$h(x) = (2x-3)^2$ written as a composition of simpler functions.

Let $g(x) = 2x-3$

& $f(x) = x^2 \longrightarrow f(\boxed{2x-3}) = \boxed{2x-3}^2$

Then $(f \circ g)(x) = f(g(x)) = f(2x-3) = (2x-3)^2$

#s 51-62 Let $f(x) = x-2$, $g(x) = \sqrt{x}$ & $h(x) = \frac{1}{x}$

#s 63-72 .. $f(x) = |x|$, $g(x) = x-7$, $h(x) = x^2$. Then...

65 $H(x) = (x-7)^2 = h(g(x)) = (h \circ g)(x)$

$h \circ g$

Homework Policy:
Get it in before I grade the stuff, you're fine

§ 2.5 Inverse functions

A function is a relation that assigns exactly one y-value (output) to each x-value (inputs)

A one-to-one function has the requirement that no y-value corresponds to more than one x-value. This makes the inverse relation a FUNCTION.

$$f = \{ (x,y) \mid x \in D \}$$

$$f^{-1} = \{ (y,x) \mid y \in R \} = \text{"f inverse"}$$

$f = \{ (1,2), (1,3), (2,3), (3,5) \}$ Not function

$g = \{ (1,2), (2,3), (3,3), (5,4) \}$ Is a function
Not 1-to-1

Not 1-to-1

Notice
$g^{-1} = \{ (2,1), (3,2), (3,3), (4,5) \}$ Inverse Relation is not a func.

$h = \{ (1,2), (2,3), (5,6) \}$ 1-to-1 func.

$h^{-1} = \{ (2,1), (3,2), (6,5) \}$ is a 1-to-1 func.

1-to-1 :

$\boxed{\text{If } f(x_1) = f(x_2) \text{ then } x_1 = x_2}$

Flunks horizontal line test.

Can't happen if it's 1-to-1.

$x_1 \qquad x_2$

If $f(x_1) = f(x_2)$ then $x_1 = x_2$

This is how we test for 1-to-1, algebraically.

§2.5 # 18

$h(x) = 4x - 9$ is 1-to-1.

Proof: Suppose $h(x_1) = h(x_2)$. Then

(Solve for x_1)

$$4x_1 - 9 = 4x_2 - 9$$
$$+9 = +9$$
$$4x_1 = 4x_2$$
$$\frac{4x_1}{4} = \frac{4x_2}{4}$$
$$x_1 = x_2 \quad \blacksquare$$

$f(x) = x^2$ is NOT 1-to-1

Proof: Suppose $f(x_1) = f(x_2)$

#'s 17, 20, 22 like this

$\sqrt{(-3)^2} = \sqrt{9} = 3$

$\sqrt{3^2} = 3$

$$x_1^2 = x_2^2$$
$$\sqrt{x_1^2} = \sqrt{x_2^2}$$
$$|x_1| = |x_2|$$

$x_1 = x_2$ OR $x_1 = -x_2$

Not 1-to-1

Two possibilities

One like #20

$f(x) = \frac{x-1}{x-5}$ is 1-to-1

Suppose $\boxed{f(x_1) = f(x_2)}$. Then

$$\frac{x_1-1}{x_1-5} = \frac{x_2-1}{x_2-5}$$

$$(x_1-1)(x_2-5) = (x_2-1)(x_1-5)$$

$$x_1x_2 - 5x_1 - x_2 + 5 = x_2x_1 - 5x_2 - x_1 + 5$$

$$-5x_1 - x_2 = -5x_2 - x_1$$

$$+x_1 = +x_1$$

$$-4x_1 - x_2 = -5x_2$$

$$+x_2 = +x_2$$

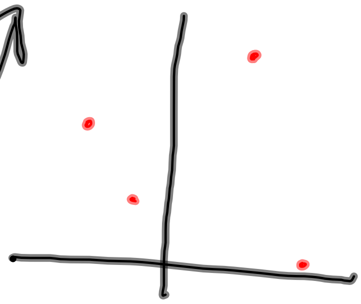
$$-4x_1 = -4x_2$$

$$x_1 = x_2 \quad \blacksquare$$

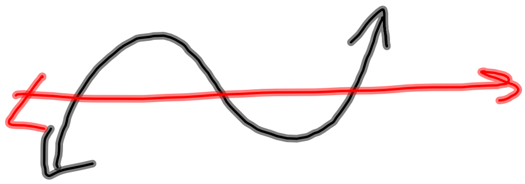
Horizontal Line Test

Yes

Yes



Yes



No

No

Finding an inverse by "reversing a composition"

$f(x) = 3x^2 - 7$

$x \xrightarrow{\text{square}} x^2 \xrightarrow{\text{times } 3} 3x^2 \xrightarrow{\text{minus } 7} 3x^2 - 7$

