Why doesn't $3x+2=|y|$ define y as a function of x?

Solve for x:

$3x = |y|-2$

$x = \frac{1}{3}(|y|-2)$

Can you see that two different y-values correspond to a single x-value?

$y=1$: $x = \frac{1}{3}(|1|-2) = \frac{1}{3}(1-2) = \frac{1}{3}(-1) = -\frac{1}{3}$

This gives $(-\frac{1}{3}, 1)$

$y=-1$: $x = \cdots = -\frac{1}{3}$. This gives $(-\frac{1}{3}, -1)$

Same 1^{st} component for 2 different 2^{nd} components. Not a function.

One x-value corresponding to more than one y-value.

$f(x) = 3x^2 + 2x$ Find the difference quotient, DQ.

$(x+1)^2 = (x+1)(x+1) = x^2 + x + x + 1 = x^2 + 2x + 1$

$(x+h)^2 = (x+h)(x+h) = x^2 + xh + hx + h^2$

$= x^2 + xh + xh + h^2$

$\boxed{(x+h)^2 = x^2 + 2xh + h^2}$

$DQ = \dfrac{f(x+h) - f(x)}{h} = \dfrac{f(x_2) - f(x_1)}{x_2 - x_1} = \dfrac{y_2 - y_1}{x_2 - x_1}$

$x_2 = x_1 + h$

$\boxed{x_2 = x + h}$

$\boxed{x_1 = x}$

$f(x) = 3x^2 + 2x$

$$DQ = \frac{f(x+h) - f(x)}{h} = \frac{\overbrace{3(x+h)^2 + 2(x+h)}^{f(x+h)} - \overbrace{[3x^2 + 2x]}^{f(x)}}{h}$$

$$= \frac{3(x^2 + 2xh + h^2) + 2x + 2h - 3x^2 - 2x}{h}$$

$$= \frac{3x^2 + 6xh + 3h^2 + 2h - 3x^2}{h}$$

$$= \frac{6xh + 3h^2 + 2h}{h} = \frac{\cancel{h}(6x + 3h + 2)}{\cancel{h}}$$

$$= \boxed{6x + 3h + 2} = DQ.$$

Not for us!

Calculus Preview.
Sleight-of-hand:
Let $h \to 0$

$$f(x) = 3x + 7$$

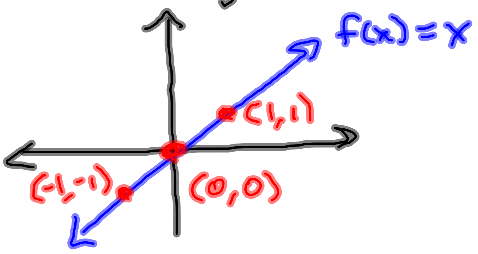
$$DQ = \frac{f(x+h) - f(x)}{h} = \frac{3(x+h) + 7 - [3x+7]}{h}$$

$$= \frac{3x + 3h + 7 - 3x - 7}{h} = \frac{3h}{h} = \boxed{3 = DQ}$$

§2.2 #s 4, 7, 8, 9, 36, 38, 40

Common / Basic Functions

Identity Function : $f(x) = x$ (The line $y = x$)



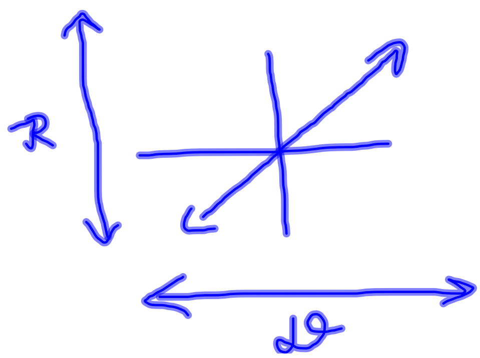
In the sequel, we shift, stretch, reflect this. I'll want you to track where the 3 "key points" go.

