Questions on old stuff

Rigid:

- $f(x+h)$ left h $(x,y) \longrightarrow (x-h, y)$
- $f(x-h)$ right h $(x,y) \longrightarrow (x+h, y)$
- $f(x)+h$ up h $(x,y) \longrightarrow (x, y+h)$
- $f(x)-h$ down h $(x,y) \longrightarrow (x, y-h)$
- $f(-x)$ flip about y-axis $(x,y) \longrightarrow (-x, y)$
- $-f(x)$ flip about x-axis $(x,y) \longrightarrow (x, -y)$

Non-rigid:

- $h f(x)$ vertical "stretch" by factor of h. Multiply y-values by h
- $f(hx)$ horizontal "stretch" by factor of $\frac{1}{h}$. Multiply x-values by $\frac{1}{h}$

$g(x) = (3x+7)^2$

$= (3(x+\frac{7}{3}))^2$

I'll have to check this, 2 be sure.

$x^2 \longrightarrow (x+\frac{7}{3})^2 \longrightarrow (3(x+\frac{7}{3}))^2$

left $\frac{7}{3}$ $(x,y) \longrightarrow (\frac{1}{3}x, y)$

I think.

$x^2 \longrightarrow (3x)^2 \longrightarrow (3(x+\frac{7}{3}))^2$

$(x,y) \longrightarrow (\frac{1}{3}x, y) \longrightarrow (3(x+\frac{7}{3}))^2$

$f(x) \longrightarrow f(3x) \longrightarrow f(3(x+\frac{7}{3}))$

students like this

$2-x$

$x \longrightarrow x+2 \longrightarrow -x+2 = 2-x$

left 2 reflect about y-axis

Another way: $2-x = -(-2+x) = -(x-2)$

I like this

$x \longrightarrow -x \longrightarrow -(x-2) = 2-x$

reflect about y-axis right 2

The hangup on tests is getting from x to 2-x in steps that are prescribed.

Most common error:

$x \longrightarrow -x \longrightarrow -x+2$

flip left 2

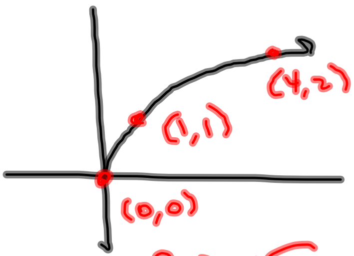
Nope. You've actually replaced x by x-2. This puts it in the proper terms.

$-x \longrightarrow -x+2 = -(x-2)$

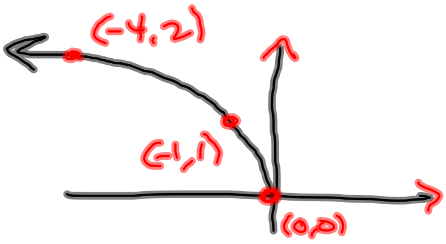
$g(x) = \sqrt{7-x} = \sqrt{-(x-7)}$

$\sqrt{x} \longrightarrow \sqrt{-x} \longrightarrow \sqrt{-(x-7)}$

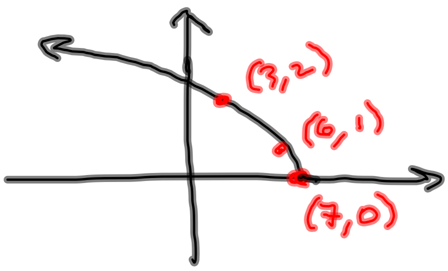
Scratch: $7 - x = -(-7+x) = -(x-7)$



$f(x) = \sqrt{x}$



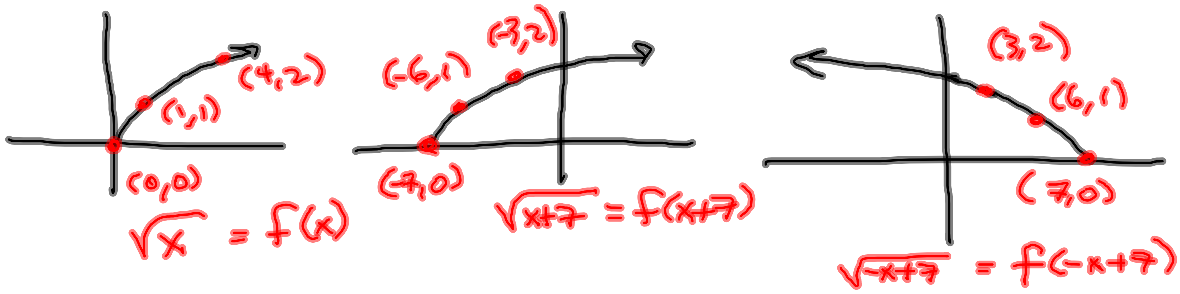
$\sqrt{-x}$



$\sqrt{-(x-7)}$

Alternative: Think of $7-x$ as $-x+7$

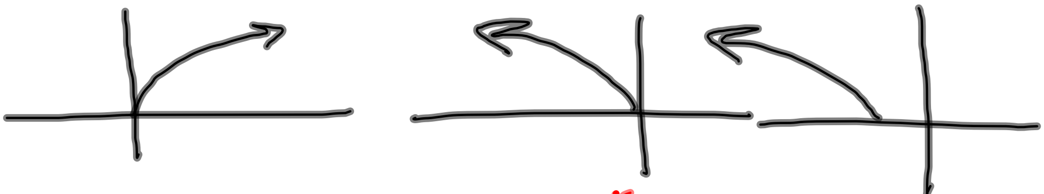
$$\sqrt{x} \longrightarrow \sqrt{x+7} \longrightarrow \sqrt{-x+7}$$



Common Error:

$\sqrt{x} \longrightarrow \sqrt{-x} \longrightarrow \sqrt{-x+7}$

Flip horiz. Left 7



OK NOT OK

§2.4 #s 16, 20, 22, 24, 28, 51, 52, 55, 56, 66

Operations on Functions.