$\sqrt{\frac{x+7}{3}} \xleftarrow{\sqrt{\ }} \frac{x+7}{3} \xleftarrow{\text{divided by } 3} x+7 \xleftarrow{\text{plus } 7} x$

Claim: $f^{-1}(x) = \sqrt{\frac{x+7}{3}}$ $\qquad f^{-1}(\square) = \sqrt{\frac{\square + 7}{3}}$

f^{-1} sends $3x^2 - 7$ back to x

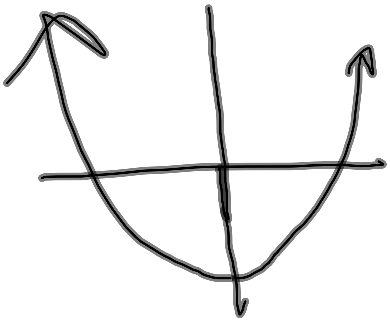
$(f^{-1} \circ f)(x) = x$

Look: $f^{-1}(f(x)) = f^{-1}(\boxed{3x^2-7})$

$= \sqrt{\frac{\boxed{3x^2-7} + 7}{3}} = \sqrt{\frac{(3x^2-7)+7}{3}} = \sqrt{\frac{3x^2}{3}} = \sqrt{x^2}$

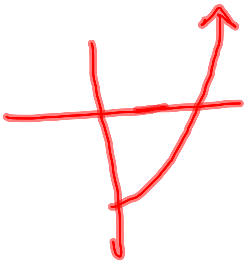
$= |x|$ Almost. Why didn't this work?

Phillip says $3x^2-7$ isn't 1-to-1.



Here's the fix:

Restrict x to $[0,\infty)$



The right half is 1-to-1.

As long as $x \geq 0$, $|x| = x$, and we have

$f^{-1}(x) = \sqrt{\frac{x+7}{3}}$ as long as $x \geq 0$.

This will explain why some exercises have that weird $x \geq 0$ goin' on.

This shows that if

$f(x) = 3x^2 - 7$, then $f^{-1}(x) = \sqrt{\frac{x+7}{3}}$

1st we found f^{-1}

The rest was CONFIRMING f^{-1} by checking that $(f^{-1} \circ f)(x) = x$

(with appropriate restriction on x).

Keep $x \geq 0$

$(f \circ f^{-1})(x)$

$f(☺) = 3☺^2 - 7$

$= f\left(\sqrt{\frac{x+7}{3}}\right) = 3\left(\sqrt{\frac{x+7}{3}}\right)^2 - 7$

