§2.1 36, 43, 46 Questions.

An equation defines y as a function of x if you can solve for y and get a single expression in x

$\boxed{\text{NONEXAMPLE}}$ $x + y^2 = 7$

$y^2 = -x + 7$

$\Longrightarrow y = \pm\sqrt{-x+7}$ $y = \sqrt{-x+7}$ OR $y = -\sqrt{-x+7}$

Each x must result in exactly one y. But we have two expressions, here.

Note that $x = 0 \Longrightarrow$

$\boxed{\text{NOT A FUNCTION}}$

$y = \sqrt{7}$ OR $y = -\sqrt{7}$

This gives two ordered pairs in the RELATION.

$(0, -\sqrt{7}), (0, \sqrt{7})$

No function can assign two outputs to a single input.

$\boxed{\text{Another nonexample}}$ $y = |3x+2|$ oops. IS a function.

Here's a NONexample:

$$|y| = 3x+2$$

$$|y| = \begin{cases} y & \text{if } y \geq 0 \\ -y & \text{if } y < 0 \end{cases}$$

$$3x+2 = |y|$$

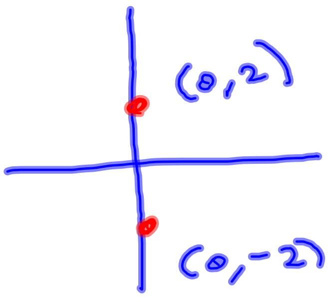
$$|x| = 3$$

$$x = 3 \quad \text{OR} \quad x = -3$$

$$|y| = 3x+2$$

$$y = 3x+2 \quad \text{OR} \quad y = -(3x+2)$$

$x = 0$: $y = 2$ OR $y = -2$



This gives 2 ordered pairs with same input, but two different outputs.

$(0,2)$, $(0,-2)$

Flunks Vertical Line Test.

other than y^2, $|y|$, y^4, y^6, $\dots$, y^{2n},

any time you see an equation in x & y,

you typically can solve for y and get

<u>one</u> answer.

33 $\quad x = 3y - 9$ $\qquad$ Yes. Is a function

$$3y - 9 = x$$

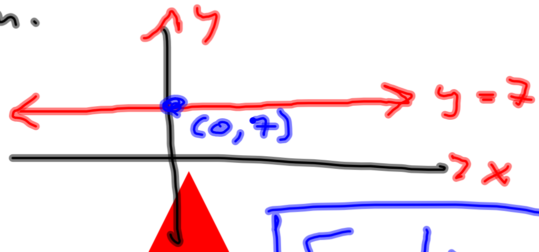
$$3y = x + 9$$

$$y = \frac{x+9}{3}$$

one # out for every x I put in.

#s 43-54 Determine the domain and range of each relation.

(45) $\{(x,y) \mid y = 7\}$



$(3,7) \in$ this relation

$(-11,7)$ is in this relation

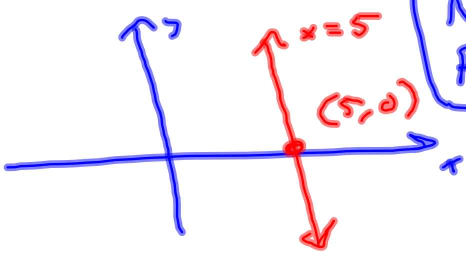
$(\pi, 7)$ " " " "

$\boxed{\text{Function}}$

Domain = set of inputs that give real outputs. $= (-\infty, \infty)$

Range = Set of all outputs $= \{7\}$

(46) $\{(x,y) \mid x = 5\}$

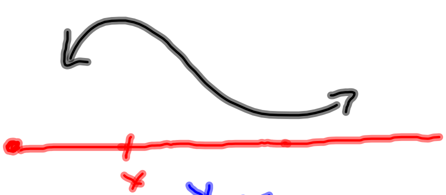


$\boxed{\text{Not a Function}}$

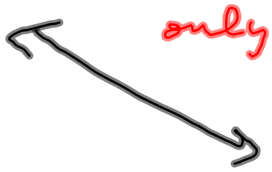
Domain $= \{5\} \neq 5$

Range $= (-\infty, \infty)$

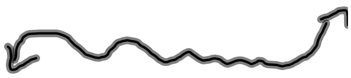
For each x, there is only one y.