$\mathcal{D} = (-\infty, \infty) = \{x \mid f(x) \text{ makes sense}\}$

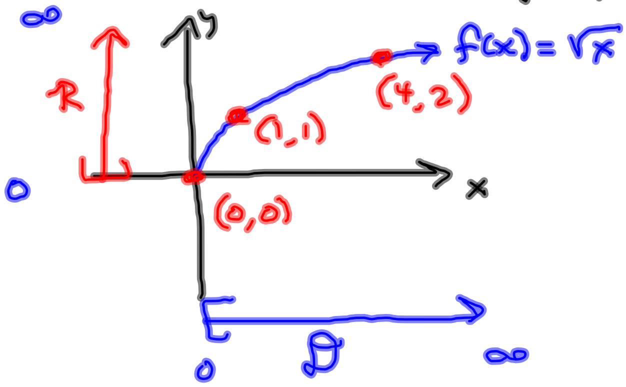
$\mathcal{R} = (-\infty, \infty) = \{y \mid y = f(x) \text{ for some } x \in \mathcal{D}\}$

$\mathcal{D}$ = DOMAIN

$\mathcal{R}$ = Range



Square Root Function: $f(x) = \sqrt{x}$



Increasing on its domain.

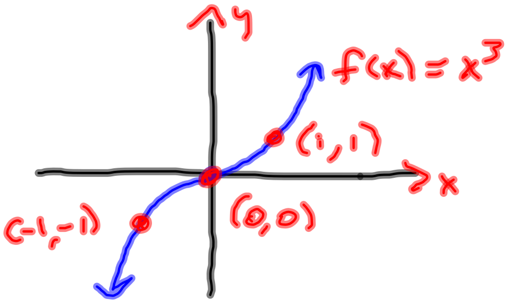
$\mathcal{D} = [0, \infty)$

$\mathcal{R} = [0, \infty)$

f is increasing on an interval if for every $x_1 < x_2$ in the interval, $f(x_1) < f(x_2)$.

decreasing, constant.

$f(x) = x^3$ is the cube function.

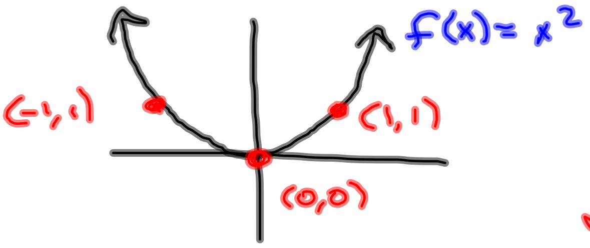


Increasing.

$\mathcal{D} = (-\infty, \infty)$

$\mathcal{R} = (-\infty, \infty)$

$f(x) = x^2$ is the square function.



