MAT 099-G14 Test 2 Name KEY

100 Points Covers Chapter 2 NO GRAPHING CALCULATORS!!!

Do your own work. SHOW your work. When in doubt about how stupid I am, assume the worst.

1. Solve the following equations and inequalities. For equations, give the solution as a set. For inequalities, give the solution in set-builder notation and interval notation.

a. (10 pts) $3x-2\geq 5x+8$

$-2x \geq 10$

$\{x \mid x \leq -5\} = (-\infty, -5]$

b. (10 pts) $|3x-2|=4$

$3x-2 = \pm 4$

$3x = 2 \pm 4$

$x = \frac{2\pm 4}{3}$ $\quad \frac{6}{3}=2$, $\frac{-2}{3}$

$\{-\frac{2}{3}, 2\}$

c. (10 pts) $|2-3x|\geq 4$

$2-3x \geq 4$ OR $2-3x \leq -4$

$-3x \geq 2$ OR $-3x \leq -6$

$\{x \mid x \leq -\frac{2}{3}$ OR $x \geq 2\}$

$= (-\infty, -\frac{2}{3}] \cup [2, \infty)$

d. (10 pts) $\left|\frac{2}{3}x-2\right|<\frac{7}{10}$

LCD $= 3\cdot 2\cdot 5$

$\left|\frac{2x}{3}\cdot\frac{2\cdot 5}{2\cdot 5} - \frac{2}{1}\cdot\frac{30}{30}\right| < \frac{7}{10}\cdot\frac{3}{3}$

$|20x-60| < 21$

$20x-60 < 21$ and $20x-60 > -21$

$20x < 81$ and $20x > 39$

$\{x \mid x < \frac{81}{20}$ and $x > \frac{39}{20}\}$

$= (\frac{39}{20}, \frac{81}{20})$

e. (5 pts) $\left|\frac{12}{39}x+\frac{9}{7}\right|\leq -4$

$\emptyset$

$\{x \mid x < -\frac{481}{28}$ and $x > \frac{247}{28}\}$

$= \emptyset$

f. (5 pts) $\left|\frac{12}{39}x+\frac{9}{7}\right|> -4$

$\mathbb{R}$

$\{x \mid x > -\frac{481}{28}$ OR $x < \frac{247}{28}\} = \mathbb{R}$

$= (-\frac{481}{28}, \infty) \cup (-\infty, \frac{247}{28}) = (-\infty, \infty)$

$= \mathbb{R}$

2. Solve the following equations:

a. (5 pts) $x^2 - 9 = 0$

$(x-3)(x+3) = 0$

$x-3=0$ OR $x+3=0$

$x=3$ OR $x=-3$

$x \in \{-3, 3\}$

b. (5 pts) $x^2 - 10x + 25 = 0$

$x^2 - 2 \cdot 5 \cdot x + 5^2 = 0$

$(x-5)^2 = 0$

$|x-5| = 0$

$x-5=0$

$\boxed{x=5}$

2 | 90
3 | 45
3 | 15
5

c. (5 pts) $10x^2 - 63x - 90 = 0$

M1: $a=10, b=-63, c=-90$

$b^2-4ac = (-63)^2 - 4(10)(-90)$

$= 7569$

$\sqrt{7569} = 87$

$x = \frac{63 \pm 87}{2(10)}$

$\frac{63+87}{20} = \frac{150}{20} = \frac{15}{2} = x$

$\frac{63-87}{20} = \frac{-6}{20} = -\frac{6}{5} = x$

M2: $(10)(-90) = -900$

$= -2 \cdot 5 \cdot 2 \cdot 3 \cdot 3 \cdot 5$

$-73+10$ -730 highest

$-83+20$ -1660 Lowest

$-77+14$ -1078 Lowest

$-76+13$ -988 Lowest

$-75+12$ -900 ✓

M3: $(-3 \cdot 5 \cdot 5)(2 \cdot 2 \cdot 3)$ ✓

$-63x = -75x + 12x$

$10x^2 - 75x + 12x - 90$

etc (M2).

3. (5 pts) Solve $s = -\frac{1}{2}gt^2 + vt + h$ for v.