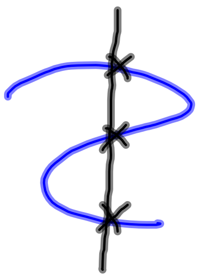
Yes



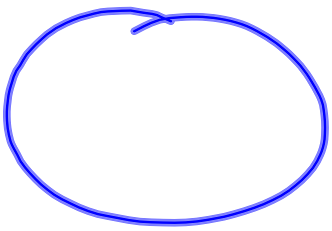
Yes



Yes



No



No

Vertical Line Test

Let $f = \{(1,2), (-5, 11), (6, 24)\}$

& $g = 5x - 3$

Evaluation

$\boxed{f(-5) = 11}$

Equation-Solving.

Find x such that $f(x) = 24$.

It's $x = 6$

$g(2) = 5(2) - 3$

$= 10 - 3$

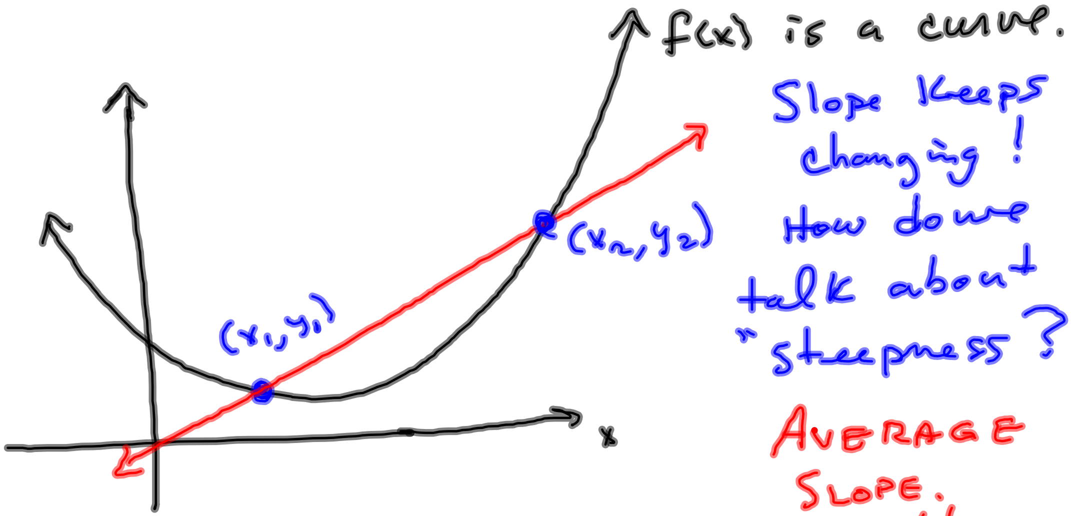
$= \boxed{7 = g(2)}$

Find x such that $g(x) = 8$. This means

$5x - 3 = 8$

$5x = 11$

$\boxed{x = \frac{11}{5}}$



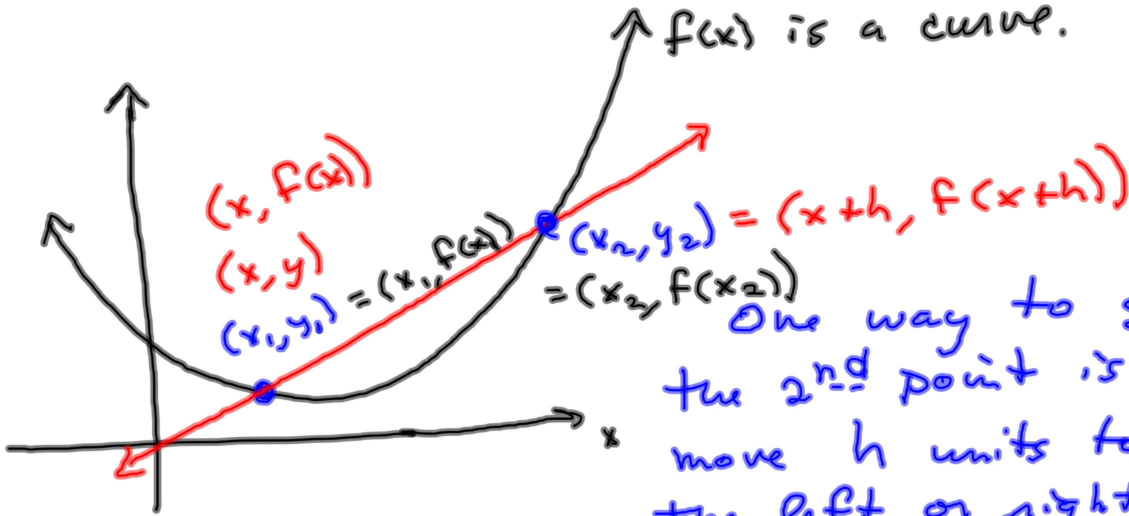
Slope keeps changing! How do we talk about "steepness?"

AVERAGE SLOPE. Todd says.

$m_{AVG} = \frac{y_2 - y_1}{x_2 - x_1}$ is a way to estimate the slope at a point. * The closer x_2 is to x_1, the closer our average slope will be to the actual slope

In Calculus we take the second point infinitely close to the 1^{st}, without touching.

For Now, we just work with the mechanics.



One way to get the 2nd point is to move h units to the left or right from the 1st point.

$$m_{AVG} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{f(x_2) - f(x_1)}{x_2 - x_1} = \frac{f(x+h) - f(x)}{h}$$

$$\underbrace{x+h}_{\text{just moved } h \text{ units away from } x} - x$$