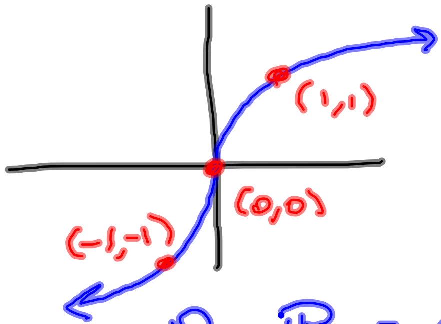
Increasing $[0, \infty)$

Decreasing $(-\infty, 0]$

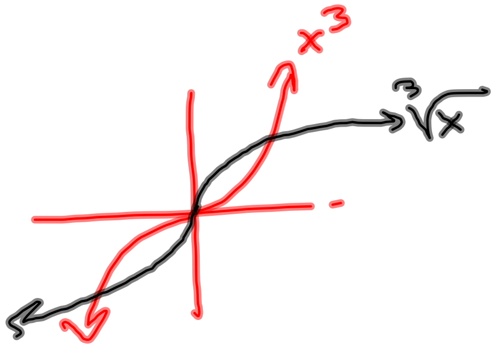
$\mathcal{D} = (-\infty, \infty)$

$\mathcal{R} = [0, \infty)$

Cube Root Function



$f(x) = \sqrt[3]{x}$
$= x^{\frac{1}{3}}$

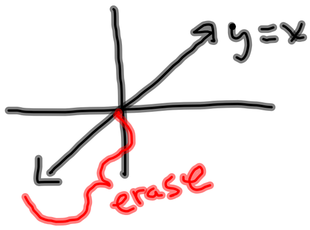


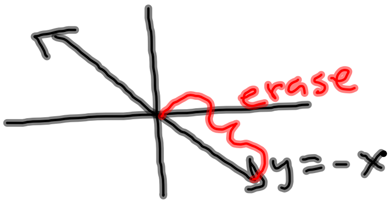
$\mathcal{D} = \mathbb{R} = (-\infty, \infty)$
$\mathcal{R} = \mathbb{R}$

Absolute Value Function

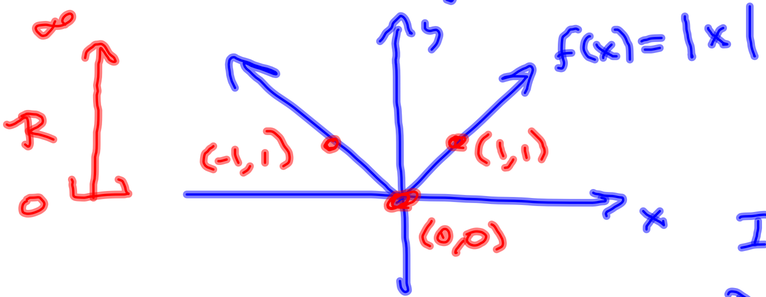
$$f(x) = |x| = \begin{cases} x & \text{if } x \geq 0 \\ -x & \text{if } x < 0 \end{cases}$$

$x=0$ is the boundary of the pieces.





Put it together:



$\mathcal{D} = (-\infty, \infty)$

$\mathcal{R} = [0, \infty)$

Increasing: $[0, \infty)$

Decreasing: $(-\infty, 0]$

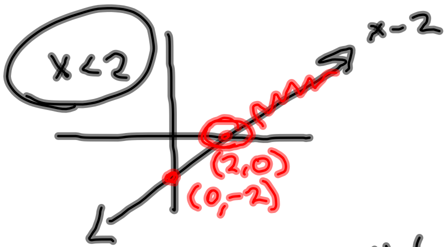
Cassandra

$$f(x)=\begin{cases} x-2 & \text{if } x<2 \\ x+1 & \text{if } x\geq 2 \end{cases}$$