$f \pm g$, $f \cdot g$ } $\mathcal{D}$ = Intersection of the two domains.

$\frac{f}{g}$ } $\mathcal{D}$ = Intersection of the two AND we can't have $g(x) = 0$

$f \circ g$ Composition

$(f \circ g)(x) = f(g(x))$

Feed x to g & then feed g(x) to f.

$\mathcal{D} = \{x \mid x \in \mathcal{D}(g) \text{ and } g(x) \in \mathcal{D}(f)\}$

The arithmetic stuff

$f(x) = \frac{1}{x-2}$, $g(x) = \sqrt{3x+1}$

$(f+g)(x) = f(x) + g(x) = \boxed{\frac{1}{x-2} + \sqrt{3x+1}}$

$\mathcal{D}(f) = \{x \mid x \neq 2\} = (-\infty, 2) \cup (2, \infty)$

$\mathcal{D}(g)$: Need $3x+1 \geq 0$

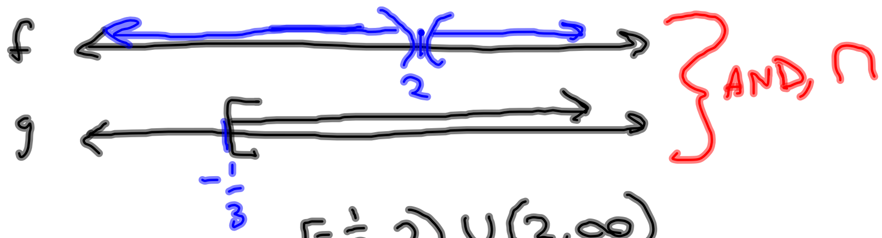
$3x \geq -1$

$x \geq -\frac{1}{3}$

$\mathcal{D}(g) = \{x \mid x \geq -\frac{1}{3}\} = [\frac{1}{3}, \infty)$

$\mathcal{D}(f+g) = \mathcal{D}(f) \cap \mathcal{D}(g) = [-\frac{1}{3}, 2) \cup (2, \infty)$

$\cap \rightarrow$ AND



$[-\frac{1}{3}, 2) \cup (2, \infty)$

What about $\frac{f}{g}$? $\frac{\frac{1}{x-2}}{\sqrt{3x+1}} = \left(\frac{f}{g}\right)(x)$

$\mathcal{D} = (-\frac{1}{3}, 2) \cup (2, \infty)$

Throw out the point(s) where $g(x) = 0$.

#'s 15-22 $f = \{(-3,1), (0,4), (2,0)\}$

$g = \{(-3,2), (1,2), (2,6), (4,0)\}$

$h = \{(2,4), (1,0)\}$

$\mathcal{D}(f) = \{-3, 0, 2\}$

$\mathcal{D}(g) = \{-3, 1, 2, 4\}$

$\mathcal{D}(h) = \{2, 1\}$

#17 Find $f-g$

$\mathcal{D}(f-g) = \{-3, 2\} = \mathcal{D}(f) \cap \mathcal{D}(g)$

$(f-g)(-3) = f(-3) - g(-3) = 1 - 2 = -1 \rightarrow (-3,-1)$

$(f-g)(2) = f(2) - g(2) = 0 - 6 = -6 \rightarrow (2,-6)$

So, $f-g = \{(-3,-1), (2,-6)\}$

$f = \{(-3,1), (0,4), (2,0)\}$

$g = \{(-3,2), (1,2), (2,6), (4,0)\}$

$h = \{(2,4), (1,0)\}$

#31 $f \circ g$

$(f \circ g)(x) = f(g(x))$

$$\mathcal{D}(f \circ g) = \{x \mid x \in \mathcal{D}(g) \text{ and } g(x) \in \mathcal{D}(f)\}$$

$$= \{-3, 1, 4\}$$

$f(g(-3)) = f(2) = 0 \rightarrow (-3,0)$

$f(g(1)) = f(2) = 0 \rightarrow (1,0)$

$f(g(4)) = f(0) = 4 \rightarrow (4,4)$

$\therefore \; f \circ g = \{(-3,0), (1,0), (4,4)\}$

Thanks, Phillip.

$h(x) = \frac{1}{x}$, $g(x) = \sqrt{x}$ $h(☆) = \frac{1}{☆}$

Find $h \circ g$ and its domain.

$(h \circ g)(x) = h(g(x)) = h(\sqrt{x}) = \frac{1}{\sqrt{x}} = (h \circ g)(x)$

$\mathcal{D}(h \circ g) = \{x \mid x \in \mathcal{D}(g) \text{ AND } g(x) \in \mathcal{D}(h)\}$

$\mathcal{D}(g) = [0, \infty)$ (from needing $x \geq 0$ in $\sqrt{x}$)

Need: $g(x) \in \mathcal{D}(h)$.

Need $\sqrt{x} \in \mathcal{D}(h)$

$\mathcal{D}(h) = \{x \mid x \neq 0\} = (-\infty, 0) \cup (0, \infty)$

$\mathcal{D}(h(g(x))) = \{x \mid x \in \mathcal{D}(g) \text{ AND } g(x) \neq 0\} = (0, \infty)$

$g(x) = 0$

$\sqrt{x} = 0$

$\sqrt{x}^2 = 0^2$

$x = 0$ — Throw $x = 0$ out of $\mathcal{D}(g)$.