

Do your own work. SHOW your work. When in doubt about how stupid I am, assume the worst.

1. Solve the following equations and inequalities. For equations, give the solution as a set. For inequalities, give the solution in set-builder notation and interval notation.

a. (10 pts) $3x - 2 \geq 5x + 8$

b. (10 pts) $|3x - 2| = 4$

$$-2x \geq 10$$

$$3x - 2 = \pm 4$$

$$3x = 2 \pm 4$$

$$x = \frac{2 \pm 4}{3} \rightarrow \begin{matrix} \frac{6}{3} = 2 \\ \frac{-2}{3} \end{matrix}$$

$$\left\{ -\frac{2}{3}, 2 \right\}$$

LCD = 3 \cdot 2 = 5

c. (10 pts) $|2 - 3x| \geq 4$

d. (10 pts) $\left| \frac{2}{3}x - 2 \right| < \frac{7}{10}$

$$2 - 3x \geq 4 \quad \text{OR} \quad 2 - 3x \leq -4$$

$$-3x \geq 2 \quad \text{OR} \quad -3x \leq -6$$

$$\left| \frac{2x \cdot 2 \cdot 5}{2 \cdot 5} - \frac{2 \cdot 30}{1 \cdot 30} \right| < \frac{7}{10} \cdot \frac{3}{3}$$

$$|20x - 60| < 21$$

$$20x - 60 < 21 \quad \text{and} \quad 20x - 60 > -21$$

$$20x < 81 \quad \text{and} \quad 20x > 39$$

$$\left\{ x \mid x < \frac{81}{20} \quad \text{and} \quad x > \frac{39}{20} \right\}$$

$$= \left(\frac{39}{20}, \frac{81}{20} \right) \quad \text{AND}$$

f. (5 pts) $\left| \frac{12}{39}x + \frac{9}{7} \right| > -4$

\mathbb{R}

$$\left\{ x \mid x > -\frac{481}{28} \quad \text{OR} \quad x < \frac{247}{28} \right\} = \mathbb{R}$$

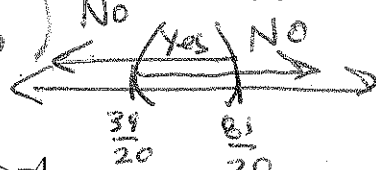
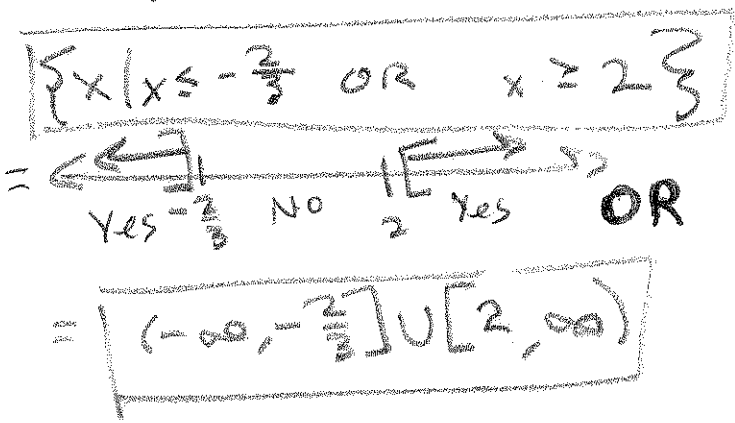
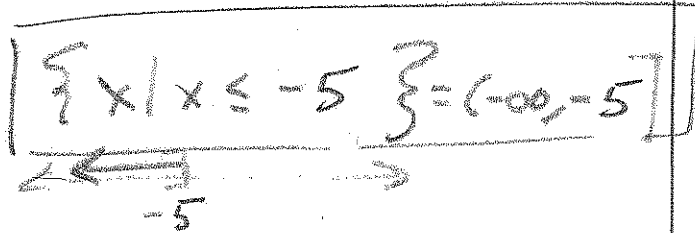
$$= \left(-\frac{481}{28}, \infty \right) \cup \left(-\infty, \frac{247}{28} \right) = (-\infty, \infty) = \mathbb{R}$$

e. (5 pts) $\frac{12}{39}x + \frac{9}{7} \leq -4$

\emptyset

$$\left\{ x \mid x < -\frac{481}{28} \quad \text{and} \quad x > \frac{247}{28} \right\}$$

$$= \emptyset$$



2. Solve the following equations:

a. (5 pts) $x^2 - 9 = 0$

$$(x-3)(x+3) = 0$$

$$x-3=0 \text{ OR } x+3=0$$

$$x=3 \text{ OR } x=-3$$

$$x \in \{-3, 3\}$$

b. (5 pts) $x^2 - 10x + 25 = 0$

$$x^2 - 2 \cdot 5 \cdot x + 5^2 = 0$$

$$(x-5)^2 = 0$$

$$|x-5| = 0$$

$$x-5 = 0$$

$$\boxed{x=5}$$

2 | 90
3 | 45
3 | 15
5

c. (5 pts) $10x^2 - 63x - 90 = 0$

M1 $a=10, b=-63, c=-90$

$$b^2 - 4ac = (-63)^2 - 4(10)(-90)$$

$$= 7569$$

$$\sqrt{7569} = 87$$

$$x = \frac{63 \pm 87}{2(10)}$$

$$\begin{aligned} \frac{63+87}{20} &= \frac{15}{2} = x \\ \frac{63-87}{20} &= -\frac{6}{5} = x \end{aligned}$$