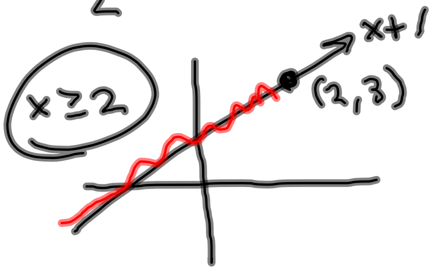
open dot: $2-2=0$: $(2,0)$

closed dot: $2+1=3$: $(2,3)$

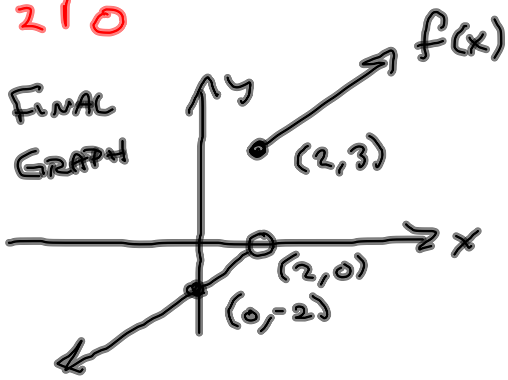
Boundary: $x=2$



x	y = x-2
0	-2
2	0



FINAL GRAPH



cook one up where the two pieces touch

$-2x+5$ $\quad$ $3x+K$

want them to touch at $x=1$

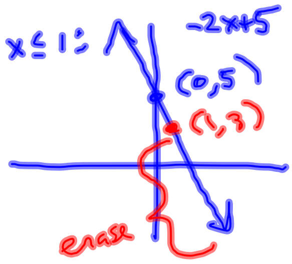
That means

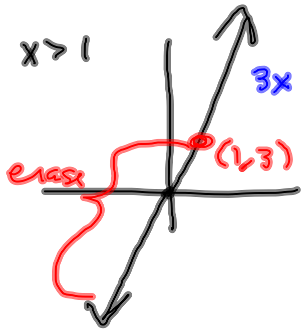
$$-2(1)+5 = 3(1)+K$$
$$-2+5 = 3+K$$
$$3 = 3+K$$
$$0 = K$$

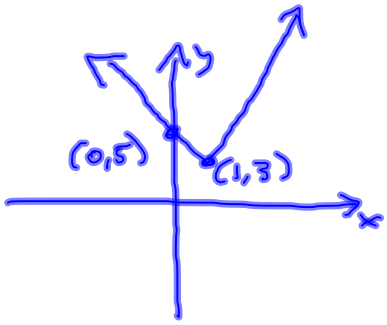
$$f(x)=\begin{cases} -2x+5 & \text{if } x\le 1 \\ 3x & \text{if } x>1 \end{cases}$$

$x=1$: $-2(1)+5=3 \rightsquigarrow (1,3)$ ●

$x=1$: $3(1)=3 \rightsquigarrow (1,3)$ ○







what's the domain of $\sqrt{x}$? $x\in[0,\infty)$

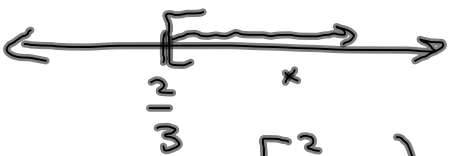
" " " " $\sqrt{☺}$? $☺\in[0,\infty)$

what's the domain of $y=\sqrt{3x-2}$?

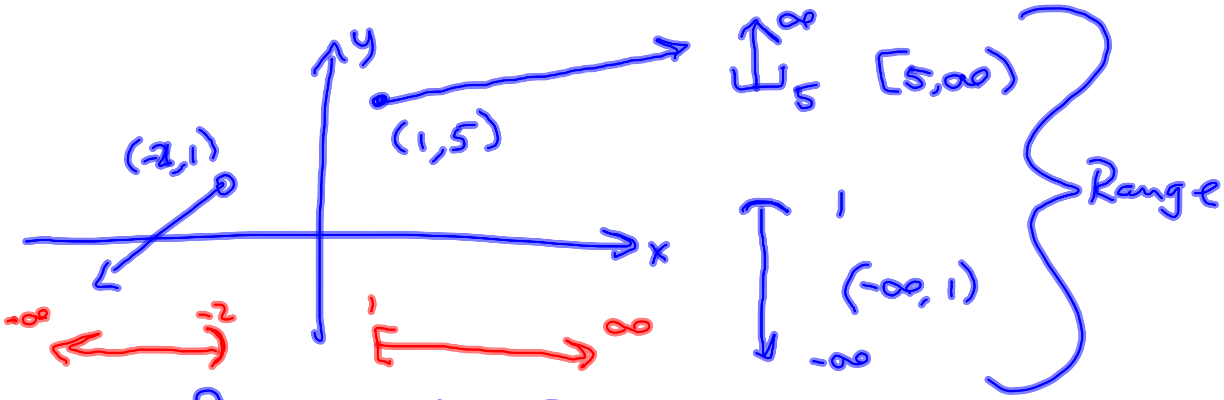
Need $3x-2\in[0,\infty)$

$\therefore$ $3x-2\ge 0$

$$3x\ge 2$$
$$x\ge \frac{2}{3}$$



$[\frac{2}{3},\infty)$



$\mathcal{D} = (-\infty, -2) \cup [1, \infty)$

$\mathcal{R} = (-\infty, 1) \cup [5, \infty)$