $P(x)$ then

p is a factor of a_0 and

q " " " " a_n

$6x^2 + 19x + 15$ has possible rational zeros:

$\pm 1, \pm 3, \pm 5, \pm 15,$

$\pm \frac{1}{2}, \pm \frac{3}{2}, \pm \frac{5}{2}, \pm \frac{15}{2}$

$\pm \frac{1}{3}, \pm \frac{3}{3}, \pm \frac{5}{3}, \pm \frac{15}{3}$

$\pm \frac{1}{6}, \pm \frac{2}{6}, \pm \frac{3}{6}, \pm \frac{5}{6}, \pm \frac{15}{6} = \pm \frac{5}{2}$

OK, Phillip wiseguy
2 is NOT a
factor of 15. Shouldn't be
a $\pm \frac{2}{6}$, here.

Rational Zeros Theorem.

$P(x) = a_n x^n + a_{n-1}x^{n-1} + \dots + a_2 x^2 + a_1 x + a_0$

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$\pm \frac{1}{3}, \pm \frac{3}{3}, \pm \frac{5}{3}, \pm \frac{15}{3}$

$\pm \frac{1}{6}, \pm \frac{2}{6}, \pm \frac{3}{6}, \pm \frac{5}{6}, \pm \frac{15}{6} = \pm \frac{5}{2}$

OK, Phillip Wiseguy
2 is NOT a
factor of 15. Shouldn't be
a $\pm \frac{2}{6}$, here.

55 Find all real & imaginary zeros for each polynomial

$x^3-9x^2+26x-24$

$a_n = a_3 = 1$: the q's

$a_0 = -24$: the p's

$\frac{p}{q}$: $\pm 1, \pm 2, \pm 3, \pm 4, \pm 6, \pm 8, \pm 12, \pm 24$

$$\begin{array}{r|rrrr} 1 & 1 & -9 & 26 & -24 \\ & & 1 & -8 & 18 \\ \hline & 1 & -8 & 18 & \text{Nope} \end{array}$$

$$\begin{array}{r|rrrr} -1 & 1 & -9 & 26 & -24 \\ & & -1 & 10 & -36 \\ \hline & 1 & -10 & 36 & \text{Nope} \end{array}$$

$$\begin{array}{r|rrrr} 2 & 1 & -9 & 26 & -24 \\ & & 2 & -14 & 24 \\ \hline & 1 & -7 & 12 & 0 \end{array}$$ Sweet!

This says $x^3-9x^2+26x-24$

$= (x-2)(x^2-7x+12)$

$x^2-7x+12=0 \Longrightarrow$

$(x-3)(x-4)=0 \Longrightarrow$

$x=3,4$

Zeros are

$x = 2, 3, 4$

Factored form

$(x-2)(x-3)(x-4)$

§3.2 #s 3, 11, 15, 19, 25, 29, 37, 41,

47, 53, $\underbrace{55, 58, 65, 77}_{\text{Integrate all the skills.}}$

Friday

Wed. is a good goal.