$= \cancel{3}\left(\frac{x+7}{\cancel{3}}\right) - 7 = x + 7 - 7 = x$

$(f \circ f^{-1})(x) = x$

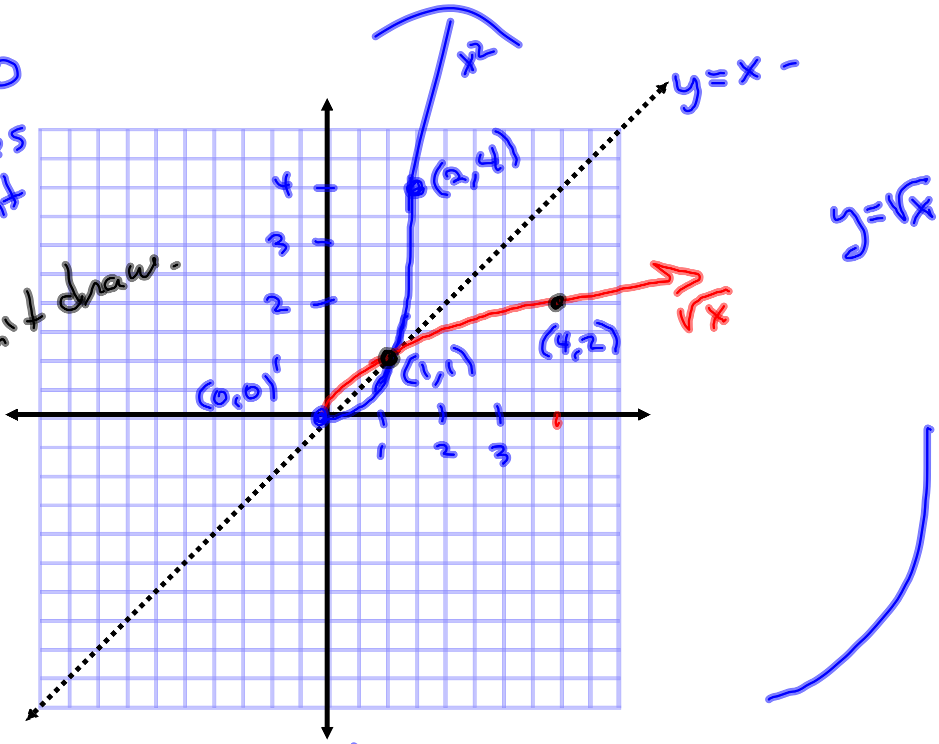
$(f^{-1} \circ f)(x) = x$

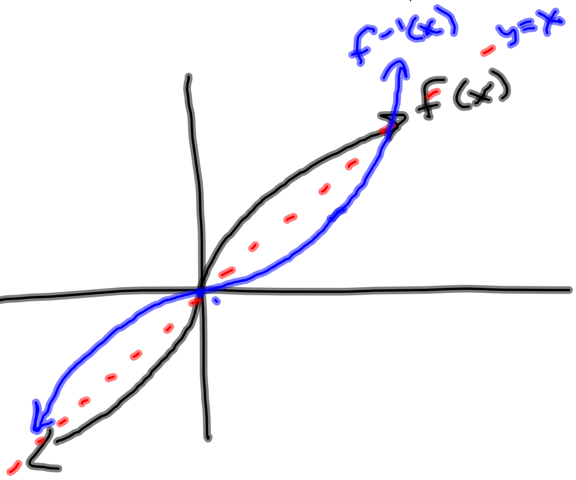
x^2, $\sqrt{x}$

Keep $x \geq 0$

This illustrates symmetry about the line $y=x$.

And that I can't draw.





Switch & Solve for finding f^{-1}:

Switch x & y. Solve for y.

$f(x) = 3x+2$ Find f^{-1}

$y = 3x+2$

$x = 3y+2$

$3y+2 = x$

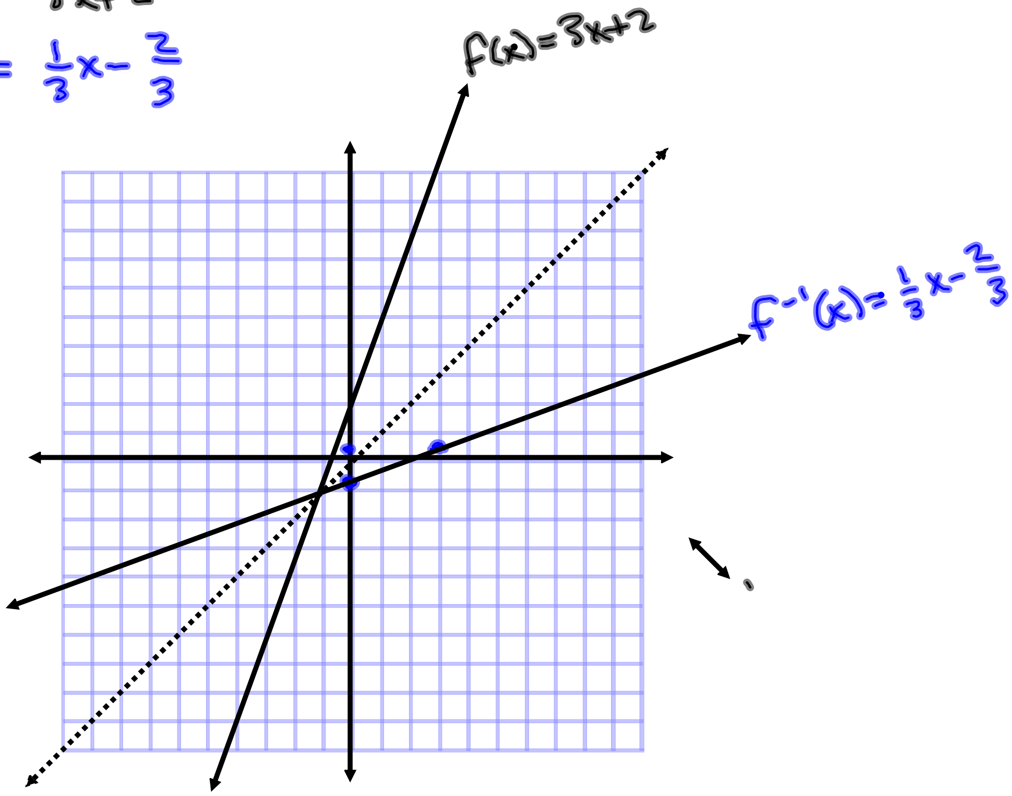
$3y = x-2$

$y = \frac{x-2}{3} = \frac{1}{3}x - \frac{2}{3}$

$f^{-1}(x) = \frac{1}{3}x - \frac{2}{3}$

$f(x) = 3x+2$

$f^{-1}(x) = \frac{1}{3}x - \frac{2}{3}$



§2.5 I #s 6, 7, 12, 13, 17, 20, 22, 24, 41

Assume $x_1 = x_2$
Solve for x_1

§2.5 II #s 50c, 50j, 58, 60, 68, 74

SI Thurs. 4-5p BH 129

Thurs. 7-8p BH 106

Fri. 1:30-2:30 BH 129