$f(x) = x^2 - 2$

$f(x-5) = (x-5)^2 - 2$

$f(\boxed{}) = \boxed{}^2 - 2$

$f(\boxed{x-5}) = \boxed{x-5}^2 - 2$

$f(x+h) = (x+h)^2 - 2$

$(x+h)^2 \stackrel{?}{=} x^2+2xh+h^2$

$(x+h)(x+h) = x^2 + xh + hx + h^2$

For $f(x) = x^2-2$, find the difference quotient.

$$\frac{f(x+h)-f(x)}{h} = \frac{(x+h)^2-2-(x^2-2)}{h}$$

$$= \frac{x^2+2xh+h^2-2-x^2+2}{h}$$

$$= \frac{2xh+h^2}{h} = \frac{h(2x+h)}{h} = \boxed{2x+h} = m_{avg}$$

Ben, Crystal & everybody but steve.

#107 $R(x) = 20000x - 500x^2$ is revenue from ticket sales as a function of the price per ticket.

Find the difference quotient when $x = 18$, $h = 0.1$. Interpret.

One approach:

$$\frac{R(x+h) - R(x)}{h} =$$

$$\frac{20000(x+h) - 500(x+h)^2 - (20000x - 500x^2)}{h}$$

$$= \frac{20000x + 20000h - 500(x^2 + 2xh + h^2) - 20000x + 500x^2}{h}$$

Brian

$$= \frac{+20000h - 500x^2 - 1000xh - 500h^2 + 500x^2}{h}$$

$$= \frac{20000h - 1000xh - 500h^2}{h}$$

$$= \frac{\cancel{h}(20000 - 1000x - 500h)}{\cancel{h}}$$

$$= 20000 - 1000x - 500h.$$

Let $h = .1$, $x = 18$. Then we have

$$20000 - 18000 - 50$$
$$= 2000 - 50$$
$$= 1950$$

This means for every \$1 increase in price, they get \$1950 more.