M2 $(10)(-90) = -900$
 $= -2 \cdot 5 \cdot 2 \cdot 3 \cdot 3 \cdot 5$

-73+10 -730 Higher!
 -83+20 -1660 Lower!
 -77+14 -1078 Lower!
 -76+13 -988 Lower!
 -75+12 -900 ✓

3. (5 pts) Solve $s = -\frac{1}{2}gt^2 + vt + h$ for v .

$$-\frac{1}{2}gt^2 + vt + h = s$$

$$vt = s + \frac{1}{2}gt^2 - h$$

$$v = \frac{s + \frac{1}{2}gt^2 - h}{t}$$

OR $\frac{2s + gt^2 - 2h}{2t}$

$$10x^2 - 75x + 12x - 90$$

$$5x(2x-15) + 6(2x-15)$$

$$= (2x-15)(5x+6)$$

$$x = \frac{15}{2} \text{ OR } x = -\frac{6}{5}$$

M3 $(-3 \cdot 5 \cdot 5)(2 \cdot 2 \cdot 3)$

$$-63x = -75x + 12x \checkmark$$

$$10x^2 - 75x + 12x - 90$$

etc (M2).

4. (10 pts) Combine into one fraction in lowest terms: $\frac{5}{24} + \frac{7}{30}$

2 | 24
2 | 12
2 | 6
3

2 | 30
3 | 15
5

$$= \frac{5}{24} \cdot \frac{5}{5} + \frac{7}{30} \cdot \frac{2 \cdot 2}{2 \cdot 2} = \frac{25+28}{120} = \boxed{\frac{53}{120}}$$

LCM = 2 \cdot 2 \cdot 2 \cdot 3 \cdot 5

5. Simplify. Assume all variables represent nonzero real numbers. Your final answer should contain only positive exponents.

a. (10 pts) $(x^2y^8)^3(x^{-5}y^2)^{-7} = (x^{(2)(3)} y^{(8)(3)}) (x^{(-5)(-7)} y^{(2)(-7)})$

$= (x^6 y^{24}) (x^{35} y^{-14})$

$= x^{6+35} y^{24-14}$

$= x^{41} y^{10}$

b. (10 pts) $\frac{(14^2 x^2 y^3)^2}{(35^3 x^{-2} y^{-5})^4} = \frac{((2 \cdot 7)^2 x^2 y^3)^2}{((5 \cdot 7)^3 x^{-2} y^{-5})^4} = \frac{(2^2 \cdot 7^2 x^2 y^3)^2}{(5^3 \cdot 7^3 x^{-2} y^{-5})^4}$

$= \frac{2^4 7^4 x^4 y^6}{5^{12} 7^{12} x^{-8} y^{-20}}$

$= 2^4 7^4 x^4 y^6 5^{-12} 7^{-12} x^8 y^{20}$

$= 2^4 7^{-8} 5^{-12} x^{-4} y^{26}$

$= 2^4 7^{-8} 5^{-12} x^{-4} y^{26}$

$\frac{2^4 y^{26}}{7^8 5^{12} x^4}$

OR $\frac{16y^{26}}{1.4 \times 10^{15} x^4}$

1,407,422,119,140,625

$7^8 5^{12} \approx 1.407422119 \times 10^{15}$

Answer up to 3 bonus questions for up to 15 points. I will grade the first 3 you do work on, unless you tell me to omit them.

1. (5 pts) Consider the equation $ax^2 + bx + c = 0$. Write the discriminant.

$b^2 - 4ac$



2. (5 pts) What's the solution of the equation $ax^2 + bx + c = 0$?

$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

3. (5 pts) Factor $420x^2 - 332x - 1155$ into the product of two binomials.

$a = 420, b = -332, c = -1155$

$b^2 - 4ac = (-332)^2 - 4(420)(-1155) = 2050624$

$\Rightarrow \sqrt{b^2 - 4ac} = 1432$

$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{332 \pm 1432}{2(420)} = \frac{332 \pm 1432}{840}$

$\frac{1764}{840} = \frac{21}{10}$

$\frac{-1100}{840} = \frac{-35}{42}$

The 3 gives:

$420(x - \frac{21}{10})(x + \frac{35}{42}) = (10x - 21)(42x + 35)$

2 | 24
2 | 12
2 | 6
3

3 | 375
5 | 125
5 | 25
5

4. (5 pts) Factor $24x^3 + 375y^6$

$= 3(8x^3 + 125y^6) = 3((2x)^3 + (5y^2)^3)$

$= 3(2x + 5y^2)((2x)^2 - (2x)(5y^2) + (5y^2)^2)$

$= 3(2x + 5y^2)(4x^2 - 10xy^2 + 25y^4)$

5. (5 pts) Use Pascal's triangle to expand $(3x - 2y)^4$

$= (3x)^4 - (3x)^3(2y) + (3x)^2(2y)^2 - 3x(2y)^3 + (2y)^4$

1
1 1
1 2 1
1 3 3 1
1 4 6 4 1

$= 81x^4 - 54x^3y + 36x^2y^2 - 24xy^3 + 16y^4$

6. (5 pts) Factor $x^2 - 20x - 10$ (It doesn't factor over the rationals! Your 'ac' method won't work!).

$a = 1, b = -20, c = -10$

$\sqrt{4ac} = 2\sqrt{110}$

$b^2 - 4ac = (-20)^2 - 4(1)(-10) = 440$
 $x = \frac{20 \pm 2\sqrt{110}}{2} = 10 \pm \sqrt{110}$

$= 400 + 40$
 $= 440$

$(x - (10 + \sqrt{110}))(x - (10 - \sqrt{110}))$