$-\frac{1}{2}gt^2 + vt + h = s$

$vt = s + \frac{1}{2}gt^2 - h$

$v = \frac{s + \frac{1}{2}gt^2 - h}{t}$

OR $\frac{2s + gt^2 - 2h}{2t}$

$10x^2 - 75x + 12x - 90$

$5x(2x-15) + 6(2x-15)$

$= (2x-15)(5x+6)$

$x = \frac{15}{2}$ OR $x = -\frac{6}{5}$

4. (10 pts) Combine into one fraction in lowest terms: $\frac{5}{24} + \frac{7}{30}$

2 | 24
2 | 12
2 | 6
3

2 | 30
3 | 15
5

LCD $= 2 \cdot 2 \cdot 2 \cdot 3 \cdot 5$

$= \frac{5}{24} \cdot \frac{5}{5} + \frac{7}{30} \cdot \frac{2 \cdot 2}{2 \cdot 2} = \frac{25+28}{120} = \boxed{\frac{53}{120}}$

5. Simplify. Assume all variables represent nonzero real numbers. Your final answer should contain only positive exponents.

a. (10 pts) $\left(x^2y^8\right)^3\left(x^{-5}y^2\right)^{-7} = \left(x^{(2)(3)}y^{(8)(3)}\right)\left(x^{(-5)(-7)}y^{(2)(-7)}\right)$

$= \left(x^6y^{24}\right)\left(x^{35}y^{-14}\right)$

$= x^{6+35}y^{24-14}$

$= \boxed{x^{41}y^{10}}$

b. (10 pts) $\dfrac{\left(14^2x^2y^3\right)^2}{\left(35^3x^{-2}y^{-5}\right)^4} = \dfrac{\left((2\cdot7)^2x^2y^3\right)^2}{\left((5\cdot7)^3x^{-2}y^{-5}\right)^4} = \dfrac{\left(2^2\cdot7^2x^2y^3\right)^2}{\left(5^3\cdot7^3x^{-2}y^{-5}\right)^4}$

$= \dfrac{2^47^4x^4y^6}{5^{12}7^{12}x^{-8}y^{-20}} = 2^47^4x^4y^65^{-12}7^{-12}x^8y^{20}$

$= 2^47^{-8}5^{-12}x^{-4}y^{26} = \boxed{\dfrac{2^4y^{26}}{7^85^{12}x^4}}$ OR $\approx \dfrac{16y^{26}}{1.4\times10^{15}x^4}$

$7^85^{12} \approx 1.407422119 \times 10^{15}$

1,407,422,119,140,625

Answer up to 3 bonus questions for up to 15 points. I will grade the first 3 you do work on, unless you tell me to omit them.

1. (5 pts) Consider the equation $ax^2 + bx + c = 0$. Write the discriminant.

$b^2 - 4ac$

2. (5 pts) What's the solution of the equation $ax^2 + bx + c = 0$?

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

3. (5 pts) Factor $420x^2 - 332x - 1155$ into the product of two binomials.

$a = 420,\ b = -332,\ c = -1155$

$b^2 - 4ac = (-332)^2 - 4(420)(-1155) = 2050624$

$\Rightarrow \sqrt{b^2 - 4ac} = 1432$

$x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \dfrac{332 \pm 1432}{2(420)} = \dfrac{332 \pm 1432}{840}$

$\dfrac{1764}{840} = \dfrac{21}{10}$

$\dfrac{-1100}{840} = -\dfrac{35}{42}$

This gives:

$420\left(x - \frac{21}{10}\right)\left(x + \frac{35}{42}\right) = \boxed{(10x - 21)(42x + 35)}$

$2|24$ $3|375$
$2|12$ $5|125$
$2|6$ $5|25$
3 5

4. (5 pts) Factor $24x^3 + 375y^6$

$= 3(8x^3 + 125y^6) = 3((2x)^3 + (5y^2)^3)$

$= 3(2x + 5y^2)((2x)^2 - (2x)(5y^2) + (5y^2)^2)$

$= \boxed{3(2x + 5y^2)(4x^2 - 10xy^2 + 25y^4)}$

5. (5 pts) Use Pascal's triangle to expand $(3x - 2y)^4$

$= (3x)^4 - (3x)^3(2y) + (3x)^2(2y)^2 - 3x(2y)^3 + (2y)^4$

1 1
1 2 1
1 3 3 1
1 4 6 4 1

$= \boxed{81x^4 - 54x^3y + 36x^2y^2 - 24xy^3 + 16y^4}$

6. (5 pts) Factor $x^2 - 20x - 10$ (It doesn't factor over the rationals! Your 'ac' method won't work!).

$a = 1,\ b = -20,\ c = -10$

$b^2 - 4ac = (-20)^2 - 4(1)(-10)$

$= 400 + 40$

$= 440$

$\sqrt{440} = 2\sqrt{110}$

$x = \dfrac{20 \pm 2\sqrt{110}}{2} = 10 \pm \sqrt{110}$

$\boxed{(x - (10 + \sqrt{110}))(x - (10 - \sqrt{110}))}$